PHYSICS

FOR SCIENTISTS AND ENGINEERS SECOND EDITION

A STRATEGIC APPROACH

ActivPhysics OnLine™ Activities

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1.2 Analyzing Motion Using Graphs
1.3 Predicting Motion from Graphs
1.4 Predicting Motion from Equations
1.5 Problem-Solving Strategies for Kinematics
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1.8 Seat Belts Save Lives
1.9 Screeching to a Halt
1.10 Pole-Vaulter Lands
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WITH MODERN PHYSICS

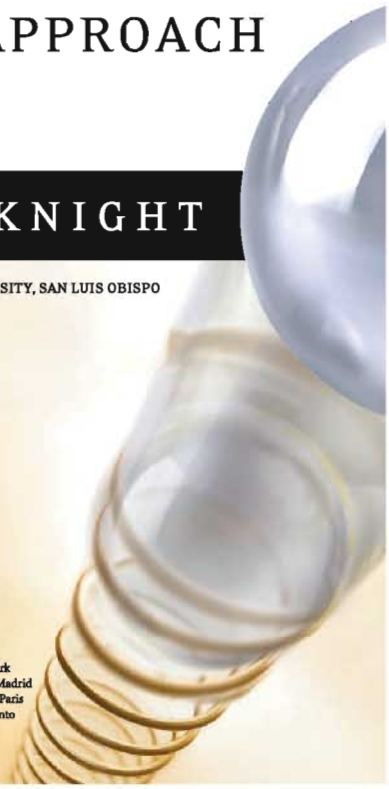
PHYSICS

FOR SCIENTISTS AND ENGINEERS SECOND EDITION

A STRATEGIC APPROACH

RANDALL D. KNIGHT

CALIFORNIA POLYTECHNIC STATE UNIVERSITY, SAN LUIS OBISPO



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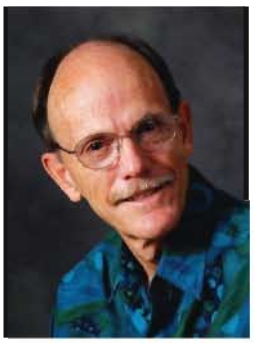
About the Author



Randy Knight has taught introductory physics for over 25 years at Ohio State University and California Polytechnic University, where he is currently Professor of Physics. Professor Knight received a bachelor's degree in physics from Washington University in St. Louis and a Ph.D. in physics from the University of California, Berkeley. He was a post-doctoral fellow at the Harvard-Smithsonian Center for Astrophysics before joining the faculty at Ohio State University. It was at Ohio State that he began to learn about the research in physics education that, many years later, led to this book.

Professor Knight's research interests are in the field of lasers and spectroscopy, and he has published over 25 research papers. He also directs the environmental studies program at Cal Poly, where, in addition to introductory physics, he teaches classes on energy, oceanography, and environmental issues. When he's not in the classroom or in front of a computer, you can find Randy hiking, sea kayaking, playing the piano, or spending time with his wife Sally and their seven cats.

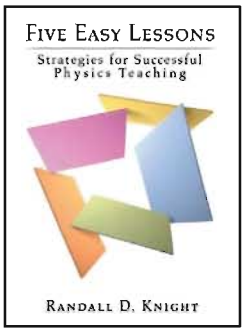
Preface to the Instructor

In 2003 we published *Physics for Scientists and Engineers: A Strategic Approach.* This was the first comprehensive introductory textbook built from the ground up on research into how students can more effectively learn physics. The development and testing that led to this book had been partially funded by the National Science Foundation. This first edition quickly became the most widely adopted new physics textbook in more than 30 years, meeting widespread critical acclaim from professors and students. In this second edition, we build on the research-proven instructional techniques introduced in the first edition and the extensive feedback from thousands of users to take student learning even further.

Objectives

My primary goals in writing *Physics for Scientists and Engineers: A Strategic Approach* have been:

- To produce a textbook that is more focused and coherent, less encyclopedic.
- To move key results from physics education research into the classroom in a way that allows instructors to use a range of teaching styles.
- To provide a balance of quantitative reasoning and conceptual understanding, with special attention to concepts known to cause student difficulties.
- To develop students' problem-solving skills in a systematic manner.
- To support an active-learning environment.



These goals and the rationale behind them are discussed at length in my small paperback book, *Five Easy Lessons: Strategies for Successful Physics Teaching* (Addison-Wesley, 2002). Please request a copy from your local Addison-Wesley sales representative if it is of interest to you (ISBN 0-8053-8702-1).

Textbook Organization

The 43-chapter extended edition (ISBN 0-321-51333-9/978-0-321-51333-5) of *Physics for Scientists and Engineers* is intended for a three-semester course. Most of the 37-chapter standard edition (ISBN 0-321-51661-3/978-0-321-51661-9), ending with relativity, can be covered in two semesters, although the judicious omission of a few chapters will avoid rushing through the material and give students more time to develop their knowledge and skills.

There's a growing sentiment that quantum physics is quickly becoming the province of engineers, not just scientists, and that even a two-semester course should include a reasonable introduction to quantum ideas. The *Instructor Guide* outlines a couple of routes through the book that allow most of the quantum physics chapters to be included in a two-semester course. I've written the book with the hope that an increasing number of instructors will choose one of these routes.

- **Extended edition,** with modern physics (ISBN 0-321-51333-9/978-0-321-51333-5): Chapters 1–43.
- **Standard edition** (ISBN 0-321-51661-3/978-0-321-51661-9): Chapters 1–37.
- **Volume 1** (ISBN 0-321-51662-1/978-0-321-51662-6) covers mechanics: Chapters 1–15.
- **Volume 2** (ISBN 0-321-51663-X/978-0-321-51663-3) covers thermodynamics: Chapters 16–19.
- **Volume 3** (ISBN 0-321-51664-8/978-0-321-51664-0) covers waves and optics: Chapters 20–25.
- **Volume 4** (ISBN 0-321-51665-6/978-0-321-51665-7) covers electricity and magnetism, plus relativity: Chapters 26–37.
- **Volume 5** (ISBN 0-321-51666-4/978-0-321-51666-4) covers relativity and quantum physics: Chapters 37–43.
- **Volumes 1–5** boxed set (ISBN 0-321-51637-0/978-0-321-51637-4).

The full textbook is divided into seven parts: Part I: *Newton's Laws*, Part II: *Conservation Laws*, Part III: *Applications of Newtonian Mechanics*, Part IV: *Thermodynamics*, Part V: *Waves and Optics*, Part VI: *Electricity and Magnetism*, and Part VII: *Relativity and Quantum Mechanics*. Although I recommend covering the parts in this order (see below), doing so is by no means essential. Each topic is self-contained, and Parts III–VI can be rearranged to suit an instructor's needs. To facilitate a reordering of topics, the full text is available in the five individual volumes listed in the margin.

Organization Rationale: Thermodynamics is placed before waves because it is a continuation of ideas from mechanics. The key idea in thermodynamics is energy, and moving from mechanics into thermodynamics allows the uninterrupted development of this important idea. Further, waves introduce students to functions of two variables, and the mathematics of waves is more akin to electricity and magnetism than to mechanics. Thus moving from waves to fields to quantum physics provides a gradual transition of ideas and skills.

The purpose of placing optics with waves is to provide a coherent presentation of wave physics, one of the two pillars of classical physics. Optics as it is presented in introductory physics makes no use of the properties of electromagnetic fields. There's little reason other than historical tradition to delay optics until after E&M. The documented difficulties that students have with optics are difficulties with waves, not difficulties with electricity and magnetism. However, the optics chapters are easily deferred until the end of Part VI for instructors who prefer that ordering of topics.

What's New in the Second Edition

This second edition reaffirms the goals and objectives of the first edition. At the same time, the extensive feedback we've received from scores of instructors has led to numerous changes and improvements to the text, the figures, and the end-of-chapter problems. These include:

- More streamlined presentations. We have shortened each chapter by one page, on average, by tightening the language and reducing superfluous material.
- Conceptual questions. By popular request, the end of each chapter now includes a section of conceptual questions similar to those in the *Student Workbook*.
- Pencil sketches. Each chapter contains several hand-drawn sketches in key worked examples to provide students with explicit examples of the types of drawings they should make in their own problem solving.
- New and revised end-of-chapter problems. Problems have been revised to incorporate the unprecedented use of data and feedback from more than 100,000 students working these problems in MasteringPhysics™. More than 20% of the end-of-chapter problems are new or significantly revised, including an increased number of problems requiring calculus.

Significant chapter and content changes include the following:

- Two-dimensional kinematics has been brought forward to Chapter 4, immediately following the chapter on vectors. This chapter also covers circular-motion kinematics in detail (rather than delaying circular-motion kinematics to the chapter on rotational dynamics) to give a more integrated understanding of kinematics.
- Newton's third law (Chapter 7) now immediately follows and is more closely linked to the chapter on dynamics in one dimension. Revised interaction diagrams are simpler to draw and conceptually more powerful.
- The mechanisms of heat transfer (conduction, convection, and radiation) have been included in Chapter 17 (Work, Heat, and the First Law of Thermodynamics).

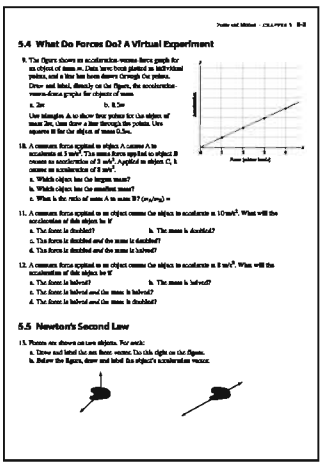
- Spherical mirrors are now covered in Chapter 23 (Ray Optics), and the entirely new Chapter 24 (Optical Instruments) treats cameras, microscopes, telescopes, and vision. This is the only new chapter in the second edition.
- Dielectrics have been added to the section on capacitors in Chapter 30 (Potential and Field), and electric current (Chapter 31) now follows the presentation of electric potential.
- Some topics in Chapters 34 (Electromagnetic Induction) and 35 (Electromagnetic Fields and Waves) have been rearranged for a more logical presentation of ideas.
- Blackbody radiation and Wien's law have been added to Chapter 38 (The End of Classical Physics).

Pedagogical Features

Your Instructor's Professional Copy contains a 10-page illustrated overview of the pedagogical features in this second edition. The *Preface to the Student* demonstrates how these features are designed to help your students.

The Student Workbook

A key component of *Physics for Scientists and Engineers: A Strategic Approach* is the accompanying *Student Workbook*. The workbook bridges the gap between textbook and homework problems by providing students the opportunity to learn and practice skills prior to using those skills in quantitative end-of-chapter problems, much as a musician practices technique separately from performance pieces. The workbook exercises, which are keyed to each section of the textbook, focus on developing specific skills, ranging from identifying forces and drawing free-body diagrams to interpreting wave functions.



5.4 What Do Forces Do? A Virtual Experiment

9. The figure shows an acceleration versus force graph for an object of mass m. Data have been plotted as individual points, and a line has been drawn through the points. Draw and label, directly on the figure, the acceleration versus-force graphs for objects of mass
a. 2m
b. 0.5m
Use triangles A to show four points for the object of mass 2m, then draw a line through the points. Use squares B for the object of mass 0.5m.

10. A constant force applied to object A causes A to accelerate at 5 m/s². The same force applied to object B causes an acceleration of 3 m/s². Applied to object C, it causes an acceleration of 2 m/s².
a. Which object has the largest mass?
b. Which object has the smallest mass?
c. What is the ratio of mass A to mass B? (m_A/m_B) =

11. A constant force applied to an object causes the object to accelerate at 10 m/s². What will the acceleration of this object be if
a. The force is doubled?
b. The mass is doubled?
c. The force is doubled *and* the mass is doubled?
d. The force is doubled *and* the mass is halved?

12. A constant force applied to an object causes the object to accelerate at 8 m/s². What will the acceleration of this object be if
a. The force is halved?
b. The mass is halved?
c. The force is halved *and* the mass is halved?
d. The force is halved *and* the mass is doubled?

5.5 Newton's Second Law

13. Forces are shown on two objects. For each:
a. Draw and label the net force vector. Use this sight on the figure.
b. Below the figure, draw and label the object's acceleration vector.

The *Student Workbook* exercises, which are generally qualitative and/or graphical, draw heavily upon the physics education research literature. The exercises deal with issues known to cause student difficulties and employ techniques that have proven to be effective at overcoming those difficulties. The workbook exercises can be used in class as part of an active-learning teaching strategy, in recitation sections, or as assigned homework. More information about effective use of the *Student Workbook* can be found in the *Instructor Guide*.

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Instructor Supplements

- The **Instructor Guide for Physics for Scientists and Engineers** (ISBN 0-321-51636-2/978-0-321-51636-7) offers detailed comments and suggested teaching ideas for every chapter, an extensive review of what has been learned from physics education research, and guidelines for using active-learning techniques in your classroom.
- The **Instructor Solutions Manuals, Chapters 1–19** (ISBN 0-321-51621-4/978-0-321-51621-3) and **Chapters 20–43** (ISBN 0-321-51657-5/978-0-321-51657-2), written by the author and Professors Scott Nutter (Northern Kentucky University) and Larry Smith (Snow College), provide *complete* solutions to all the end-of-chapter problems. The solutions follow the four-step Model/Visualize/Solve/Assess

procedure used in the Problem-Solving Strategies and in all worked examples. The full text of each solution is available as an editable Word document and as a pdf file on the *Media Manager CD-ROMs* for your own use or for posting on your password-protected course website.

- The cross-platform **Media Manager CD-ROMs** (ISBN 0-321-51624-9/978-0-321-51624-4) provide invaluable and easy-to-use resources for your class, including jpg files of all the figures, photos, tables, key (boxed) equations, chapter summaries, and knowledge structures from the textbook. In addition, all Tactics Boxes, Problem-Solving Strategies, and key equations are provided in an editable Word format. Also included are Word versions and pdf files of the *Instructor Guide* and the *Instructor Solutions Manuals*, and complete *Student Workbook* answers as pdf files. PowerPoint Lecture Outlines and Classroom Response System "Clicker" Questions (including reading quizzes) formatted for PRS systems are provided as well. A simple browser interface allows you to quickly identify the material you need, add it to your shopping cart, and export for editing or directly into your PPT lectures. An **ActivPhysics CD-ROM,** providing a comprehensive library of more than 280 applets from *ActivPhysics OnLine,* and the **Computerized Test Bank CD-ROM** (see below) are also included.
- The online **Instructor Resource Center** (www.aw-bc.com/irc) provides updates to files on the Media Manager CD-ROMs. To obtain a login name and password, contact your Pearson Addison-Wesley sales representative.
- **MasteringPhysics™** (www.masteringphysics.com) is the most widely used and educationally proven physics homework, tutorial, and assessment system available. It is designed to assign, assess, and track each student's progress using a wide diversity of extensively pre-tested problems. Icons throughout the book indicate that *MasteringPhysics™* offers specific tutorials for all the textbook's Tactics Boxes and Problem-Solving Strategies, as well as all the end-of-chapter problems, Test Bank items, and Reading Quizzes. *MasteringPhysics™* provides instructors with a fast and effective way to assign uncompromising, wide-ranging online homework assignments of just the right difficulty and duration. The powerful post-assignment diagnostics allow instructors to assess the progress of their class as a whole or to quickly identify individual students' areas of difficulty.
- **ActivPhysics OnLine™** (accessed through the Self Study area within www.masteringphysics.com) provides a comprehensive library of more than 420 tried and tested *ActivPhysics* applets. In addition, it provides a suite of highly regarded applet-based tutorials developed by education pioneers Professors Alan Van Heuvelen and Paul D'Alessandris. The *ActivPhysics* icons that appear throughout the book direct students to specific interactive exercises that complement the textbook discussion.

  The online exercises are designed to encourage students to confront misconceptions, reason qualitatively about physical processes, experiment quantitatively, and learn to think critically. They cover all topics from mechanics to electricity and magnetism and from optics to modern physics. The highly acclaimed *ActivPhysics OnLine* companion workbooks help students work through complex concepts and understand them more clearly. More than 280 applets from the *ActivPhysics OnLine* library are also available on the Instructor *Media Manager CD-ROMs*.
- The **Printed Test Bank** (ISBN 0-321-51622-2/978-0-321-51622-0) and cross-platform **Computerized Test Bank** (included with the Media Manager CD-ROMs), prepared by Dr. Peter W. Murphy, contain more than 1500 high-quality problems, with a range of multiple-choice, true/false, short-answer, and regular homework-type questions. In the computerized version, more than half of the questions have numerical values that can be randomly assigned for each student.
- The **Transparency Acetates** (ISBN 0-321-51623-0/978-0-321-51623-7) provide more than 200 key figures from *Physics for Scientists and Engineers* for classroom presentation.

Student Supplements

- The **Student Solutions Manuals Chapters 1–19** (ISBN 0-321-51354-1/978-0-321-51356-0) and **Chapters 20–43** (ISBN 0-321-51356-8/978-0-321-51356-4), written by the author and Professors Scott Nutter (Northern Kentucky University) and Larry Smith (Snow College), provide *detailed* solutions to more than half of the odd-numbered end-of-chapter problems. The solutions follow the four-step Model/Visualize/Solve/Assess procedure used in the Problem-Solving Strategies and in all worked examples.
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Preface to the Student

From Me to You

The most incomprehensible thing about the universe is that it is comprehensible.
—Albert Einstein

The day I went into physics class it was death.
—Sylvia Plath, *The Bell Jar*

Let's have a little chat before we start. A rather one-sided chat, admittedly, because you can't respond, but that's OK. I've talked with many of your fellow students over the years, so I have a pretty good idea of what's on your mind.

What's your reaction to taking physics? Fear and loathing? Uncertainty? Excitement? All of the above? Let's face it, physics has a bit of an image problem on campus. You've probably heard that it's difficult, maybe downright impossible unless you're an Einstein. Things that you've heard, your experiences in other science courses, and many other factors all color your *expectations* about what this course is going to be like.

It's true that there are many new ideas to be learned in physics and that the course, like college courses in general, is going to be much faster paced than science courses you had in high school. I think it's fair to say that it will be an *intense* course. But we can avoid many potential problems and difficulties if we can establish, here at the beginning, what this course is about and what is expected of you—and of me!

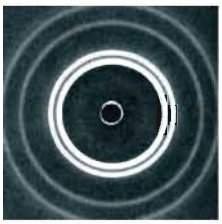
Just what is physics, anyway? Physics is a way of thinking about the physical aspects of nature. Physics is not better than art or biology or poetry or religion, which are also ways to think about nature; it's simply different. One of the things this course will emphasize is that physics is a human endeavor. The ideas presented in this book were not found in a cave or conveyed to us by aliens; they were discovered and developed by real people engaged in a struggle with real issues. I hope to convey to you something of the history and the process by which we have come to accept the principles that form the foundation of today's science and engineering.

You might be surprised to hear that physics is not about "facts." Oh, not that facts are unimportant, but physics is far more focused on discovering *relationships* that exist between facts and *patterns* that exist in nature than on learning facts for their own sake. As a consequence, there's not a lot of memorization when you study physics. Some—there are still definitions and equations to learn—but less than in many other courses. Our emphasis, instead, will be on thinking and reasoning. This is important to factor into your expectations for the course.

Perhaps most important of all, *physics is not math!* Physics is much broader. We're going to look for patterns and relationships in nature, develop the logic that relates different ideas, and search for the reasons *why* things happen as they do. In doing so, we're going to stress qualitative reasoning, pictorial and graphical reasoning, and reasoning by analogy. And yes, we will use math, but it's just one tool among many.

It will save you much frustration if you're aware of this physics–math distinction up front. Many of you, I know, want to find a formula and plug numbers into it—

(a) X-ray diffraction pattern



(b) Electron diffraction pattern

that is, to do a math problem. Maybe that worked in high school science courses, but it is *not* what this course expects of you. We'll certainly do many calculations, but the specific numbers are usually the last and least important step in the analysis.

Physics is about recognizing patterns. For example, the top photograph is an x-ray diffraction pattern showing how a focused beam of x rays spreads out after passing through a crystal. The bottom photograph shows what happens when a focused beam of electrons is shot through the same crystal. What does the obvious similarity in these two photographs tell us about the nature of light and the nature of matter?

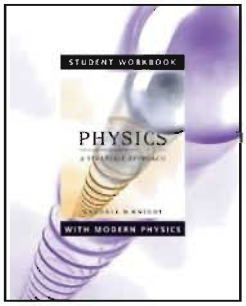
As you study, you'll sometimes be baffled, puzzled, and confused. That's perfectly normal and to be expected. Making mistakes is OK too *if* you're willing to learn from the experience. No one is born knowing how to do physics any more than he or she is born knowing how to play the piano or shoot basketballs. The ability to do physics comes from practice, repetition, and struggling with the ideas until you "own" them and can apply them yourself in new situations. There's no way to make learning effortless, at least for anything worth learning, so expect to have some difficult moments ahead. But also expect to have some moments of excitement at the joy of discovery. There will be instants at which the pieces suddenly click into place and you *know* that you understand a powerful idea. There will be times when you'll surprise yourself by successfully working a difficult problem that you didn't think you could solve. My hope, as an author, is that the excitement and sense of adventure will far outweigh the difficulties and frustrations.

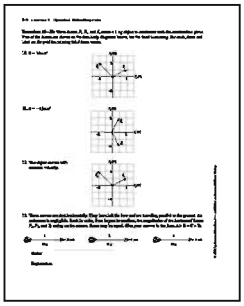
Getting the Most Out of Your Course

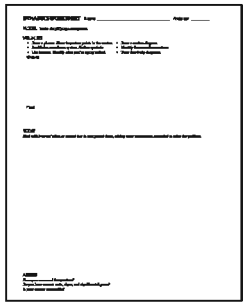
Many of you, I suspect, would like to know the "best" way to study for this course. There is no best way. People are different, and what works for one student is less effective for another. But I do want to stress that *reading the text* is vitally important. Class time will be used to clarify difficulties and to develop tools for using the knowledge, but your instructor will *not* use class time simply to repeat information in the text. The basic knowledge for this course is written down on these pages, and the *number-one expectation* is that you will read carefully and thoroughly to find and learn that knowledge.

Despite there being no best way to study, I will suggest *one* way that is successful for many students. It consists of the following four steps:

1. **Read each chapter *before* it is discussed in class.** I cannot stress too strongly how important this step is. Class attendance is much more effective if you are prepared. When you first read a chapter, focus on learning new vocabulary, definitions, and notation. There's a list of terms and notations at the end of each chapter. Learn them! You won't understand what's being discussed or how the ideas are being used if you don't know what the terms and symbols mean.
2. **Participate actively in class.** Take notes, ask and answer questions, and participate in discussion groups. There is ample scientific evidence that *active participation* is much more effective for learning science than passive listening.
3. **After class, go back for a careful re-reading of the chapter.** In your second reading, pay closer attention to the details and the worked examples. Look for the *logic* behind each example (I've highlighted this to make it clear), not just at what formula is being used. Do the *Student Workbook* exercises for each section as you finish your reading of it.
4. **Finally, apply what you have learned to the homework problems at the end of each chapter.** I strongly encourage you to form a study group with two or three classmates. There's good evidence that students who study regularly with a group do better than the rugged individualists who try to go it alone.







Did someone mention a workbook? The companion *Student Workbook* is a vital part of the course. Its questions and exercises ask you to reason *qualitatively,* to use graphical information, and to give explanations. It is through these exercises that you will learn what the concepts mean and will practice the reasoning skills appropriate to the chapter. You will then have acquired the baseline knowledge and confidence you need *before* turning to the end-of-chapter homework problems. In sports or in music, you would never think of performing before you practice, so why would you want to do so in physics? The workbook is where you practice and work on basic skills.

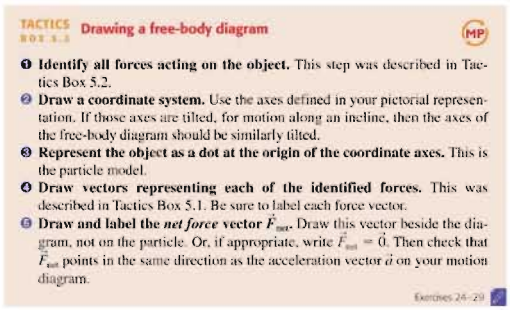
Many of you, I know, will be tempted to go straight to the homework problems and then thumb through the text looking for a formula that seems like it will work. That approach will not succeed in this course, and it's guaranteed to make you frustrated and discouraged. Very few homework problems are of the "plug and chug" variety where you simply put numbers into a formula. To work the homework problems successfully, you need a better study strategy—either the one outlined above or your own—that helps you learn the concepts and the relationships between the ideas.

A traditional guideline in college is to study two hours outside of class for every hour spent in class, and this text is designed with that expectation. Of course, two hours is an average. Some chapters are fairly straightforward and will go quickly. Others likely will require much more than two study hours per class hour.

Getting the Most Out of Your Textbook

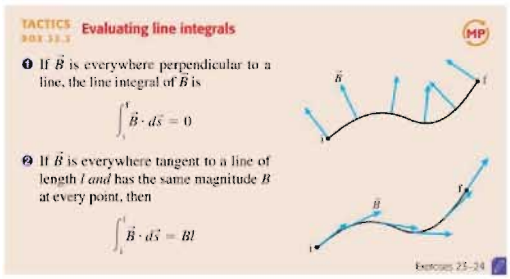
Your textbook provides many features designed to help you learn the concepts of physics and solve problems more effectively.

- **TACTICS BOXES** give step-by-step procedures for particular skills, such as interpreting graphs or drawing special diagrams. Tactics Box steps are explicitly illustrated in subsequent worked examples, and these are often the starting point of a full *Problem-Solving Strategy.*


TACTICS BOX 5.3 Drawing a free-body diagram

❶ **Identify all forces acting on the object.** This step was described in Tactics Box 5.2.
❷ **Draw a coordinate system.** Use the axes defined in your pictorial representation. If those axes are tilted, for motion along an incline, then the axes of the free-body diagram should be similarly tilted.
❸ **Represent the object as a dot at the origin of the coordinate axes.** This is the particle model.
❹ **Draw vectors representing each of the identified forces.** This was described in Tactics Box 5.1. Be sure to label each force vector.
❺ **Draw and label the *net force* vector $\vec{F}_{net}$.** Draw this vector beside the diagram, not on the particle. Or, if appropriate, write $\vec{F}_{net} = \vec{0}$. Then check that $\vec{F}_{net}$ points in the same direction as the acceleration vector $\vec{a}$ on your motion diagram.

Exercises 24–29


TACTICS BOX 33.3 Evaluating line integrals

❶ If $\vec{B}$ is everywhere perpendicular to a line, the line integral of $\vec{B}$ is

$$\int_i^f \vec{B} \cdot d\vec{s} = 0$$

❷ If $\vec{B}$ is everywhere tangent to a line of length l *and* has the same magnitude B at every point, then

$$\int_i^f \vec{B} \cdot d\vec{s} = Bl$$

Exercises 23–24

- **PROBLEM-SOLVING STRATEGIES** are provided for each broad class of problems—problems characteristic of a chapter or group of chapters. The strategies follow a consistent four-step approach to help you develop confidence and proficient problem-solving skills: **MODEL, VISUALIZE, SOLVE, ASSESS.**

PROBLEM-SOLVING STRATEGY 6.2 **Dynamics problems** (MP)

MODEL Make simplifying assumptions.

VISUALIZE Draw a **pictorial representation.**

- Show important points in the motion with a sketch, establish a coordinate system, define symbols, and identify what the problem is trying to find. This is the process of translating words into symbols.
- Use a motion diagram to determine the object's acceleration vector $\vec{a}$.
- Identify all forces acting on the object and show them on a free-body diagram.
- It's OK to go back and forth between these steps as you visualize the situation.

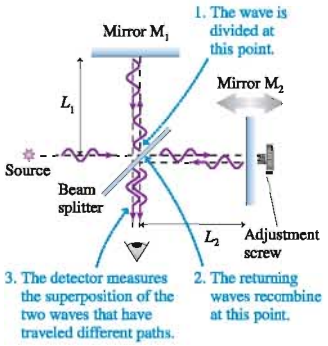
SOLVE The mathematical representation is based on Newton's second law:

$$\vec{F}_{net} = \sum_i \vec{F}_i = m\vec{a}$$

The vector sum of the forces is found directly from the free-body diagram. Depending on the problem, either

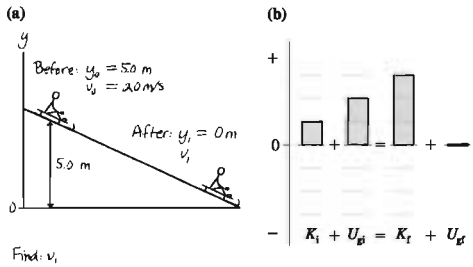
- Solve for the acceleration, then use kinematics to find velocities and positions; or
- Use kinematics to determine the acceleration, then solve for unknown forces.

ASSESS Check that your result has the correct units, is reasonable, and answers the question.



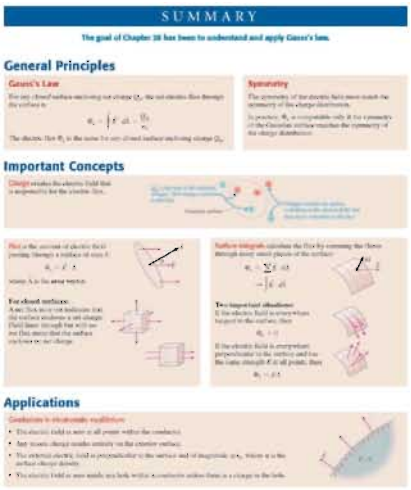
Annotated **FIGURE** showing the operation of the Michelson interferometer.

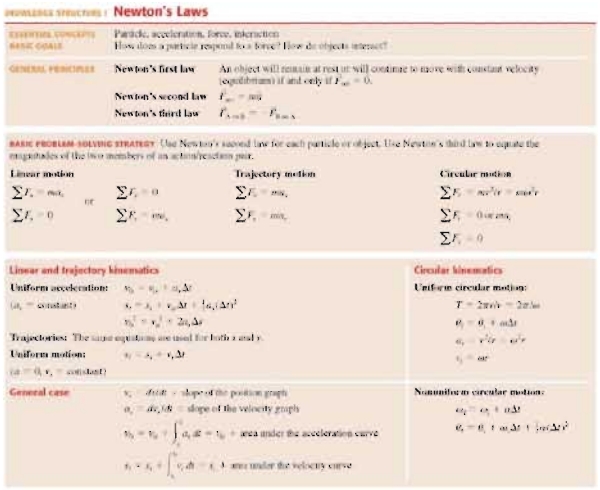
- Worked **EXAMPLES** illustrate good problem-solving practices through the consistent use of the four-step problem-solving approach and, where appropriate, the Tactics Box steps. The worked examples are often very detailed and carefully lead you through the *reasoning* behind the solution as well as the numerical calculations. A careful study of the reasoning will help you apply the concepts and techniques to the new and novel problems you will encounter in homework assignments and on exams.
- **NOTE ▶** paragraphs alert you to common mistakes and point out useful tips for tackling problems.
- **STOP TO THINK** questions embedded in the chapter allow you to quickly assess whether you've understood the main idea of a section. A correct answer will give you confidence to move on to the next section. An incorrect answer will alert you to re-read the previous section.
- Blue annotations on figures help you better understand what the figure is showing. They will help you to interpret graphs; translate between graphs, math, and pictures; grasp difficult concepts through a visual analogy; and develop many other important skills.
- *Pencil sketches* provide practical examples of the figures you should draw yourself when solving a problem.



Pencil-sketch **FIGURE** showing a toboggan going down a hill and its energy bar chart.

- The learning goals and links that begin each chapter outline what to focus on in the chapter ahead and what you need to remember from previous chapters.
  - ▶ **Looking Ahead** lists key concepts and skills you will learn in the coming chapter.
  - ◀ **Looking Back** highlights important topics you should review from previous chapters.
- Schematic *Chapter Summaries* help you organize what you have learned into a hierarchy, from general principles (top) to applications (bottom). Side-by-side pictorial, graphical, textual, and mathematical representations are used to help you translate between these key representations.
- *Part Overviews and Summaries* provide a global framework for what you are learning. Each part begins with an overview of the chapters ahead and concludes with a broad summary to help you to connect the concepts presented in that set of chapters. **KNOWLEDGE STRUCTURE** tables in the Part Summaries, similar to the Chapter Summaries, help you to see the forest rather than just the trees.





Now that you know more about what is expected of you, what can you expect of me? That's a little trickier because the book is already written! Nonetheless, the book was prepared on the basis of what I think my students throughout the years have expected—and wanted—from their physics textbook. Further, I've listened to the extensive feedback I have received from thousands of students like you, and their instructors, who used the first edition of this book.

You should know that these course materials—the text and the workbook—are based on extensive research about how students learn physics and the challenges they face. The effectiveness of many of the exercises has been demonstrated through extensive class testing. I've written the book in an informal style that I hope you will find appealing and that will encourage you to do the reading. And, finally, I have endeavored to make clear not only that physics, as a technical body of knowledge, is relevant to your profession but also that physics is an exciting adventure of the human mind.

I hope you'll enjoy the time we're going to spend together.

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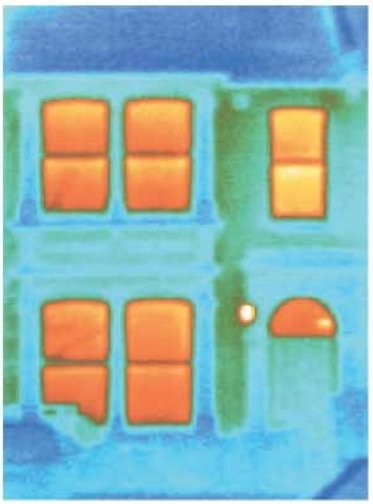

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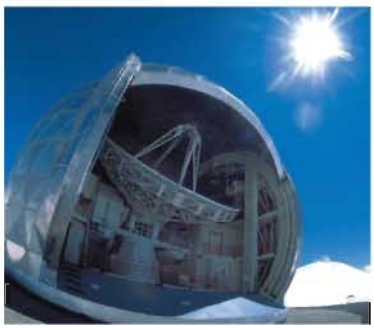

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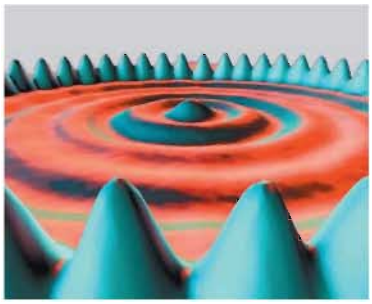

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Introduction

Journey into Physics

Said Alice to the Cheshire cat,
"Cheshire-Puss, would you tell me, please, which way I ought to go from here?"
"That depends a good deal on where you want to go," said the Cat.
"I don't much care where—" said Alice.
"Then it doesn't matter which way you go," said the Cat.
—Lewis Carroll, *Alice in Wonderland*

Have you ever wondered about questions such as

Why is the sky blue?

Why is glass an insulator but metal a conductor?

What, really, is an atom?

These are the questions of which physics is made. Physicists try to understand the universe in which we live by observing the phenomena of nature—such as the sky being blue—and by looking for patterns and principles to explain these phenomena. Many of the discoveries made by physicists, from electromagnetic waves to nuclear energy, have forever altered the ways in which we live and think.

You are about to embark on a journey into the realm of physics. It is a journey in which you will learn about many physical phenomena and find the answers to questions such as the ones posed above. Along the way, you will also learn how to use physics to analyze and solve many practical problems.

As you proceed, you are going to see the methods by which physicists have come to understand the laws of nature. The ideas and theories of physics are not arbitrary; they are firmly grounded in experiments and measurements. By the time you finish this text, you will be able to recognize the *evidence* upon which our present knowledge of the universe is based.

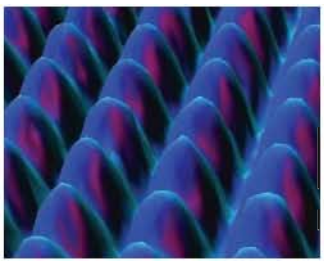
Which Way Should We Go?

We are rather like Alice in Wonderland, here at the start of the journey, in that we must decide which way to go. Physics is an immense body of knowledge, and without specific goals it would not much matter which topics we study. But unlike Alice, we *do* have some particular destinations that we would like to visit.

The physics that provides the foundation for all of modern science and engineering can be divided into three broad categories:

- Particles and energy.
- Fields and waves.
- The atomic structure of matter.

A particle, in the sense that we'll use the term, is an idealization of a physical object. We will use particles to understand how objects move and how they interact with each other. One of the most important properties of a particle or a collection of particles is *energy*. We will study energy both for its value in understanding physical processes and because of its practical importance in a technological society.



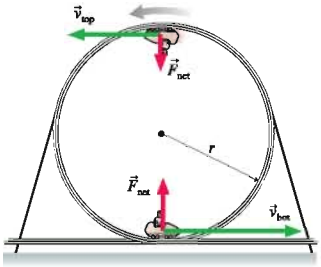
A scanning tunneling microscope allows us to "see" the individual atoms on a surface. One of our goals is to understand how an image such as this is made.

Particles are discrete, localized objects. Although many phenomena can be understood in terms of particles and their interactions, the long-range interactions of gravity, electricity, and magnetism are best understood in terms of *fields,* such as the gravitational field and the electric field. Rather than being discrete, fields spread continuously through space. Much of the second half of this book will be focused on understanding fields and the interactions between fields and particles.

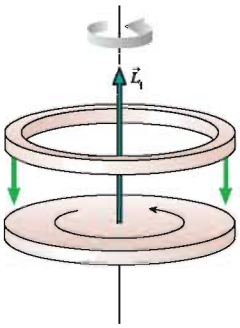
Certainly one of the most significant discoveries of the past 500 years is that matter consists of atoms. Atoms and their properties are described by quantum physics, but we cannot leap directly into that subject and expect that it would make any sense. To reach our destination, we are going to have to study many other topics along the way—rather like having to visit the Rocky Mountains if you want to drive from New York to San Francisco. All our knowledge of particles and fields will come into play as we end our journey by studying the atomic structure of matter.

The Route Ahead

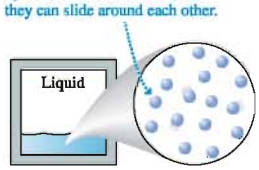
Here at the beginning, we can survey the route ahead. Where will our journey take us? What scenic vistas will we view along the way?



Parts I and II, *Newton's Laws* and *Conservation Laws,* form the basis of what is called *classical mechanics.* Classical mechanics is the study of motion. (It is called *classical* to distinguish it from the modern theory of motion at the atomic level, which is called *quantum mechanics.*) The first two parts of this textbook establish the basic language and concepts of motion. Part I will look at motion in terms of *particles* and *forces.* We will use these concepts to study the motion of everything from accelerating sprinters to orbiting satellites. Then, in Part II, we will introduce the ideas of *momentum* and *energy.* These concepts—especially energy—will give us a new perspective on motion and extend our ability to analyze motion.

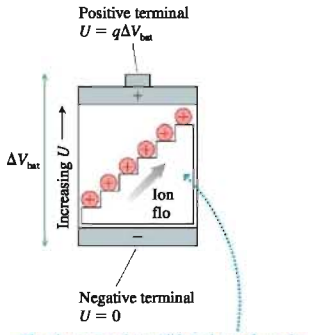


Part III, *Applications of Newtonian Mechanics,* will pause to look at four important applications of classical mechanics: Newton's theory of gravity, rotational motion, oscillatory motion, and the motion of fluids. Only oscillatory motion is a prerequisite for later chapters. Your instructor may choose to cover some or all of the other chapters, depending upon the time available, but your study of Parts IV–VII will not be hampered if these chapters are omitted.



Part IV, *Thermodynamics,* extends the ideas of particles and energy to systems such as liquids and gases that contain vast numbers of particles. Here we will look for connections between the *microscopic* behavior of large numbers of atoms and the *macroscopic* properties of bulk matter. You will find that some of the properties of gases that you know from chemistry, such as the ideal gas law, turn out to be direct consequences of the underlying atomic structure of the gas. We will also expand the concept of energy and study how energy is transferred and utilized.

Waves are ubiquitous in nature, whether they be large-scale oscillations like ocean waves, the less obvious motions of sound waves, or the subtle undulations of light waves and matter waves that go to the heart of the atomic structure of matter. In **Part V,** *Waves and Optics,* we will emphasize the unity of wave physics and find that many diverse wave phenomena can be analyzed with the same concepts and mathematical language. It is here we will begin to accumulate evidence that the theory of classical mechanics is inadequate to explain the observed behavior of atoms, and we will end this section with some atomic puzzles that seem to defy understanding.



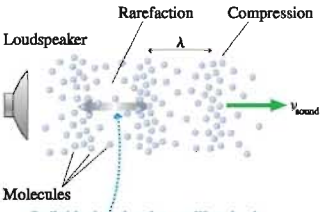
The charge escalator "lifts" charge from the negative side to the positive side. Charge q gains energy $\Delta U = q\Delta V_{bat}$.

Part VI, *Electricity and Magnetism,* is devoted to the *electromagnetic force,* one of the most important forces in nature. In essence, the electromagnetic force is the "glue" that holds atoms together. It is also the force that makes this the "electronic age." We'll begin this part of the journey with simple observations of static electricity. Bit by bit, we'll be led to the basic ideas behind electrical circuits, to magnetism, and eventually to the discovery of electromagnetic waves.

Part VII is *Relativity and Quantum Physics.* We'll start by exploring the strange world of Einstein's theory of *relativity,* a world in which space and time aren't quite what they appear to be. Then we will enter the microscopic domain of *atoms,* where the behaviors of light and matter are at complete odds with what our common sense tells us is possible. Although the mathematics of quantum theory quickly gets beyond the level of this text, and time will be running out, you will see that the quantum theory of atoms and nuclei explains many of the things that you learned simply as rules in chemistry.

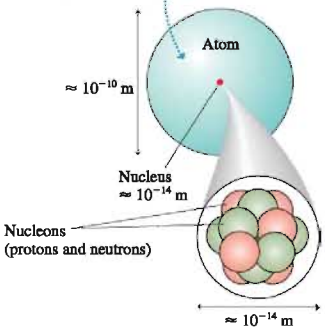
We will not have visited all of physics on our travels. There just isn't time. Many exciting topics, ranging from quarks to black holes, will have to remain unexplored. But this particular journey need not be the last. As you finish this text, you will have the background and the experience to explore new topics further in more advanced courses or for yourself.

With that said, let us take the first step.



Individual molecules oscillate back and forth with displacement D. As they do so, the compressions propagate forward at speed v_{sound}. Because compressions are regions of higher pressure, a sound wave can be thought of as a pressure wave.

This picture of an atom would need to be 10 m in diameter if it were drawn to the same scale as the dot representing the nucleus.



PART I

Newton's Laws

Motion can be exhilarating and beautiful. These sailboats driving across the bay are responding to forces of wind, water, and the weight of the crew as they balance precariously on the edge.

OVERVIEW

Why Things Change

Each of the seven parts of this book opens with an overview to give you a look ahead, a glimpse at where your journey will take you in the next few chapters. It's easy to lose sight of the big picture while you're busy negotiating the terrain of each chapter. In Part I, the big picture, in a word, is *change*.

Simple observations of the world around you show that most things change, few things remain the same. Some changes, such as aging, are biological. Others, such as sugar dissolving in your coffee, are chemical. We're going to study change that involves *motion* of one form or another—the motion of balls, cars, and rockets.

There are two big questions we must tackle:

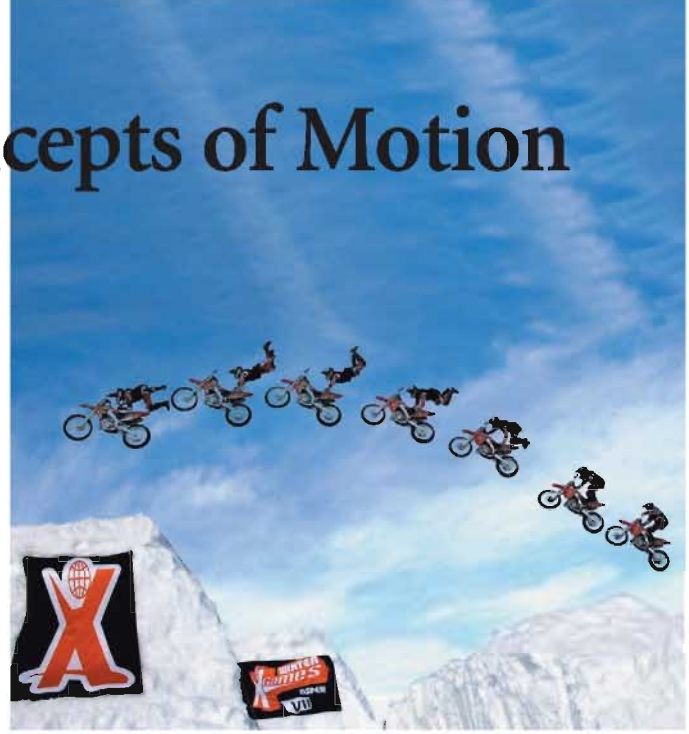
- **How do we describe motion?** It is easy to say that an object moves, but it's not obvious how we should measure or characterize the motion if we want to analyze it mathematically. The mathematical description of motion is called *kinematics,* and it is the subject matter of Chapters 1 through 4.
- **How do we explain motion?** Why do objects have the particular motion they do? Why, when you toss a ball upward, does it go up and then come back down rather than keep going up? Are there "laws of nature" that allow us to predict an object's motion? The explanation of motion in terms of its causes is called *dynamics,* and it is the topic of Chapters 5 through 8.

Two key ideas for answering these questions are *force* (the "cause") and *acceleration* (the "effect"). A variety of pictorial and graphical tools will be developed in Chapters 1 through 5 to help you develop an *intuition* for the connection between force and acceleration. You'll then put this knowledge to use in Chapters 5 through 8 as you analyze motion of increasing complexity.

Another important tool will be the use of *models.* Reality is extremely complicated. We would never be able to develop a science if we had to keep track of every little detail of every situation. A model is a simplified description of reality—much as a model airplane is a simplified version of a real airplane—used to reduce the complexity of a problem to the point where it can be analyzed and understood. We will introduce several important models of motion, paying close attention, especially in these earlier chapters, to where simplifying assumptions are being made, and why.

The "laws of motion" were discovered by Isaac Newton roughly 350 years ago, so the study of motion is hardly cutting-edge science. Nonetheless, it is still extremely important. Mechanics—the science of motion—is the basis for much of engineering and applied science, and many of the ideas introduced here will be needed later to understand things like the motion of waves and the motion of electrons through circuits. Newton's mechanics is the foundation of much of contemporary science, thus we will start at the beginning.

1 Concepts of Motion



Motion takes many forms. Some are simple. Others, like this, are complex.

▶ Looking Ahead

The goal of Chapter 1 is to introduce the fundamental concepts of motion. In this chapter you will learn to:

- Draw and interpret motion diagrams.
- Describe motion with vectors.
- Use the concepts of position, velocity, and acceleration.
- Use multiple representations of motion.
- Analyze and interpret motion problems.
- Draw and analyze motion graphs.

The universe in which we live is one of change and motion. This snowboarder was clearly in motion when the photograph was taken. In the course of a day you probably walk, run, bicycle, or drive your car, all forms of motion. The clock hands are moving inexorably forward as you read this text. The pages of this book may look quite still, but a microscopic view would reveal jostling atoms and whirling electrons. The stars look as permanent as anything, yet the astronomer's telescope reveals them to be ceaselessly moving within galaxies that rotate and orbit yet other galaxies.

Motion is a theme that will appear in one form or another throughout this entire book. Although we all have intuition about motion, based on our experiences, some of the important aspects of motion turn out to be rather subtle. So rather than jumping immediately into a lot of mathematics and calculations, this first chapter focuses on *visualizing* motion and becoming familiar with the *concepts* needed to describe a moving object. We will use mathematical ideas when needed, because they increase the precision of our thoughts, but we will defer actual calculations until Chapter 2. Our goal is to lay the foundations for understanding motion.

1.1 Motion Diagrams

The quest to understand motion dates to antiquity. The ancient Babylonians, Chinese, and Greeks were especially interested in the celestial motions of the night sky. The Greek philosopher and scientist Aristotle wrote extensively about the nature of moving objects. However, our modern understanding of motion did not begin until Galileo (1564–1642) first formulated the concepts of motion in mathematical terms.

Socrates: ***The nature of motion appears to be the question with which we begin.***
Plato, 375 BCE

FIGURE 1.1 Four basic types of motion.

Translational motion

Circular motion

Projectile motion

Rotational motion

And it took Newton (1642–1727) and the invention of calculus to put the concepts of motion on a firm and rigorous footing.

As a starting point, let's define **motion** as the change of an object's position with time. Examples of motion are easy to list. Bicycles, baseballs, cars, airplanes, and rockets are all objects that move. The path along which an object moves, which might be a straight line or might be curved, is called the object's **trajectory.**

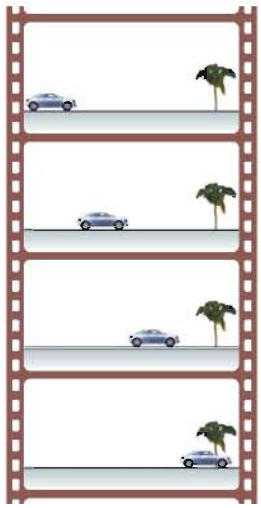
FIGURE 1.1 shows four basic types of motion that we will study in this book. Rotational motion is somewhat different from the other three in that rotation is a change of the object's *angular* position. We'll defer rotational motion until later and, for now, focus on motion along a line, circular motion, and projectile motion.

Making a Motion Diagram

An easy way to study motion is to make a movie of a moving object. A movie camera, as you probably know, takes photographs at a fixed rate, typically 30 photographs every second. Each separate photo is called a *frame,* and the frames are all lined up one after the other in a *filmstrip.* As an example, FIGURE 1.2 shows four frames from the movie of a car going past. Not surprisingly, the car is in a somewhat different position in each frame.

Suppose we cut the individual frames of the filmstrip apart, stack them on top of each other, and project the entire stack at once onto a screen for viewing. The

FIGURE 1.2 Four frames from the movie of a car.



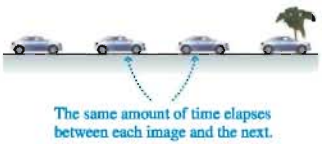


FIGURE 1.3 A motion diagram of the car shows all the frames simultaneously.

result is shown in **FIGURE 1.3.** This composite photo, showing an object's position at several *equally spaced instants of time,* is called a **motion diagram.** As simple as motion diagrams seem, they will turn out to be a powerful tool for analyzing motion.

NOTE ▶ It's important to keep the camera in a *fixed position* as the object moves by. Don't "pan" it to track the moving object. ◀

Now let's take our camera out into the world and make a few motion diagrams. The following table shows how we can see important aspects of the motion in a motion diagram.

Examples of motion diagrams

	An object that occupies only a *single position* in a motion diagram is *at rest.* **A stationary ball on the ground.**
	Images that are *equally spaced* indicate an object moving with *constant speed.* **A skateboarder rolling down the sidewalk.**
	An *increasing distance* between the images shows that the object is *speeding up.* **A sprinter starting the 100 meter dash.**
	A *decreasing distance* between the images shows that the object is *slowing down.* **A car stopping for a red light.**
	A more complex motion shows aspects of both slowing down (as the ball rises) and speeding up (as the ball falls). **A jump shot from center court.**

We have defined several concepts (at rest, constant speed, speeding up, and slowing down) in terms of how the moving object appears in a motion diagram. These are called **operational definitions,** meaning that the concepts are defined in terms of a particular procedure or operation performed by the investigator. For example, we could answer the question "Is the airplane speeding up?" by checking whether or not the images in the plane's motion diagram are getting farther apart. Many of the concepts in physics will be introduced as operational definitions. This reminds us that physics is an experimental science.

STOP TO THINK 1.1 Which car is going faster, A or B? Assume there are equal intervals of time between the frames of both movies.

NOTE ▶ Each chapter will have several *Stop to Think* questions. These questions are designed to see if you've understood the basic ideas that have been presented. The answers are given at the end of the chapter, but you should make a serious effort to think about these questions before turning to the answers. If you answer correctly, and are sure of your answer rather than just guessing, you can proceed to the next section with confidence. But if you answer incorrectly, it would be wise to reread the preceding sections before proceeding onward. ◀

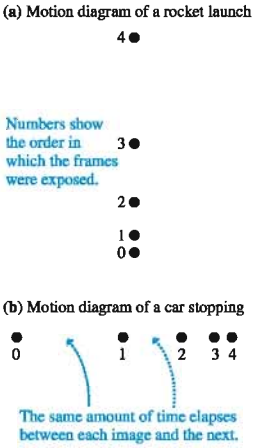
1.2 The Particle Model

For many objects, such as cars and rockets, the motion of the object *as a whole* is not influenced by the "details" of the object's size and shape. To describe the object's motion, all we really need to keep track of is the motion of a single point, such as a white dot painted on the side of the object.

If we restrict our attention to objects undergoing **translational motion,** which is the motion of an object along a trajectory, we can consider the object *as if* it were just a single point, without size or shape. We can also treat the object *as if* all of its mass were concentrated into this single point. An object that can be represented as a mass at a single point in space is called a **particle.** A particle has no size, no shape, and no distinction between top and bottom or between front and back.

If we treat an object as a particle, we can represent the object in each frame of a motion diagram as a simple dot rather than having to draw a full picture. FIGURE 1.4 shows how much simpler motion diagrams appear when the object is represented as a particle. Note that the dots have been numbered 0, 1, 2, . . . to tell the sequence in which the frames were exposed.

FIGURE 1.4 Motion diagrams in which the object is represented as a particle.



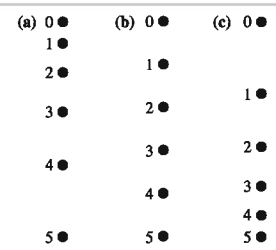
Using the Particle Model

Treating an object as a particle is, of course, a simplification of reality. As we noted in the overview, such a simplification is called a *model.* Models allow us to focus on the important aspects of a phenomenon by excluding those aspects that play only a minor role. The **particle model** of motion is a simplification in which we treat a moving object as if all of its mass were concentrated at a single point.

The particle model is an excellent approximation of reality for the motion of cars, planes, rockets, and similar objects. People are somewhat more complex, because of moving arms and legs, but the motion of a person's body as a whole is still described reasonably well within the particle model. In later chapters, we'll find that the motion of more complex objects, which cannot be treated as a single particle, can often be analyzed as if the object were a collection of particles.

Not all motions can be reduced to the motion of a single point. Consider a rotating gear. The center of the gear doesn't move at all, and each tooth on the gear is moving in a different direction. Rotational motion is qualitatively different than translational motion, and we'll need to go beyond the particle model later when we study rotational motion.

STOP TO THINK 1.2 Three motion diagrams are shown. Which is a dust particle settling to the floor at constant speed, which is a ball dropped from the roof of a building, and which is a descending rocket slowing to make a soft landing on Mars?



1.3 Position and Time

As we look at a motion diagram, it would be useful to know *where* the object is (i.e., its *position*) and *when* the object was at that position (i.e., the *time*). These are easy measurements to make.

Position measurements can be made by laying a coordinate system grid over a motion diagram. You can then measure the (x, y) coordinates of each point in the motion diagram. Of course, the world does not come with a coordinate system attached. A coordinate system is an artificial grid that *you* place over a problem in order to analyze the motion. You place the origin of your coordinate system wherever you wish, and different observers of a moving object might all choose to use different origins. Likewise, you can choose the orientation of the x-axis and y-axis to be helpful for that particular problem. The conventional choice is for the x-axis to point to the right and the y-axis to point upward, but there is nothing sacred about this choice. We will soon have many occasions to tilt the axes at an angle.

Time, in a sense, is also a coordinate system, although you may never have thought of time this way. You can pick an arbitrary point in the motion and label it "$t = 0$ seconds." This is simply the instant you decide to start your clock or stopwatch, so it is the origin of your time coordinate. Different observers might choose to start their clocks at different moments. A movie frame labeled "$t = 4$ seconds" was taken 4 seconds after you started your clock.

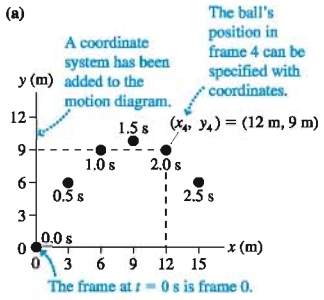
We typically choose $t = 0$ to represent the "beginning" of a problem, but the object may have been moving before then. Those earlier instants would be measured as negative times, just as objects on the x-axis to the left of the origin have negative values of position. Negative numbers are not to be avoided; they simply locate an event in space or time *relative to an origin.*

To illustrate, FIGURE 1.5a shows an xy-coordinate system and time information superimposed over the motion diagram of a basketball. You can see that the ball's position is $(x_4, y_4) = (12\text{ m}, 9\text{ m})$ at time $t_4 = 2.0\text{ s}$. Notice how we've used subscripts to indicate the time and the object's position in a specific frame of the motion diagram.

NOTE ▶ The first frame is labeled 0 to correspond with time $t = 0$. That is why the fifth frame is labeled 4. ◀

Another way to locate the ball is to draw an arrow from the origin to the point representing the ball. You can then specify the length and direction of the arrow. An arrow drawn from the origin to an object's position is called the **position vector** of the object, and it is given the symbol $\vec{r}$. FIGURE 1.5b shows the position vector $\vec{r}_4 = (15\text{ m}, 37°)$.

The position vector $\vec{r}$ does not tell us anything different than the coordinates (x, y). It simply provides the information in an alternative form. Although you're more familiar with coordinates than with vectors, you will find that vectors are a useful way to describe many concepts in physics.



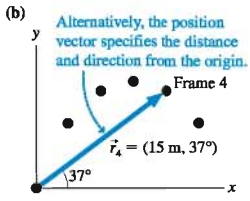


FIGURE 1.5 Position and time measurements made on the motion diagram of a basketball.

A Word About Vectors and Notation

Before continuing, let's take a closer look at what a vector is. Vectors will be studied thoroughly in Chapter 3, so all we need for now is a little basic information. Some physical quantities, such as time, mass, and temperature, can be described completely by a single number with a unit. For example, the mass of an object is 6 kg and its temperature is 30°C. When a physical quantity is described by a single number (with a unit), we call it a **scalar quantity.** A scalar can be positive, negative, or zero.

Many other quantities, however, have a directional quality and cannot be described by a single number. To describe the motion of a car, for example, you must specify not only how fast it is moving, but also the *direction* in which it is moving. A **vector quantity** is a quantity having both a *size* (the "How far?" or "How fast?") and a *direction* (the "Which way?"). The size or length of a vector is called its *magnitude.* The magnitude of a vector can be positive or zero, but it cannot be negative.

When we want to represent a vector quantity with a symbol, we need somehow to indicate that the symbol is for a vector rather than for a scalar. We do this by drawing an arrow over the letter that represents the quantity. Thus $\vec{r}$ and $\vec{A}$ are symbols for vectors, whereas r and A, without the arrows, are symbols for scalars. In handwritten work you must draw arrows over all symbols that represent vectors. This may seem strange until you get used to it, but it is very important because we will often use both r and $\vec{r}$, or both A and $\vec{A}$, in the same problem, and they mean different things! Without the arrow, you will be using the same symbol with two different meanings and will likely end up making a mistake. Note that the arrow over the symbol always points to the right, regardless of which direction the actual vector points. Thus we write $\vec{r}$ or $\vec{A}$, never $\overleftarrow{r}$ or $\overleftarrow{A}$.

NOTE ▶ Some textbooks represent vectors with boldface type, such as **r** or **A**. This book will consistently display the vector arrow over vector symbols, just as you should do in handwritten work. ◀

Change in Position

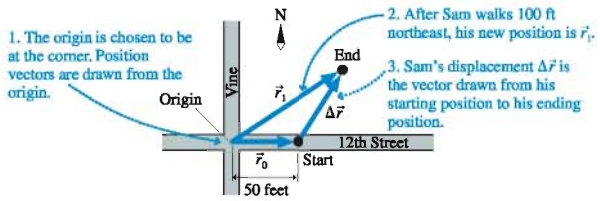
Consider the following:

> Sam is standing 50 feet (ft) east of the corner of 12th Street and Vine. He then walks northeast for 100 ft to a second point. What is Sam's change of position?

FIGURE 1.6 shows Sam's motion in terms of position vectors. Because we're free to place the origin of our coordinate system wherever we wish, we've placed it at the intersection. Sam's initial position is the vector $\vec{r}_0$ drawn from the origin to the point where he starts walking. Vector $\vec{r}_1$ is his position after he finishes walking. You can see that Sam has changed position, and a *change* of position is called a **displacement.** His displacement is the vector labeled $\Delta\vec{r}$. The Greek letter delta (Δ) is used in math and science to indicate the *change* in a quantity. Here it indicates a change in the position $\vec{r}$.

NOTE ▶ $\Delta\vec{r}$ is a *single* symbol. You cannot cancel out or remove the Δ in algebraic operations. ◀

FIGURE 1.6 Sam undergoes a displacement $\Delta\vec{r}$ from position $\vec{r}_0$ to position $\vec{r}_1$.



Displacement is a vector quantity; it requires both a length and a direction to describe it. Specifically, the displacement $\Delta\vec{r}$ is a vector drawn *from* a starting position *to* an ending position. Sam's displacement is written

$$\Delta\vec{r} = (100 \text{ ft, northeast})$$

where we've given both the length and the direction. The length, or magnitude, of a displacement vector is simply the straight-line distance between the starting and ending positions.

If you start 10 ft from a door and walk directly away from the door for 5 ft, you end up 15 ft from the door. The *procedure* by which you learn this is to *add* your change in position (5 ft) to your initial position (10 ft).

Similarly, we can answer the question "Where does Sam end up?" if we *add* his change in position (his displacement $\Delta\vec{r}$) to his initial position, the vector $\vec{r}_0$. Sam's final position in Figure 1.6, vector $\vec{r}_1$, can be seen as a combination of vector $\vec{r}_0$ *plus* vector $\Delta\vec{r}$. In fact, $\vec{r}_1$ is the *vector sum* of vectors $\vec{r}_0$ and $\Delta\vec{r}$. This is written

$$\vec{r}_1 = \vec{r}_0 + \Delta\vec{r} \qquad (1.1)$$

Notice, however, that we are adding vector quantities, not numbers. Vector addition is a different process from "regular" addition. We'll explore vector addition more thoroughly in Chapter 3, but for now you can add two vectors $\vec{A}$ and $\vec{B}$ with the three-step procedure shown in Tactics Box 1.1.

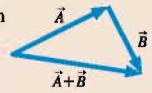
TACTICS BOX 1.1 Vector addition MP

To add $\vec{B}$ to $\vec{A}$:

❶ Draw $\vec{A}$.

❷ Place the tail of $\vec{B}$ at the tip of $\vec{A}$.

❸ Draw an arrow from the tail of $\vec{A}$ to the tip of $\vec{B}$. This is vector $\vec{A} + \vec{B}$.

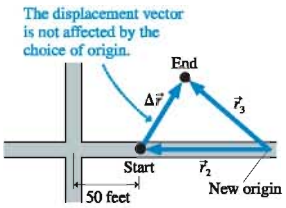


If you examine Figure 1.6, you'll see that the steps of Tactics Box 1.1 are exactly how $\vec{r}_0$ and $\Delta\vec{r}$ are added to give $\vec{r}_1$.

NOTE ▶ A vector is not tied to a particular location on the page. You can move a vector around as long as you don't change its length or the direction it points. Vector $\vec{B}$ is not changed by sliding it to where its tail is at the tip of $\vec{A}$. ◀

In Figure 1.6, we chose *arbitrarily* to put the origin of the coordinate system at the corner. While this might be convenient, it certainly is not mandatory. FIGURE 1.7 shows a different choice of where to place the origin. Notice something interesting. The initial and final position vectors $\vec{r}_0$ and $\vec{r}_1$ have become new vectors $\vec{r}_2$ and $\vec{r}_3$, but the displacement vector $\Delta\vec{r}$ has not changed! **The displacement is a quantity that is independent of the coordinate system.** In other words, the arrow drawn from the one position of an object to the next is the same no matter what coordinate system you choose. This independence gives the displacement $\Delta\vec{r}$ more *physical significance* than the position vectors themselves have.

FIGURE 1.7 Sam's displacement $\Delta\vec{r}$ is unchanged by using a different coordinate system.



This observation suggests that the displacement, rather than the actual position, is what we want to focus on as we analyze the motion of an object. Equation 1.1 told us that $\vec{r}_1 = \vec{r}_0 + \Delta\vec{r}$. This is easily rearranged to give a more precise definition of displacement: **The displacement $\Delta\vec{r}$ of an object as it moves from an initial position $\vec{r}_i$ to a final position $\vec{r}_f$ is**

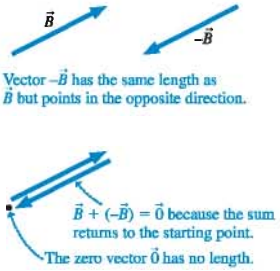
$$\Delta\vec{r} = \vec{r}_f - \vec{r}_i \qquad (1.2)$$

Graphically, $\Delta\vec{r}$ is a vector arrow drawn *from* position $\vec{r}_i$ *to* position $\vec{r}_f$. The displacement vector is independent of the coordinate system.

NOTE ▶ To be more general, we've written Equation 1.2 in terms of an *initial position* and a *final position*, indicated by subscripts i and f. We'll frequently use i and f when writing general equations, then use specific numbers or values, such as 0 and 1, when working a problem. ◀

This definition of $\Delta\vec{r}$ involves *vector subtraction*. With numbers, subtraction is the same as the addition of a negative number. That is, $5 - 3$ is the same as $5 + (-3)$. Similarly, we can use the rules for vector addition to find $\vec{A} - \vec{B} = \vec{A} + (-\vec{B})$ if we first define what we mean by $-\vec{B}$. As FIGURE 1.8 shows, the negative of vector $\vec{B}$ is a vector with the same length but pointing in the opposite direction. This makes sense because $\vec{B} - \vec{B} = \vec{B} + (-\vec{B}) = \vec{0}$, where $\vec{0}$, a vector with zero length, is called the **zero vector.**

FIGURE 1.8 The negative of a vector.

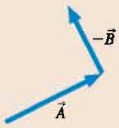


TACTICS BOX 1.2 Vector subtraction MP

To subtract $\vec{B}$ from $\vec{A}$:

❶ Draw $\vec{A}$.

❷ Place the tail of $-\vec{B}$ at the tip of $\vec{A}$.



❸ Draw an arrow from the tail of $\vec{A}$ to the tip of $-\vec{B}$. This is vector $\vec{A} - \vec{B}$.

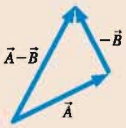


FIGURE 1.9 uses the vector subtraction rules of Tactics Box 1.2 to prove that the displacement $\Delta\vec{r}$ is simply the vector connecting the dots of a motion diagram.

FIGURE 1.9 Using vector subtraction to find $\Delta\vec{r}$.

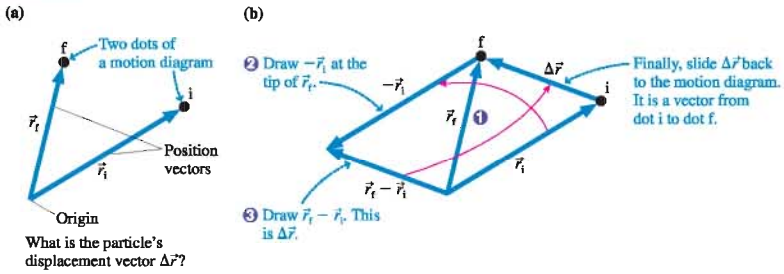
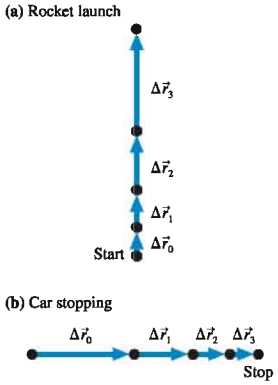


FIGURE 1.10 Motion diagrams with the displacement vectors.



Application to Motion Diagrams

The first step in analyzing a motion diagram is to determine all of the displacement vectors. As Figure 1.9 shows, the displacement vectors are simply the arrows connecting each dot to the next. Label each arrow with a *vector* symbol $\Delta\vec{r}_n$, starting with $n = 0$. FIGURE 1.10 shows the motion diagrams of Figure 1.4 redrawn to include the displacement vectors. You do not need to show the position vectors.

NOTE ▶ When an object either starts from rest or ends at rest, the initial or final dots are *as close together* as you can draw the displacement vector arrow connecting them. In addition, just to be clear, you should write "Start" or "Stop" beside the initial or final dot. It is important to distinguish stopping from merely slowing down. ◀

Now we can conclude, more precisely than before, that, as time proceeds:

- An object is speeding up if its displacement vectors are increasing in length.
- An object is slowing down if its displacement vectors are decreasing in length.

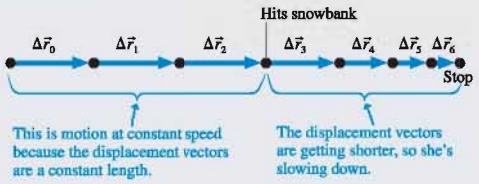
EXAMPLE 1.1 Headfirst into the snow

Alice is sliding along a smooth, icy road on her sled when she suddenly runs headfirst into a large, very soft snowbank that gradually brings her to a halt. Draw a motion diagram for Alice. Show and label all displacement vectors.

MODEL Use the particle model to represent Alice as a dot.

VISUALIZE FIGURE 1.11 shows Alice's motion diagram. The problem statement suggests that Alice's speed is very nearly constant until she hits the snowbank. Thus her displacement vectors are of equal length as she slides along the icy road. She begins slowing when she hits the snowbank, so the displacement vectors then get shorter until she stops. It is reasonable to assume that her stopping distance in the snow is less than the distance she had slid along the road, but we do not want to make her stop *too* quickly.

FIGURE 1.11 Alice's motion diagram.



A stopwatch is used to measure a time interval.

Change in Time

It's also useful to consider a *change* in time. For example, the clock readings of two frames of film might be t_1 and t_2. The specific values are arbitrary because they are timed relative to an arbitrary instant that you chose to call $t = 0$. But the **time interval** $\Delta t = t_2 - t_1$ is *not* arbitrary. It represents the elapsed time for the object to move from one position to the next. All observers will measure the same value for Δt, regardless of when they choose to start their clocks.

The time interval $\Delta t = t_f - t_i$ measures the elapsed time as an object moves from an initial position $\vec{r}_i$ at time t_i to a final position $\vec{r}_f$ at time t_f. The value of Δt is independent of the specific clock used to measure the times.

To summarize the main idea of this section, we have added coordinate systems and clocks to our motion diagrams in order to measure *when* each frame was exposed and *where* the object was located at that time. Different observers of the motion may choose different coordinate systems and different clocks. Thus a particular value of the position $\vec{r}$ or the time t is arbitrary because each is measured relative to an arbitrarily chosen origin. However, all observers find the *same* values for the displacements $\Delta\vec{r}$ and the time intervals Δt because these are independent of the specific coordinate system used to measure them.

1.4 Velocity

It's no surprise that, during a given time interval, a speeding bullet travels farther than a speeding snail. To extend our study of motion so that we can compare the bullet to the snail, we need a way to measure how fast or how slowly an object moves.

One quantity that measures an object's fastness or slowness is its **average speed,** defined as the ratio

$$\text{average speed} = \frac{\text{distance traveled}}{\text{time interval spent traveling}} \tag{1.3}$$

If you drive 15 miles (mi) in 30 minutes $\left(\frac{1}{2}\text{ hour}\right)$, your average speed is

$$\text{average speed} = \frac{15\text{ mi}}{\frac{1}{2}\text{ hour}} = 30\text{ mph} \tag{1.4}$$

Although the concept of speed is widely used in our day-to-day lives, it is not a sufficient basis for a science of motion. To see why, imagine you're trying to land a jet plane on an aircraft carrier. It matters a great deal to you whether the aircraft carrier is moving at 20 mph (miles per hour) to the north or 20 mph to the east. Simply knowing that the boat's speed is 20 mph is not enough information! The difficulty with speed is that it tells us nothing about the direction in which an object is moving.

It's the displacement $\Delta\vec{r}$, a vector quantity, that tells us not only the distance traveled by a moving object, but also the *direction* of motion. Consequently, a more useful ratio for an object undergoing a displacement $\Delta\vec{r}$ during the time interval Δt is the ratio $\Delta\vec{r}/\Delta t$. This ratio is a vector, because $\Delta\vec{r}$ is a vector, so it has both a magnitude and a direction. The size, or magnitude, of this ratio is very similar to the definition of speed: The ratio will be larger for a fast object than for a slow object. But in addition to measuring how fast an object moves, this ratio is a vector that points in the same direction as $\Delta\vec{r}$. That is, it points in the direction of motion.

It is convenient to give this ratio a name. We call it the **average velocity,** and it has the symbol $\vec{v}_{\text{avg}}$. **The average velocity of an object during the time interval Δt, in which the object undergoes a displacement $\Delta\vec{r}$, is the vector**

$$\vec{v}_{\text{avg}} = \frac{\Delta\vec{r}}{\Delta t} \tag{1.5}$$

An object's average velocity vector points in the same direction as the displacement vector $\Delta\vec{r}$. This is the direction of motion.

NOTE ▶ In everyday language we do not make a distinction between speed and velocity, but in physics *the distinction is very important.* In particular, speed is simply "How fast," whereas velocity is "How fast, and in which direction." As we go along we will be giving other words more precise meaning in physics than they have in everyday language. ◀

As an example, FIGURE 1.12a shows two ships that start from the same position and move 5 miles in 15 minutes. Both ships have a speed of 20 mph, but their velocities are different. Because their displacements during Δt are $\Delta\vec{r}_{\text{A}} = (5\text{ mi, north})$ and $\Delta\vec{r}_{\text{B}} = (5\text{ mi, east})$, we can write their velocities as

$$\begin{aligned}\vec{v}_{\text{avg A}} &= (20\text{ mph, north})\\ \vec{v}_{\text{avg B}} &= (20\text{ mph, east})\end{aligned} \tag{1.6}$$

Notice how the velocity *vectors* in FIGURE 1.12b point in the direction of motion.

NOTE ▶ Our goal in this chapter is to *visualize* motion with motion diagrams. Strictly speaking, the vector we have defined in Equation 1.5, and the vector we will show on motion diagrams, is the *average* velocity $\vec{v}_{\text{avg}}$. But to allow the motion diagram to be a useful tool, we will drop the subscript and refer to the average

The victory goes to the runner with the highest average speed.

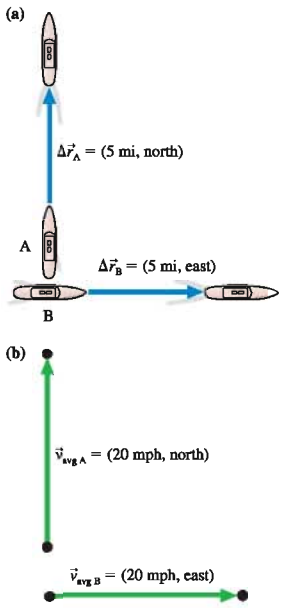


FIGURE 1.12 The displacement vectors and velocities of ships A and B.

velocity as simply $\vec{v}$. Our definitions and symbols, which somewhat blur the distinction between average and instantaneous quantities, are adequate for visualization purposes, but they're not the final word on the subject. We will refine these definitions in Chapter 2, where our goal will be to develop the mathematics of motion. ◀

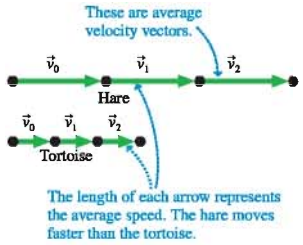
Motion Diagrams with Velocity Vectors

The velocity vector, as we've defined it, points in the same direction as the displacement $\Delta\vec{r}$, and the length of $\vec{v}$ is directly proportional to the length of $\Delta\vec{r}$. Consequently, the vectors connecting each dot of a motion diagram to the next, which we previously labeled as displacement vectors, could equally well be identified as velocity vectors.

This idea is illustrated in FIGURE 1.13, which shows four frames from the motion diagram of a tortoise racing a hare. The vectors connecting the dots are now labeled as velocity vectors $\vec{v}$. **The length of a velocity vector represents the average speed with which the object moves between the two points.** Longer velocity vectors indicate faster motion. You can see from the diagram that the hare moves faster than the tortoise.

Notice that the hare's velocity vectors do not change; each has the same length and direction. We say the hare is moving with *constant velocity*. The tortoise is also moving with its own constant velocity.

FIGURE 1.13 Motion diagram of the tortoise racing the hare.



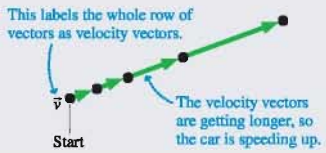
EXAMPLE 1.2 Accelerating up a hill

The light turns green and a car accelerates, starting from rest, up a 20° hill. Draw a motion diagram showing the car's velocity.

MODEL Use the particle model to represent the car as a dot.

VISUALIZE The car's motion takes place along a straight line, but the line is neither horizontal nor vertical. Because a motion diagram is made from frames of a movie, it will show the object moving with the correct orientation—in this case, at an angle of 20°. FIGURE 1.14 shows several frames of the motion diagram, where we see the car speeding up. The car starts from rest, so the first arrow is drawn as short as possible and the first dot is labeled "Start." The displacement vectors have been drawn from each dot to the next, but then they have been identified and labeled as average velocity vectors $\vec{v}$.

FIGURE 1.14 Motion diagram of a car accelerating up a hill.



NOTE ▶ Rather than label every single vector, it's easier to give one label to the entire row of velocity vectors. You can see this in Figure 1.14. ◀

EXAMPLE 1.3 It's a hit!

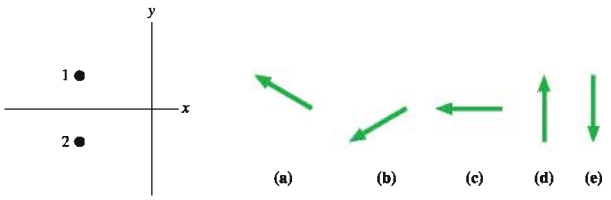
Jake hits a ball at a 60° angle above horizontal. It is caught by Jim. Draw a motion diagram of the ball.

MODEL This example is typical of how many problems in science and engineering are worded. The problem does not give a clear statement of where the motion begins or ends. Are we interested in the motion of the ball just during the time it is in the air between Jake and Jim? What about the motion *as* Jake hits it (ball rapidly speeding up) or *as* Jim catches it (ball rapidly slowing down)? Should we include Jim dropping the ball after he catches it? The point is that *you* will often be called on to make a *reasonable interpretation* of a problem statement. In this problem, the details of hitting and catching the ball are complex. The motion of the ball through the air is easier to describe, and it's a motion you might expect to learn about in a physics class. So our *interpretation* is that the motion diagram should start as the ball leaves Jake's bat (ball already moving) and should end the instant it touches Jim's hand (ball still moving). We will model the ball as a particle.

VISUALIZE With this interpretation in mind, **FIGURE 1.15** shows the motion diagram of the ball. Notice how, in contrast to the car of Figure 1.14, the ball is already moving as the motion diagram movie begins. As before, the average velocity vectors are found by connecting the dots with *straight* arrows. You can see that the average velocity vectors get shorter (ball slowing down), get longer (ball speeding up), and change direction. Each $\vec{v}$ is different, so this is *not* constant-velocity motion.

FIGURE 1.15 Motion diagram of a ball traveling from Jake to Jim.

STOP TO THINK 1.3 A particle moves from position 1 to position 2 during the interval Δt. Which vector shows the particle's average velocity?



1.5 Linear Acceleration

The goal of this chapter is to find a set of concepts with which to describe motion. Position, time, and velocity are important concepts, and at first glance they might appear to be sufficient. But that is not the case. Sometimes an object's velocity is constant, as it was in Figure 1.13. More often, an object's velocity changes as it moves, as in Figures 1.14 and 1.15. We need one more motion concept, one that will describe a *change* in the velocity.

Because velocity is a vector, it can change in two possible ways:

1. The magnitude can change, indicating a change in speed; or
2. The direction can change, indicating that the object has changed direction.

We will concentrate for now on the first case, a change in speed. The car in Figure 1.14 was an example of a situation in which the magnitude of the velocity vector changed but not the direction. We'll return to the second case in Chapter 4.

How can we measure the change of velocity in a meaningful way? When we wanted to measure changes in position, the ratio $\Delta\vec{r}/\Delta t$ was useful. This ratio is the *rate of change of position.* By analogy, consider an object whose velocity changes from $\vec{v}_1$ to $\vec{v}_2$ during the time interval Δt. Just as $\Delta\vec{r} = \vec{r}_2 - \vec{r}_1$ is the change of position, the quantity $\Delta\vec{v} = \vec{v}_2 - \vec{v}_1$ is the change of velocity. The ratio $\Delta\vec{v}/\Delta t$ is then the *rate of change of velocity.* But what does it measure?

Consider two cars, a Volkswagen Beetle and a fancy Porsche. Let them start from rest, and measure their velocities after an elapsed time of 10 seconds. The Porsche, we can assume, will have a larger $\Delta\vec{v}$. Consequently, it will have the larger value of the ratio $\Delta\vec{v}/\Delta t$. Thus this ratio appears to measure how quickly the car speeds up. It has a large magnitude for objects that speed up quickly and a small magnitude for objects that speed up slowly. The ratio $\Delta\vec{v}/\Delta t$ is called the **average acceleration,** and

The Audi TT accelerates from 0 to 60 mph in 6 s.

its symbol is $\vec{a}_{avg}$. **The average acceleration of an object during the time interval Δt, in which the object's velocity changes by $\Delta\vec{v}$, is the vector**

$$\vec{a}_{avg} = \frac{\Delta\vec{v}}{\Delta t} \qquad (1.7)$$

An object's average acceleration vector points in the same direction as the vector $\Delta\vec{v}$. Note that acceleration, like position and velocity, is a vector. Both its magnitude and its direction are important pieces of information.

Acceleration is a fairly abstract concept. Position and time are our real hands-on measurements of an object, and they are easy to understand. You can "see" where the object is located and the time on the clock. Velocity is a bit more abstract, being a relationship between the change of position and the change of time. Motion diagrams help us visualize velocity as the vector arrows connecting one position of the object to the next. Acceleration is an even more abstract idea about changes in the velocity. Yet it is essential to develop a good intuition about acceleration because it will be a key concept for understanding why objects move as they do.

NOTE ▶ As we did with velocity, we will drop the subscript and refer to the average acceleration as simply $\vec{a}$. This is adequate for visualization purposes, but not the final word on the subject. We will refine the definition of acceleration in Chapter 2. ◀

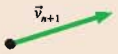
Finding the Acceleration Vectors on a Motion Diagram

Let's look at how we can determine the average acceleration vector $\vec{a}$ from a motion diagram. From its definition, Equation 1.7, we see that $\vec{a}$ points in the same direction as $\Delta\vec{v}$, the change of velocity. This critical idea is the basis for a technique to find $\vec{a}$.

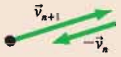
TACTICS BOX 1.3 Finding the acceleration vector MP

To find the acceleration as the velocity changes from $\vec{v}_n$ to $\vec{v}_{n+1}$:

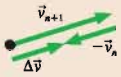
❶ Draw the velocity vector $\vec{v}_{n+1}$.



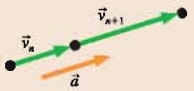
❷ Draw $-\vec{v}_n$ at the tip of $\vec{v}_{n+1}$.



❸ Draw $\Delta\vec{v} = \vec{v}_{n+1} - \vec{v}_n$
$= \vec{v}_{n+1} + (-\vec{v}_n)$
This is the direction of $\vec{a}$.



❹ Return to the original motion diagram. Draw a vector at the middle point in the direction of $\Delta\vec{v}$; label it $\vec{a}$. This is the average acceleration at the midpoint between $\vec{v}_n$ and $\vec{v}_{n+1}$.



Exercises 21–24

Most Tactics Boxes will refer you to exercises in the *Student Workbook* where you can practice the new skill.

Notice that the acceleration vector goes beside the dot, not beside the velocity vectors. This is because each acceleration vector is determined as the *difference* between the two velocity vectors on either side of a dot. The length of $\vec{a}$ does not have to be the exact length of $\Delta\vec{v}$; it is the direction of $\vec{a}$ that is most important.

The procedure of Tactics Box 1.3 can be repeated to find $\vec{a}$ at each point in the motion diagram. Note that we cannot determine $\vec{a}$ at the first and last points because we have only one velocity vector and can't find $\Delta\vec{v}$.

The Complete Motion Diagram

You've now seen several *Tactics Boxes* that help you achieve specific tasks. Tactics Boxes will appear in nearly every chapter in this book. We'll also, where appropriate, provide *Problem-Solving Strategies.* Problem solving will be discussed in more detail later in the chapter, but this is a good place for the first problem-solving strategy.

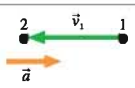
PROBLEM-SOLVING STRATEGY 1.1 Motion diagrams MP

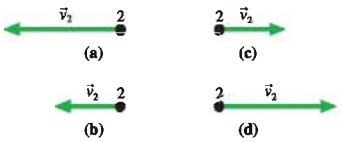
MODEL Represent the moving object as a particle. Make simplifying assumptions when interpreting the problem statement.

VISUALIZE A complete motion diagram consists of:

- The position of the object in each frame of the film, shown as a dot. Use five or six dots to make the motion clear but without overcrowding the picture. More complex motions may need more dots.
- The average velocity vectors, found by connecting each dot in the motion diagram to the next with a vector arrow. There is *one* velocity vector linking each *two* position dots. Label the row of velocity vectors $\vec{v}$.
- The average acceleration vectors, found using Tactics Box 1.3. There is *one* acceleration vector linking each *two* velocity vectors. Each acceleration vector is drawn at the dot between the two velocity vectors it links. Use $\vec{0}$ to indicate a point at which the acceleration is zero. Label the row of acceleration vectors $\vec{a}$.

STOP TO THINK 1.4 A particle undergoes acceleration $\vec{a}$ while moving from point 1 to point 2. Which of the choices shows the velocity vector $\vec{v}_2$ as the particle moves away from point 2?





Examples of Motion Diagrams

Let's look at some examples of the full strategy for drawing motion diagrams.

EXAMPLE 1.4 The first astronauts land on Mars

A spaceship carrying the first astronauts to Mars descends safely to the surface. Draw a motion diagram for the last few seconds of the descent.

MODEL Represent the spaceship as a particle. It's reasonable to assume that its motion in the last few seconds is straight down. The problem ends as the spacecraft touches the surface.

VISUALIZE FIGURE 1.16 shows a complete motion diagram as the spaceship descends and slows, using its rockets, until it comes to rest on the surface. Notice how the dots get closer together as it slows. The inset shows how the acceleration vector $\vec{a}$ is determined at one point. All the other acceleration vectors will be similar, because for each pair of velocity vectors the earlier one is longer than the later one.

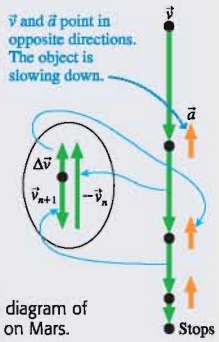


FIGURE 1.16 Motion diagram of a spaceship landing on Mars.

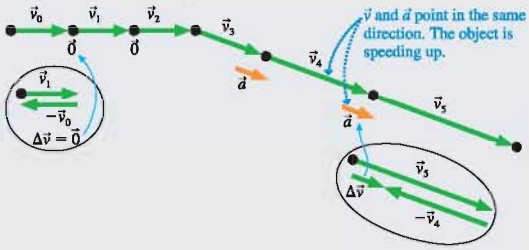
EXAMPLE 1.5 Skiing through the woods

A skier glides along smooth, horizontal snow at constant speed, then speeds up going down a hill. Draw the skier's motion diagram.

MODEL Represent the skier as a particle. It's reasonable to assume that the downhill slope is a straight line. Although the motion as a whole is not linear, we can treat the skier's motion as two separate linear motions.

VISUALIZE FIGURE 1.17 shows a complete motion diagram of the skier. The dots are equally spaced for the horizontal motion, indicating constant speed; then the dots get farther apart as the skier speeds up down the hill. The insets show how the average acceleration vector $\vec{a}$ is determined for the horizontal motion and along the slope. All the other acceleration vectors along the slope will be similar to the one shown because each velocity vector is longer than the preceding one. Notice that we've explicitly written $\vec{0}$ for the acceleration beside the dots where the velocity is constant. The acceleration at the point where the direction changes will be considered in Chapter 4.

FIGURE 1.17 Motion diagram of a skier.



Notice something interesting in Figures 1.16 and 1.17. Where the object is speeding up, the acceleration and velocity vectors point in the *same direction*. Where the object is slowing down, the acceleration and velocity vectors point in *opposite directions*. These results are always true for motion in a straight line. **For motion along a line:**

- **An object is speeding up if and only if $\vec{v}$ and $\vec{a}$ point in the same direction.**
- **An object is slowing down if and only if $\vec{v}$ and $\vec{a}$ point in opposite directions.**
- **An object's velocity is constant if and only if $\vec{a} = \vec{0}$.**

NOTE ▶ In everyday language, we use the word *accelerate* to mean "speed up" and the word *decelerate* to mean "slow down." But speeding up and slowing down are both changes in the velocity and consequently, by our definition, *both* are accelerations. In physics, *acceleration* refers to changing the velocity, no matter what the change is, and not just to speeding up. ◀

EXAMPLE 1.6 Tossing a ball

Draw the motion diagram of a ball tossed straight up in the air.

MODEL This problem calls for some interpretation. Should we include the toss itself, or only the motion after the tosser releases the ball? Should we include the ball hitting the ground? It appears that this problem is really concerned with the ball's motion through the air. Consequently, we begin the motion diagram at the moment that the tosser releases the ball and end the diagram at the moment the ball hits the ground. We will consider neither the toss nor the impact. And, of course, we will represent the ball as a particle.

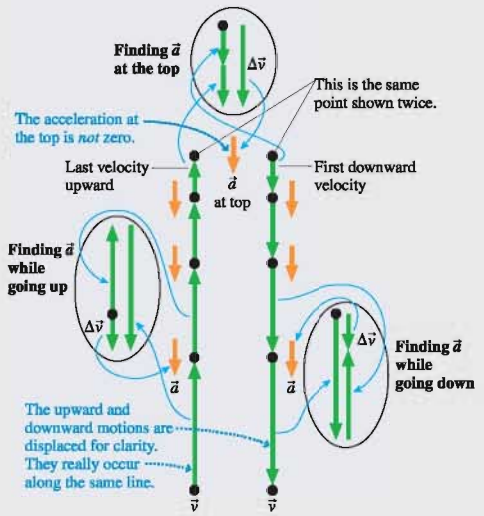
VISUALIZE We have a slight difficulty here because the ball retraces its route as it falls. A literal motion diagram would show the upward motion and downward motion on top of each other, leading to confusion. We can avoid this difficulty by horizontally separating the upward motion and downward motion diagrams. This will not affect our conclusions because it does not change any of the vectors. FIGURE 1.18 shows the motion diagram drawn this way. Notice that the very top dot is shown twice—as the end point of the upward motion and the beginning point of the downward motion.

The ball slows down as it rises. You've learned that the acceleration vectors point opposite the velocity vectors for an object that is slowing down along a line, and they are shown accordingly. Similarly, $\vec{a}$ and $\vec{v}$ point in the same direction as the falling ball speeds up. Notice something interesting: The acceleration vectors point downward both while the ball is rising *and* while it is falling. Both "speeding up" and "slowing down" occur with the *same* acceleration vector. This is an important conclusion, one worth pausing to think about.

Now let's look at the top point on the ball's trajectory. The velocity vectors are pointing upward but getting shorter as the ball approaches the top. As the ball starts to fall, the velocity vectors are pointing downward and getting longer. There must be a moment—just an instant as $\vec{v}$ switches from pointing up to pointing down—when the velocity is zero. Indeed, the ball's velocity *is* zero for an instant at the precise top of the motion!

But what about the acceleration at the top? The inset shows how the average acceleration is determined from the last upward velocity before the top point and the first downward velocity. We find that the acceleration at the top is pointing downward, just as it does elsewhere in the motion.

FIGURE 1.18 Motion diagram of a ball tossed straight up in the air.



Many people expect the acceleration to be zero at the highest point. But recall that the velocity at the top point *is* changing—from up to down. If the velocity is changing, there *must* be an acceleration. A downward-pointing acceleration vector is needed to turn the velocity vector from up to down. Another way to think about this is to note that zero acceleration would mean no change of velocity. When the ball reached zero velocity at the top, it would hang there and not fall if the acceleration were also zero!

1.6 Motion in One Dimension

As you've seen, an object's motion can be described in terms of three fundamental quantities: its position $\vec{r}$, velocity $\vec{v}$, and acceleration $\vec{a}$. These quantities are vectors, having a direction as well as a magnitude. But for motion in one dimension, the vectors are restricted to point only "forward" or "backward." Consequently, we can describe one-dimensional motion with the simpler quantities x, v_x, and a_x (or y, v_y, and a_y). However, we need to give each of these quantities an explicit *sign*, positive or negative, to indicate whether the position, velocity, or acceleration vector points forward or backward.

Determining the Signs of Position, Velocity, and Acceleration

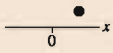
1.1 Activ ONLINE Physics

Position, velocity, and acceleration are measured with respect to a coordinate system, a grid or axis that *you* impose on a problem to analyze the motion. We will find it convenient to use an x-axis to describe both horizontal motion and motion along an inclined plane. A y-axis will be used for vertical motion. A coordinate axis has two essential features:

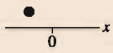
1. An origin to define zero; and
2. An x or y label to indicate the positive end of the axis.

We will adopt the convention that **the positive end of an x-axis is to the right and the positive end of a y-axis is up.** The signs of position, velocity, and acceleration are based on this convention.

TACTICS BOX 1.4 Determining the sign of the position, velocity, and acceleration MP



$x > 0$ Position to right of origin.



$x < 0$ Position to left of origin.

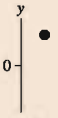
$v_x > 0$ Direction of motion is to the right.

$v_x < 0$ Direction of motion is to the left.

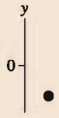
$a_x > 0$ Acceleration vector points to the right.

$a_x < 0$ Acceleration vector points to the left.

- The sign of position (x or y) tells us *where* an object is.
- The sign of velocity (v_x or v_y) tells us *which direction* the object is moving.
- The sign of acceleration (a_x or a_y) tells us which way the acceleration vector points, *not* whether the object is speeding up or slowing down.



$y > 0$
Position above origin.



$y < 0$
Position below origin.

$v_y > 0$
Direction of motion is up.

$v_y < 0$
Direction of motion is down.

$a_y > 0$
Acceleration vector points up.

$a_y < 0$
Acceleration vector points down.

Exercises 30–31

Acceleration is where things get a bit tricky. A natural tendency is to think that a positive value of a_x or a_y describes an object that is speeding up while a negative value describes an object that is slowing down (decelerating). However, this interpretation *does not work.*

Acceleration was defined as $\vec{a}_{avg} = \Delta\vec{v}/\Delta t$. The direction of $\vec{a}$ can be determined by using a motion diagram to find the direction of $\Delta\vec{v}$. The one-dimensional acceleration a_x (or a_y) is then positive if the vector $\vec{a}$ points to the right (or up), negative if $\vec{a}$ points to the left (or down).

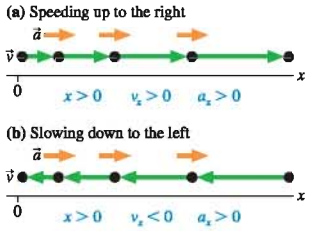
FIGURE 1.19 shows that this method for determining the sign of a does not conform to the simple idea of speeding up and slowing down. The object in Figure 1.19a has a positive acceleration ($a_x > 0$) not because it is speeding up but because the vector $\vec{a}$ points to the right. Compare this with the motion diagram of Figure 1.19b. Here the object is slowing down, but it still has a positive acceleration ($a_x > 0$) because $\vec{a}$ points to the right.

We found that an object is speeding up if $\vec{v}$ and $\vec{a}$ point in the same direction, slowing down if they point in opposite directions. For one-dimensional motion this rule becomes:

- An object is speeding up if and only if v_x and a_x have the same sign.
- An object is slowing down if and only if v_x and a_x have opposite signs.
- An object's velocity is constant if and only if $a_x = 0$.

Notice how the first two of these rules are at work in Figure 1.19.

FIGURE 1.19 One of these objects is speeding up, the other slowing down, but they both have a positive acceleration a_x.



Position-versus-Time Graphs

FIGURE 1.20 is a motion diagram, made at 1 frame per minute, of a student walking to school. You can see that she leaves home at a time we choose to call $t = 0$ min and makes steady progress for a while. Beginning at $t = 3$ min there is a period where the distance traveled during each time interval becomes less—perhaps she slowed down to speak with a friend. Then she picks up the pace, and the distances within each interval are longer.

FIGURE 1.20 The motion diagram of a student walking to school and a coordinate axis for making measurements.

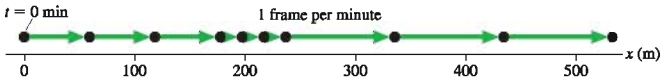


Figure 1.20 includes a coordinate axis, and you can see that every dot in a motion diagram occurs at a specific position. Table 1.1 shows the student's positions at different times as measured along this axis. For example, she is at position $x = 120$ m at $t = 2$ min.

TABLE 1.1 Measured positions of a student walking to school

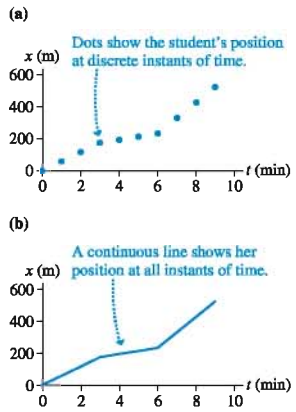
Time t (min)	Position x (m)
0	0
1	60
2	120
3	180
4	200
5	220
6	240
7	340
8	440
9	540

The motion diagram is one way to represent the student's motion. Another is to make a graph of the measurements in Table 1.1. FIGURE 1.21a is a graph of x versus t for the student. The motion diagram tells us only where the student is at a few discrete points of time, so this graph of the data shows only points, no lines.

NOTE ▶ A graph of "a versus b" means that a is graphed on the vertical axis and b on the horizontal axis. Saying "graph a versus b" is really a shorthand way of saying "graph a as a function of b." ◀

However, common sense tells us the following. First, the student was *somewhere specific* at all times. That is, there was never a time when she failed to have a well-defined position, nor could she occupy two positions at one time. (As reasonable as this belief appears to be, it will be severely questioned and found not entirely accurate when we get to quantum physics!) Second, the student moved *continuously* through all intervening points of space. She could not go from $x = 100$ m to $x = 200$ m without passing through every point in between. It is thus quite reasonable to believe that her motion can be shown as a continuous line passing through the measured points, as shown in FIGURE 1.21b. A continuous line or curve showing an object's position as a function of time is called a **position-versus-time graph** or, sometimes, just a *position graph*.

FIGURE 1.21 Position graphs of the student's motion.



NOTE ▶ A graph is *not* a "picture" of the motion. The student is walking along a straight line, but the graph itself is not a straight line. Further, we've graphed her position on the vertical axis even though her motion is horizontal. Graphs are *abstract representations* of motion. We will place significant emphasis on the process of interpreting graphs, and many of the exercises and problems will give you a chance to practice these skills. ◀

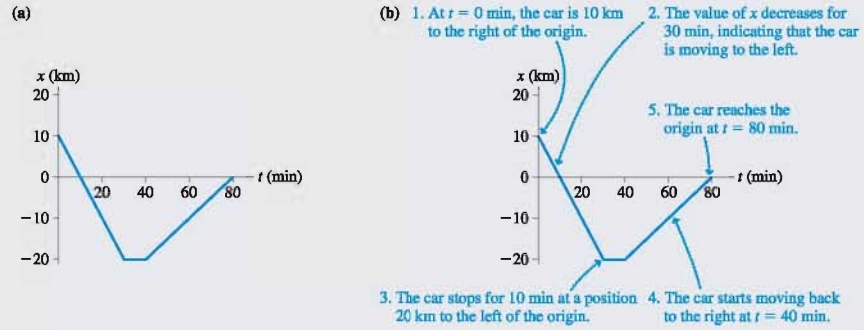
EXAMPLE 1.7 Interpreting a position graph

The graph in FIGURE 1.22a represents the motion of a car along a straight road. Describe the motion of the car.

MODEL Represent the car as a particle.

VISUALIZE As FIGURE 1.22b shows, the graph represents a car that travels to the left for 30 minutes, stops for 10 minutes, then travels back to the right for 40 minutes.

FIGURE 1.22 Position-versus-time graph of a car.



1.7 Solving Problems in Physics

Physics is not mathematics. Math problems are clearly stated, such as "What is 2 + 2?" Physics is about the world around us, and to describe that world we must use language. Now, language is wonderful—we couldn't communicate without it—but language can sometimes be imprecise or ambiguous.

The challenge when reading a physics problem is to translate the words into symbols that can be manipulated, calculated, and graphed. **The translation from words to symbols is the heart of problem solving in physics.** This is the point where ambiguous words and phrases must be clarified, where the imprecise must be made precise, and where you arrive at an understanding of exactly what the question is asking.

Richard Feynman, one of the greatest physicists of the 20th century, developed a new way to solve some difficult problems by representing complex ideas with special symbols and diagrams.

Using Symbols

Symbols are a language that allows us to talk with precision about the relationships in a problem. As with any language, we all need to agree to use words or symbols in the same way if we want to communicate with each other. Many of the ways we use symbols in science and engineering are somewhat arbitrary, often reflecting historical roots. Nonetheless, practicing scientists and engineers have come to agree on how to use the language of symbols. Learning this language is part of learning physics.

The previous section began to introduce the symbols needed to describe motion along a line—the one-dimensional position, velocity, and acceleration of an object represented by the symbols x, v_x, and a_x (or y, v_y, and a_y if the motion is vertical). The vector nature of these quantities appears through their *signs:*

- v_x (or v_y) is positive if the velocity vector $\vec{v}$ points to the right (or up). It is negative if the velocity vector $\vec{v}$ points to the left (or down).
- a_x (or a_y) is positive if the acceleration vector $\vec{a}$ points to the right (or up). It is negative if the acceleration vector $\vec{a}$ points to the left (or down).

The appropriate sign for v is usually clear. Determining the sign of a is more difficult, and this is where a motion diagram can help.

We will use subscripts to designate a particular point in the problem. Scientists usually label the starting point of the problem with the subscript "0," not the subscript "1" that you might expect. When using subscripts, make sure that all symbols referring to the same point in the problem have the *same numerical subscript.* To have the one point in a problem characterized by position x_1 but velocity v_{2x} is guaranteed to lead to confusion!

Drawing Pictures

You may have been told that the first step in solving a physics problem is to "draw a picture," but perhaps you didn't know why, or what to draw. The purpose of drawing a picture is to aid you in the words-to-symbols translation. Complex problems have far more information than you can keep in your head at one time. Think of a picture as a "memory extension," helping you organize and keep track of vital information.

Although any picture is better than none, there really is a *method* for drawing pictures that will help you be a better problem solver. It is called the **pictorial representation** of the problem. We'll add other pictorial representations as we go along, but the following procedure is appropriate for motion problems.

TACTICS BOX 1.5 Drawing a pictorial representation MP

1. **Draw a motion diagram.** The motion diagram develops your intuition for the motion and, especially important, determines whether the signs of v and a are positive or negative.
2. **Establish a coordinate system.** Select your axes and origin to match the motion. For one-dimensional motion, you want either the x-axis or the y-axis parallel to the motion.
3. **Sketch the situation.** Not just any sketch. Show the object at the *beginning* of the motion, at the *end,* and at any point where the character of the motion changes. Show the object, not just a dot, but very simple drawings are adequate.
4. **Define symbols.** Use the sketch to define symbols representing quantities such as position, velocity, acceleration, and time. *Every* variable used later in the mathematical solution should be defined on the sketch. Some will have known values, others are initially unknown, but all should be given symbolic names.
5. **List known information.** Make a table of the quantities whose values you can determine from the problem statement or that can be found quickly with simple geometry or unit conversions. Some quantities are implied by the problem, rather than explicitly given. Others are determined by your choice of coordinate system.
6. **Identify the desired unknowns.** What quantity or quantities will allow you to answer the question? These should have been defined as symbols in step 4. Don't list every unknown, only the one or two needed to answer the question.

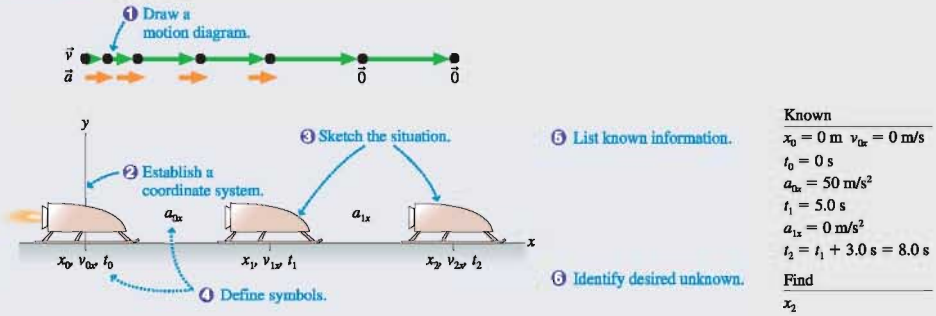
It's not an overstatement to say that a well-done pictorial representation of the problem will take you halfway to the solution. The following example illustrates how to construct a pictorial representation for a problem that is typical of problems you will see in the next few chapters.

EXAMPLE 1.8 Drawing a pictorial representation

Draw a pictorial representation for the following problem: A rocket sled accelerates at 50 m/s^2 for 5.0 s, then coasts for 3.0 s. What is the total distance traveled?

VISUALIZE The motion diagram shows an acceleration phase followed by a coasting phase. Because the motion is horizontal, the appropriate coordinate system is an x-axis. We've chosen to place the origin at the starting point. The motion has a beginning, an end, and a point where the nature of the motion changes from accelerating to coasting. These are the three sled positions sketched in **FIGURE 1.23**. The quantities x, v_x, and t are needed at each of three *points*, so these have been defined on the sketch and distinguished by subscripts. Accelerations are associated with *intervals* between the points, so only two accelerations are defined. Values for three quantities are given in the problem statement, although we need to use the motion diagram, where $\vec{a}$ points to the right, to know that $a_{0x} = +50$ m/s^2 rather than -50 m/s^2. Other quantities, such as $x_0 = 0$ m and $t_0 = 0$ s, are inferred from our choice of coordinate system. The value $v_{0x} = 0$ m/s is part of our *interpretation* of the problem. Finally, we identify x_2 as the quantity that will answer the question. We now understand quite a bit about the problem and would be ready to start a quantitative analysis.

FIGURE 1.23 A pictorial representation.



We didn't *solve* the problem; that is not the purpose of the pictorial representation. The pictorial representation is a systematic way to go about interpreting a problem and getting ready for a mathematical solution. Although this is a simple problem, and you probably know how to solve it if you've taken physics before, you will soon be faced with much more challenging problems. Learning good problem-solving skills at the beginning, while the problems are easy, will make them second nature later when you really need them.

A new building requires careful planning. The architect's visualization and drawings have to be complete before the detailed procedures of construction get under way. The same is true for solving problems in physics.

Representations

A picture is one way to *represent* your knowledge of a situation. You could also represent your knowledge using words, graphs, or equations. Each **representation of knowledge** gives us a different perspective on the problem. The more tools you have for thinking about a complex problem, the more likely you are to solve it.

There are four representations of knowledge that we will use over and over:

1. The *verbal* representation. A problem statement, in words, is a verbal representation of knowledge. So is an explanation that you write.
2. The *pictorial* representation. The pictorial representation, which we've just presented, is the most literal depiction of the situation.
3. The *graphical* representation. We will make extensive use of graphs.
4. The *mathematical* representation. Equations that can be used to find the numerical values of specific quantities are the mathematical representation.

NOTE ▶ The mathematical representation is only one of many. Much of physics is more about thinking and reasoning than it is about solving equations. ◀

A Problem-Solving Strategy

One of the goals of this textbook is to help you learn a *strategy* for solving physics problems. The purpose of a strategy is to guide you in the right direction with minimal wasted effort. The four-part problem-solving strategy shown below—**Model, Visualize, Solve, Assess**—is based on using different representations of knowledge. You will see this problem-solving strategy used consistently in the worked examples throughout this textbook, and you should endeavor to apply it to your own problem solving.

Throughout this textbook we will emphasize the first two steps. They are the *physics* of the problem, as opposed to the mathematics of solving the resulting equations. This is not to say that those mathematical operations are always easy—in many cases they are not. But our primary goal is to understand the physics.

General Problem-Solving Strategy

MODEL It's impossible to treat every detail of a situation. Simplify the situation with a model that captures the essential features. For example, the object in a mechanics problem is usually represented as a particle.

VISUALIZE This is where expert problem solvers put most of their effort.

- Draw a *pictorial representation*. This helps you visualize important aspects of the physics and assess the information you are given. It starts the process of translating the problem into symbols.
- Use a *graphical representation* if it is appropriate for the problem.
- Go back and forth between these representations; they need not be done in any particular order.

SOLVE Only after modeling and visualizing are complete is it time to develop a *mathematical representation* with specific equations that must be solved. All symbols used here should have been defined in the pictorial representation.

ASSESS Is your result believable? Does it have proper units? Does it make sense?

Textbook illustrations are obviously more sophisticated than what you would draw on your own paper. To show you a figure very much like what you *should* draw, the final example of this section is in a "pencil sketch" style. We will include one or more pencil-sketch examples in nearly every chapter to illustrate exactly what a good problem solver would draw.

EXAMPLE 1.9 Launching a weather rocket

Use the first two steps of the problem-solving strategy to analyze the following problem: A small rocket, such as those used for meteorological measurements of the atmosphere, is launched vertically with an acceleration of 30 m/s^2. It runs out of fuel after 30 s. What is its maximum altitude?

MODEL We need to do some interpretation. Common sense tells us that the rocket does not stop the instant it runs out of fuel. Instead, it continues upward, while slowing, until it reaches its maximum altitude. This second half of the motion, after running out of fuel, is like the ball that was tossed upward in the first half of Example 1.6. Because the problem does not ask about the rocket's descent, we conclude that the problem ends at the point of maximum altitude. We'll represent the rocket as a particle.

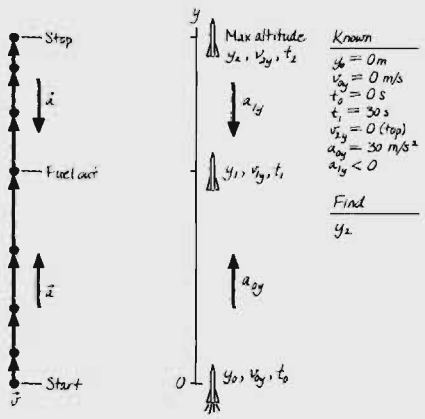
Continued

VISUALIZE FIGURE 1.24 shows the pictorial representation in pencil-sketch style. The rocket is speeding up during the first half of the motion, so $\vec{a}_0$ points upward, in the positive y-direction. Thus the initial acceleration is $a_{0y} = 30\ \text{m/s}^2$.During the second half, as the rocket slows, $\vec{a}_1$ points downward. Thus a_{1y} is a negative number.

This information is included with the known information. Although the velocity v_{2y} wasn't given in the problem statement, we know it must be zero at the very top of the trajectory. Last, we have identified y_2 as the desired unknown. This, of course, is not the only unknown in the problem, but it is the one we are specifically asked to find.

ASSESS If you've had a previous physics class, you may be tempted to assign a_{1y} the value $-9.8\ \text{m/s}^2$, the free-fall acceleration. However, that would be true only if there is no air resistance on the rocket. We will need to consider the *forces* acting on the rocket during the second half of its motion before we can determine a value for a_{1y}. For now, all that we can safely conclude is that a_{1y} is negative.

FIGURE 1.24 Pictorial representation for the rocket.



Our task in this section is not to *solve* problems—all that in due time—but to focus on what is happening in a problem. In other words, to make the translation from words to symbols in preparation for subsequent mathematical analysis. Modeling and the pictorial representation will be our most important tools.

1.8 Units and Significant Figures

Science is based upon experimental measurements, and measurements require *units*. The system of units used in science is called *le Système Internationale d'Unités*. These are commonly referred to as **SI units.** Older books often referred to *mks units*, which stands for "meter-kilogram-second," or *cgs units*, which is "centimeter-gram-second." For practical purposes, SI units are the same as mks units. In casual speaking we often refer to *metric units*, although this could mean either mks or cgs units.

TABLE 1.2 The basic SI units

Quantity	Unit	Abbreviation
time	second	s
length	meter	m
mass	kilogram	kg

All of the quantities needed to understand motion can be expressed in terms of the three basic SI units shown in Table 1.2. Other quantities can be expressed as a combination of these basic units. Velocity, expressed in meters per second or m/s, is a ratio of the length unit to the time unit.

Time

An atomic clock at the National Institute of Standards and Technology is the primary standard of time.

The standard of time prior to 1960 was based on the *mean solar day*. As time-keeping accuracy and astronomical observations improved, it became apparent that the earth's rotation is not perfectly steady. Meanwhile, physicists had been developing a device called an *atomic clock*. This instrument is able to measure, with incredibly high precision, the frequency of radio waves absorbed by atoms as they move between two closely spaced energy levels. This frequency can be reproduced with great accuracy at many laboratories around the world. Consequently, the SI unit of time—the second—was redefined in 1967 as follows:

> One *second* is the time required for 9,192,631,770 oscillations of the radio wave absorbed by the cesium-133 atom. The abbreviation for second is the letter s.

Several radio stations around the world broadcast a signal whose frequency is linked directly to the atomic clocks. This signal is the time standard, and any time-measuring equipment you use was calibrated from this time standard.

Length

The SI unit of length—the meter—also has a long and interesting history. It was originally defined as one ten-millionth of the distance from the North Pole to the equator along a line passing through Paris. There are obvious practical difficulties with implementing this definition, and it was later abandoned in favor of the distance between two scratches on a platinum-iridium bar stored in a special vault in Paris. The present definition, agreed to in 1983, is as follows:

> One *meter* is the distance traveled by light in vacuum during 1/299,792,458 of a second. The abbreviation for meter is the letter m.

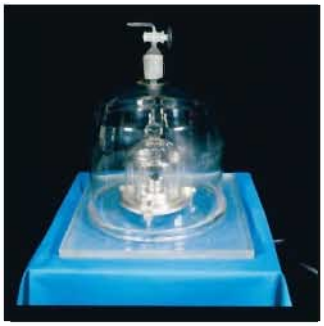
This is equivalent to defining the speed of light to be exactly 299,792,458 m/s. Laser technology is used in various national laboratories to implement this definition and to calibrate secondary standards that are easier to use. These standards ultimately make their way to your ruler or to a meter stick. It is worth keeping in mind that any measuring device you use is only as accurate as the care with which it was calibrated.

Mass

The original unit of mass, the gram, was defined as the mass of 1 cubic centimeter of water. That is why you know the density of water as 1 g/cm^3. This definition proved to be impractical when scientists needed to make very accurate measurements. The SI unit of mass—the kilogram—was redefined in 1889 as:

> One *kilogram* is the mass of the international standard kilogram, a polished platinum-iridium cylinder stored in Paris. The abbreviation for kilogram is the symbol kg.

The kilogram is the only SI unit still defined by a manufactured object. Despite the prefix *kilo*, it is the kilogram, not the gram, that is the proper SI unit.



By international agreement, this metal cylinder, stored in Paris, is the definition of the kilogram.

Using Prefixes

We will have many occasions to use lengths, times, and masses that are either much less or much greater than the standards of 1 meter, 1 second, and 1 kilogram. We will do so by using *prefixes* to denote various powers of 10. Table 1.3 lists the common prefixes that will be used frequently throughout this book. Memorize it! Few things in science are learned by rote memory, but this list is one of them. A more extensive list of prefixes is shown inside the cover of the book.

Although prefixes make it easier to talk about quantities, the proper SI units are meters, seconds, and kilograms. Quantities given with prefixed units must be converted to SI units before any calculations are done. Unit conversions are best done at the very beginning of a problem, as part of the pictorial representation.

TABLE 1.3 Common prefixes

Prefix	Power of 10	Abbreviation
mega-	10^6	M
kilo-	10^3	k
centi-	10^{-2}	c
milli-	10^{-3}	m
micro-	10^{-6}	μ
nano-	10^{-9}	n

Unit Conversions

Although SI units are our standard, we cannot entirely forget that the United States still uses English units. Many engineering calculations are done in English units. And even after repeated exposure to metric units in classes, most of us "think" in the English units we grew up with. Thus it remains important to be able to convert back and forth between SI units and English units. Table 1.4 shows several frequently used conversions, and these are worth memorizing if you do not already know them. While the English system was originally based on the length of the king's foot, it is interesting to note that today the conversion 1 in = 2.54 cm is the *definition* of the inch. In other words, the English system for lengths is now based on the meter!

TABLE 1.4 Useful unit conversions

1 in = 2.54 cm
1 mi = 1.609 km
1 mph = 0.447 m/s
1 m = 39.37 in
1 km = 0.621 mi
1 m/s = 2.24 mph

There are various techniques for doing unit conversions. One effective method is to write the conversion factor as a ratio equal to one. For example, using information in Tables 1.3 and 1.4,

$$\frac{10^{-6}\text{ m}}{1\ \mu\text{m}} = 1 \qquad \text{and} \qquad \frac{2.54\text{ cm}}{1\text{ in}} = 1$$

Because multiplying any expression by 1 does not change its value, these ratios are easily used for conversions. To convert 3.5 μm to meters we would compute

$$3.5\ \mu\text{m} \times \frac{10^{-6}\text{ m}}{1\ \mu\text{m}} = 3.5 \times 10^{-6}\text{ m}$$

Similarly, the conversion of 2 feet to meters would be

$$2.00\text{ ft} \times \frac{12\text{ in}}{1\text{ ft}} \times \frac{2.54\text{ cm}}{1\text{ in}} \times \frac{10^{-2}\text{ m}}{1\text{ cm}} = 0.610\text{ m}$$

Notice how units in the numerator and in the denominator cancel until just the desired units remain at the end. You can continue this process of multiplying by 1 as many times as necessary to complete all the conversions.

Assessment

As we get further into problem solving, we will need to decide whether or not the answer to a problem "makes sense." To determine this, at least until you have more experience with SI units, you may need to convert from SI units back to the English units in which you think. But this conversion does not need to be very accurate. For example, if you are working a problem about automobile speeds and reach an answer of 35 m/s, all you really want to know is whether or not this is a realistic speed for a car. That requires a "quick and dirty" conversion, not a conversion of great accuracy.

Table 1.5 shows several approximate conversion factors that can be used to assess the answer to a problem. Using 1 m/s $\approx$ 2 mph, you find that 35 m/s is roughly 70 mph, a reasonable speed for a car. But an answer of 350 m/s, which you might get after making a calculation error, would be an unreasonable 700 mph. Practice with these will allow you to develop intuition for metric units.

TABLE 1.5 Approximate conversion factors

1 cm $\approx \frac{1}{2}$ in
10 cm $\approx$ 4 in
1 m $\approx$ 1 yard
1 m $\approx$ 3 feet
1 km $\approx$ 0.6 mile
1 m/s $\approx$ 2 mph

NOTE ▶ These approximate conversion factors are accurate only to one significant figure. This is sufficient to assess the answer to a problem, but do *not* use the conversion factors from Table 1.5 for converting English units to SI units at the start of a problem. Use Table 1.4. ◀

Significant Figures

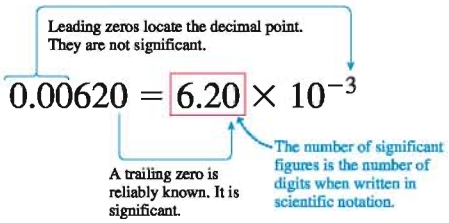
It is necessary to say a few words about a perennial source of difficulty: significant figures. Mathematics is a subject where numbers and relationships can be as precise as desired, but physics deals with a real world of ambiguity. It is important in science and engineering to state clearly what you know about a situation—no less and, especially, no more. Numbers provide one way to specify your knowledge.

If you report that a length has a value of 6.2 m, the implication is that the actual value falls between 6.15 m and 6.25 m and thus rounds to 6.2 m. If that is the case, then reporting a value of simply 6 m is saying less than you know; you are withholding information. On the other hand, to report the number as 6.213 m is wrong. Any person reviewing your work—perhaps a client who hired you—would interpret the number 6.213 m as meaning that the actual length falls between 6.2125 m and 6.2135 m, thus rounding to 6.213 m. In this case, you are claiming to have knowledge and information that you do not really possess.

The way to state your knowledge precisely is through the proper use of **significant figures.** You can think of a significant figure as being a digit that is reliably known. A

number such as 6.2 m has *two* significant figures because the next decimal place—the one-hundredths—is not reliably known. As FIGURE 1.25 shows, the best way to determine how many significant figures a number has is to write it in scientific notation.

FIGURE 1.25 Determining significant figures.



- The number of significant figures ≠ the number of decimal places.
- Changing units shifts the decimal point but does not change the number of significant figures.

Calculations with numbers follow the "weakest link" rule. The saying, which you probably know, is that "a chain is only as strong as its weakest link." If nine out of ten links in a chain can support a 1000 pound weight, that strength is meaningless if the tenth link can support only 200 pounds. Nine out of the ten numbers used in a calculation might be known with a precision of 0.01%; but if the tenth number is poorly known, with a precision of only 10%, then the result of the calculation cannot possibly be more precise than 10%. The weak link rules!

TACTICS BOX 1.6 Using significant figures MP

1. When multiplying or dividing several numbers, or taking roots, the number of significant figures in the answer should match the number of significant figures of the *least* precisely known number used in the calculation.
2. When adding or subtracting several numbers, the number of decimal places in the answer should match the *smallest* number of decimal places of any number used in the calculation.
3. It is acceptable to keep one or two extra digits during intermediate steps of a calculation, as long as the final answer is reported with the proper number of significant figures. The goal is to minimize round-off errors in the calculation. But only one or two extra digits, not the seven or eight shown in your calculator display.

Exercises 38–39

EXAMPLE 1.10 Using significant figures

An object consists of two pieces. The mass of one piece has been measured to be 6.47 kg. The volume of the second piece, which is made of aluminum, has been measured to be $4.44 \times 10^{-4}\ \text{m}^3$. A handbook lists the density of aluminum as $2.7 \times 10^3\ \text{kg/m}^3$. What is the total mass of the object?

SOLVE First, calculate the mass of the second piece:

$$m = (4.44 \times 10^{-4}\ \text{m}^3)(2.7 \times 10^3\ \text{kg/m}^3)$$
$$= 1.199\ \text{kg} = 1.2\ \text{kg}$$

The number of significant figures of a product must match that of the *least* precisely known number, which is the two-significant-figure density of aluminum. Now add the two masses:

$$\begin{array}{r} 6.47\ \text{kg} \\ +\ 1.2\ \text{kg} \\ \hline 7.7\ \text{kg} \end{array}$$

The sum is 7.67 kg, but the hundredths place is not reliable because the second mass has no reliable information about this digit. Thus we must round to the one decimal place of the 1.2 kg. The best we can say, with reliability, is that the total mass is 7.7 kg.

Some quantities can be measured very precisely—three or more significant figures. Others are inherently much less precise—only two significant figures. Examples and problems in this textbook will normally provide data to either two or three significant figures, as is appropriate to the situation. **The appropriate number of significant figures for the answer is determined by the data provided.**

NOTE ▶ Be careful! Many calculators have a default setting that shows two decimal places, such as 5.23. This is dangerous. If you need to calculate 5.23/58.5, your calculator will show 0.09 and it is all too easy to write that down as an answer. By doing so, you have reduced a calculation of two numbers having three significant figures to an answer with only one significant figure. The proper result of this division is 0.0894 or 8.94×10^{-2}. You will avoid this error if you keep your calculator set to display numbers in *scientific notation* with two decimal places. ◀

Proper use of significant figures is part of the "culture" of science and engineering. We will frequently emphasize these "cultural issues" because you must learn to speak the same language as the natives if you wish to communicate effectively. Most students know the rules of significant figures, having learned them in high school, but many fail to apply them. It is important to understand the reasons for significant figures and to get in the habit of using them properly.

TABLE 1.6 Some approximate lengths

	Length (m)
Circumference of the earth	4×10^7
New York to Los Angeles	5×10^6
Distance you can drive in 1 hour	1×10^5
Altitude of jet planes	1×10^4
Distance across a college campus	1000
Length of a football field	100
Length of a classroom	10
Length of your arm	1
Width of a textbook	0.1
Length of your little fingernail	0.01
Diameter of a pencil lead	1×10^{-3}
Thickness of a sheet of paper	1×10^{-4}
Diameter of a dust particle	1×10^{-5}

TABLE 1.7 Some approximate masses

	Mass (kg)
Large airliner	1×10^5
Small car	1000
Large human	100
Medium-size dog	10
Science textbook	1
Apple	0.1
Pencil	0.01
Raisin	1×10^{-3}
Fly	1×10^{-4}

Orders of Magnitude and Estimating

Precise calculations are appropriate when we have precise data, but there are many times when a very rough estimate is sufficient. Suppose you see a rock fall off a cliff and would like to know how fast it was going when it hit the ground. By doing a mental comparison with the speeds of familiar objects, such as cars and bicycles, you might judge that the rock was traveling at "about" 20 mph.

This is a one-significant-figure estimate. With some luck, you can distinguish 20 mph from either 10 mph or 30 mph, but you certainly cannot distinguish 20 mph from 21 mph. A one-significant-figure estimate or calculation, such as this, is called an **order-of-magnitude estimate.** An order-of-magnitude estimate is indicated by the symbol $\sim$, which indicates even less precision than the "approximately equal" symbol $\approx$. You would say that the speed of the rock is $v \sim 20$ mph.

A useful skill is to make reliable estimates on the basis of known information, simple reasoning, and common sense. This is a skill that is acquired by practice. Most chapters in this book will have homework problems that ask you to make order-of-magnitude estimates. The following example is a typical estimation problem.

Tables 1.6 and 1.7 have information that will be useful for doing estimates.

EXAMPLE 1.11 Estimating a sprinter's speed

Estimate the speed with which an Olympic sprinter crosses the finish line of the 100 m dash.

SOLVE We do need one piece of information, but it is a widely known piece of sports trivia. That is, world-class sprinters run the 100 m dash in about 10 s. Their *average* speed is $v_{avg} \approx$ (100 m)/(10 s) $\approx$ 10 m/s. But that's only average. They go slower than average at the beginning, and they cross the finish line at a speed faster than average. How much faster? Twice as fast, 20 m/s, would be $\approx$40 mph. Sprinters don't seem like they're running as fast as a 40 mph car, so this probably is too fast. Let's *estimate* that their final speed is 50% faster than the average. Thus they cross the finish line at $v \sim 15$ m/s.

STOP TO THINK 1.5 Rank in order, from the most to the least, the number of significant figures in the following numbers. For example, if b has more than c, c has the same number as a, and a has more than d, you could give your answer as b > c = a > d.

a. 82 b. 0.0052 c. 0.430 d. 4.321×10^{-10}

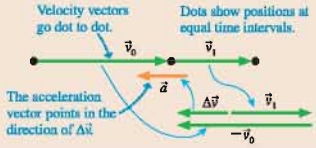
SUMMARY

The goal of Chapter 1 has been to introduce the fundamental concepts of motion.

General Strategy

Motion Diagrams

- Help visualize motion.
- Provide a tool for finding acceleration vectors.



▶ These are the average velocity and the average acceleration vectors.

Problem Solving

MODEL Make simplifying assumptions.

VISUALIZE Use:

- **Pictorial representation**
- **Graphical representation**

SOLVE Use a **mathematical representation** to find numerical answers.

ASSESS Does the answer have the proper units? Does it make sense?

Important Concepts

The **particle model** represents a moving object as if all its mass were concentrated at a single point.

Position locates an object with respect to a chosen coordinate system. Change in position is called displacement.

Velocity is the rate of change of the position vector $\vec{r}$.

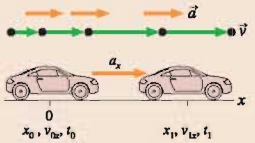
Acceleration is the rate of change of the velocity vector $\vec{v}$.

An object has an acceleration if it

- Changes speed and/or
- Changes direction.

Pictorial Representation

1. Draw a motion diagram.
2. Establish coordinates.
3. Sketch the situation.
4. Define symbols.
5. List knowns.
6. Identify desired unknown.



Known

$x_0 = v_{0x} = t_0 = 0$

$a_x = 2.0\ \text{m/s}^2$ $t_1 = 2.0\ \text{s}$

Find

x_1

Applications

For **motion along a line:**

- Speeding up: $\vec{v}$ and $\vec{a}$ point in the same direction, v_x and a_x have the same sign.
- Slowing down: $\vec{v}$ and $\vec{a}$ point in opposite directions, v_x and a_x have opposite signs.
- Constant speed: $\vec{a} = \vec{0}$, $a_x = 0$.

Acceleration a_x is positive if $\vec{a}$ points right, negative if $\vec{a}$ points left. The sign of a_x does *not* imply speeding up or slowing down.

Significant figures are reliably known digits. The number of significant figures for:

- **Multiplication, division, powers** is set by the value with the fewest significant figures.
- **Addition, subtraction** is set by the value with the smallest number of decimal places.

The appropriate number of significant figures in a calculation is determined by the data provided.

Terms and Notation

motion
trajectory
motion diagram
operational definition
translational motion
particle
particle model
position vector, $\vec{r}$
scalar quantity
vector quantity
displacement, $\Delta\vec{r}$
zero vector, $\vec{0}$
time interval, Δt
average speed
average velocity, $\vec{v}$
average acceleration, $\vec{a}$
position-versus-time graph
pictorial representation
representation of knowledge
SI units
significant figures
order-of-magnitude estimate

For homework assigned on MasteringPhysics, go to www.masteringphysics.com

Problem difficulty is labeled as | (straightforward) to ||| (challenging).

CONCEPTUAL QUESTIONS

1. How many significant figures does each of the following numbers have?
   a. 6.21 b. 62.1 c. 0.620 d. 0.062
2. How many significant figures does each of the following numbers have?
   a. 6200 b. 0.006200 c. 1.0621 d. 6.21×10^3
3. Is the particle in FIGURE Q1.3 speeding up? Slowing down? Or can you tell? Explain.

FIGURE Q1.3

4. Does the object represented in FIGURE Q1.4 have positive or negative value of a_x? Explain.
5. Does the object represented in FIGURE Q1.5 have a positive or negative value of a_y? Explain.

FIGURE Q1.4

FIGURE Q1.5

6. Determine the signs (positive or negative) of the position, velocity, and acceleration for the particle in FIGURE Q1.6.

FIGURE Q1.6

7. Determine the signs (positive or negative) of the position, velocity, and acceleration for the particle in FIGURE Q1.7.

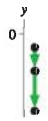


FIGURE Q1.7

FIGURE Q1.8

8. Determine the signs (positive or negative) of the position, velocity, and acceleration for the particle in FIGURE Q1.8.

EXERCISES AND PROBLEMS

Exercises

Section 1.1 Motion Diagrams

1. | A car skids to a halt to avoid hitting an object in the road. Draw a basic motion diagram, using the images from the movie, from the time the skid begins until the car is stopped.
2. | You drop a soccer ball from your third-story balcony. Draw a basic motion diagram, using the images from the movie, from the time you release the ball until it touches the ground.
3. | Two bank robbers are driving at a steady speed in their getaway car until they see the police. Then they start to speed up. Draw a basic motion diagram of the getaway car, using images from the movie, from 1 min before the robbers see the police until 1 min afterward.

Section 1.2 The Particle Model

4. | a. Write a paragraph describing the particle model. What is it, and why is it important?
   b. Give two examples of situations, different from those described in the text, for which the particle model is appropriate.
   c. Give an example of a situation, different from those described in the text, for which it would be inappropriate.

Section 1.3 Position and Time

Section 1.4 Velocity

5. | a. What is an *operational definition?*
   b. Give operational definitions of displacement and velocity. Your definition should be given mostly in words and pictures, with a minimum of symbols or mathematics.
6. | A softball player hits the ball and starts running toward first base. Draw a motion diagram, using the particle model, showing her position and her average velocity vectors during the first few seconds of her run.
7. | A softball player slides into second base. Draw a motion diagram, using the particle model, showing his position and his average velocity vectors from the time he begins to slide until he reaches the base.

Section 1.5 Linear Acceleration

8. | Give an operational definition of acceleration. Your definition should be given mostly in words and pictures, with a minimum of symbols or mathematics.
9. | a. Find the average acceleration vector at point 1 of the three-point motion diagram shown in FIGURE EX1.9.
   b. Is the object's average speed between points 1 and 2 greater than, less than, or equal to its average speed between points 0 and 1? Explain how you can tell.

FIGURE EX1.9

FIGURE EX1.10

10. | a. Find the average acceleration vector at point 1 of the three-point motion diagram shown in FIGURE EX1.10.
    b. Is the object's average speed between points 1 and 2 greater than, less than, or equal to its average speed between points 0 and 1? Explain how you can tell.
11. | FIGURE EX1.11 shows two dots of a motion diagram and vector $\vec{v}_1$. Copy this figure and add vector $\vec{v}_2$ and dot 3 if the acceleration vector $\vec{a}$ at dot 2 (a) points up and (b) points down.

FIGURE EX1.11

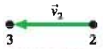


FIGURE EX1.12

12. | FIGURE EX1.12 shows two dots of a motion diagram and vector $\vec{v}_2$. Copy this figure and add vector $\vec{v}_1$ and dot 1 if the acceleration vector $\vec{a}$ at dot 2 (a) points to the right and (b) points to the left.
13. | A car travels to the left at a steady speed for a few seconds, then brakes for a stop sign. Draw a complete motion diagram of the car.
14. | A child is sledding on a smooth, level patch of snow. She encounters a rocky patch and slows to a stop. Draw a complete motion diagram of the child and her sled.
15. | A roof tile falls straight down from a two-story building. It lands in a swimming pool and settles gently to the bottom. Draw a complete motion diagram of the tile.
16. | Your roommate drops a tennis ball from a third-story balcony. It hits the sidewalk and bounces as high as the second story. Draw a complete motion diagram of the tennis ball from the time it is released until it reaches the maximum height on its bounce. Be sure to determine and show the acceleration at the lowest point.
17. | A toy car rolls down a ramp, then across a smooth, horizontal floor. Draw a complete motion diagram of the toy car.

Section 1.6 Motion in One Dimension

18. | FIGURE EX1.18 shows the motion diagram of a drag racer. The camera took one frame every 2 s.

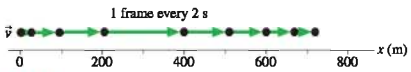


FIGURE EX1.18

   a. Measure the x-value of the racer at each dot. List your data in a table similar to Table 1.1, showing each position and the time at which it occurred.
   b. Make a position-versus-time graph for the drag racer. Because you have data only at certain instants, your graph should consist of dots that are not connected together.
19. | Write a short description of the motion of a real object for which FIGURE EX 1.19 would be a realistic position-versus-time graph.

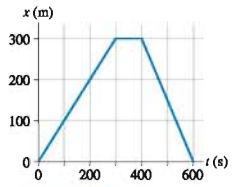


FIGURE EX1.19

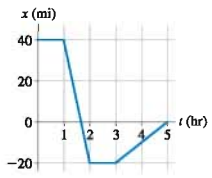


FIGURE EX1.20

20. | Write a short description of the motion of a real object for which FIGURE EX 1.20 would be a realistic position-versus-time graph.

Section 1.7 Solving Problems in Physics

21. | Draw a pictorial representation for the following problem. Do *not* solve the problem. The light turns green, and a bicyclist starts forward with an acceleration of 1.5 m/s^2. How far must she travel to reach a speed of 7.5 m/s?
22. | Draw a pictorial representation for the following problem. Do *not* solve the problem. What acceleration does a rocket need to reach a speed of 200 m/s at a height of 1.0 km?

Section 1.8 Units and Significant Figures

23. | Convert the following to SI units:
   a. 9.12 μs b. 3.42 km
   c. 44 cm/ms d. 80 km/hour
24. | Convert the following to SI units:
   a. 8.0 in b. 66 ft/s
   c. 60 mph d. 14 in^2
25. | Convert the following to SI units:
   a. 1 hour b. 1 day
   c. 1 year d. 32 ft/s^2
26. | Using the approximate conversion factors in Table 1.5, convert the following to SI units *without* using your calculator.
   a. 20 ft b. 60 mi
   c. 60 mph d. 8 in
27. | Using the approximate conversion factors in Table 1.5, convert the following SI units to English units *without* using your calculator.
   a. 30 cm b. 25 m/s
   c. 5 km d. 0.5 cm
28. | Compute the following numbers, applying the significant figure rule adopted in this textbook.
   a. 33.3×25.4 b. $33.3 - 25.4$
   c. $\sqrt{33.3}$ d. $333.3 \div 25.4$
29. | Compute the following numbers, applying the significant figure rule adopted in this textbook.
   a. 33.3^2 b. 33.3×45.1
   c. $\sqrt{22.2} - 1.2$ d. 44.4^{-1}

30. I Estimate (don't measure!) the length of a typical car. Give your answer in both feet and meters. Briefly describe how you arrived at this estimate.
31. I Estimate the height of a telephone pole. Give your answer in both feet and meters. Briefly describe how you arrived at this estimate.
32. I Estimate the average speed with which you go from home to campus via whatever mode of transportation you use most commonly. Give your answer in both mph and m/s. Briefly describe how you arrived at this estimate.
33. I Estimate the average speed with which the hair on your head grows. Give your answer in both m/s and μm/hour. Briefly describe how you arrived at this estimate.

Problems

For Problems 34 through 43, draw a complete pictorial representation. Do *not* solve these problems or do any mathematics.

34. I A Porsche accelerates from a stoplight at 5.0 m/s^2 for five seconds, then coasts for three more seconds. How far has it traveled?
35. I Billy drops a watermelon from the top of a three-story building, 10 m above the sidewalk. How fast is the watermelon going when it hits?
36. I Sam is recklessly driving 60 mph in a 30 mph speed zone when he suddenly sees the police. He steps on the brakes and slows to 30 mph in three seconds, looking nonchalant as he passes the officer. How far does he travel while braking?
37. I A speed skater moving across frictionless ice at 8.0 m/s hits a 5.0-m-wide patch of rough ice. She slows steadily, then continues on at 6.0 m/s. What is her acceleration on the rough ice?
38. I You would like to stick a wet spit wad on the ceiling, so you toss it straight up with a speed of 10 m/s. How long does it take to reach the ceiling, 3.0 m above?
39. I A student standing on the ground throws a ball straight up. The ball leaves the student's hand with a speed of 15 m/s when the hand is 1.5 m above the ground. How long is the ball in the air before it hits the ground? (The student moves her hand out of the way.)
40. I A ball rolls along a smooth horizontal floor at 10 m/s, then starts up a 20° ramp. How high does it go before rolling back down?
41. I A motorist is traveling at 20 m/s. He is 60 m from a stoplight when he sees it turn yellow. His reaction time, before stepping on the brake, is 0.50 s. What steady deceleration while braking will bring him to a stop right at the light?
42. II Ice hockey star Bruce Blades is 5.0 m from the blue line and gliding toward it at a speed of 4.0 m/s. You are 20 m from the blue line, directly behind Bruce. You want to pass the puck to Bruce. With what speed should you shoot the puck down the ice so that it reaches Bruce exactly as he crosses the blue line?
43. II You are standing still as Fred runs past you with the football at a speed of 6.0 yards per second. He has only 30 yards left to go before reaching the goal line to score the winning touchdown. If you begin running at the exact instant he passes you, what acceleration must you maintain to catch him 5.0 yards in front of the goal line?

Problems 44 through 48 show a motion diagram. For each of these problems, write a one or two sentence "story" about a *real object* that has this motion diagram. Your stories should talk about people or objects by name and say what they are doing. Problems 34 through 43 are examples of motion short stories.

44. I

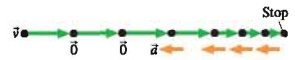


FIGURE P1.44

45. I

FIGURE P1.45

46. I

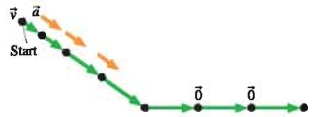


FIGURE P1.46

47. I

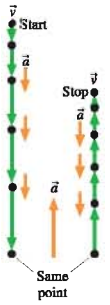


FIGURE P1.47

48. I

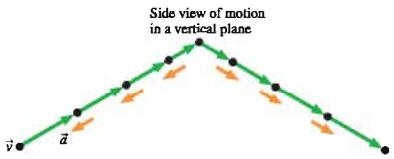


FIGURE P1.48

Problems 49 through 52 show a partial motion diagram. For each:

a. Complete the motion diagram by adding acceleration vectors.
b. Write a physics *problem* for which this is the correct motion diagram. Be imaginative! Don't forget to include enough information to make the problem complete and to state clearly what is to be found.
c. Draw a pictorial representation for your problem.

49. I

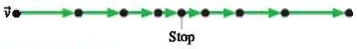


FIGURE P1.49

50. I

FIGURE P1.50

51. I

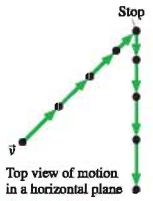


FIGURE P1.51

52. I

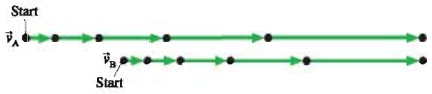


FIGURE P1.52

53. I A regulation soccer field for international play is a rectangle with a length between 100 m and 110 m and a width between 64 m and 75 m. What are the smallest and largest areas that the field could be?

54. II The quantity called *mass density* is the mass per unit volume of a substance. Express the following mass densities in SI units.
   a. Aluminum, 2.7×10^{-3} kg/cm^3
   b. Alcohol, 0.81 g/cm^3

55. I FIGURE P1.55 shows a motion diagram of a car traveling down a street. The camera took one frame every 10 s. A distance scale is provided.
   a. Measure the x-value of the car at each dot. Place your data in a table, similar to Table 1.1, showing each position and the instant of time at which it occurred.
   b. Make a position-versus-time graph for the ball. Because you have data only at certain instants of time, your graph should consist of dots that are not connected together.

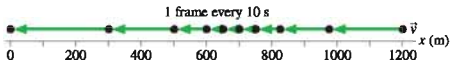


FIGURE P1.55

56. I Write a short description of a real object for which FIGURE P1.56 would be a realistic position-versus-time graph.

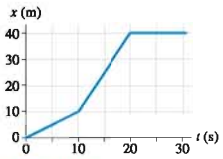


FIGURE P1.56

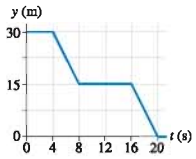


FIGURE P1.57

57. I Write a short description of a real object for which FIGURE P1.57 would be a realistic position-versus-time graph.

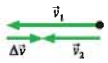
STOP TO THINK ANSWERS

Stop to Think 1.1: B. The images of B are farther apart, so it travels a larger distance than does A during the same intervals of time.

Stop to Think 1.2: a. Dropped ball. **b.** Dust particle. **c.** Descending rocket.

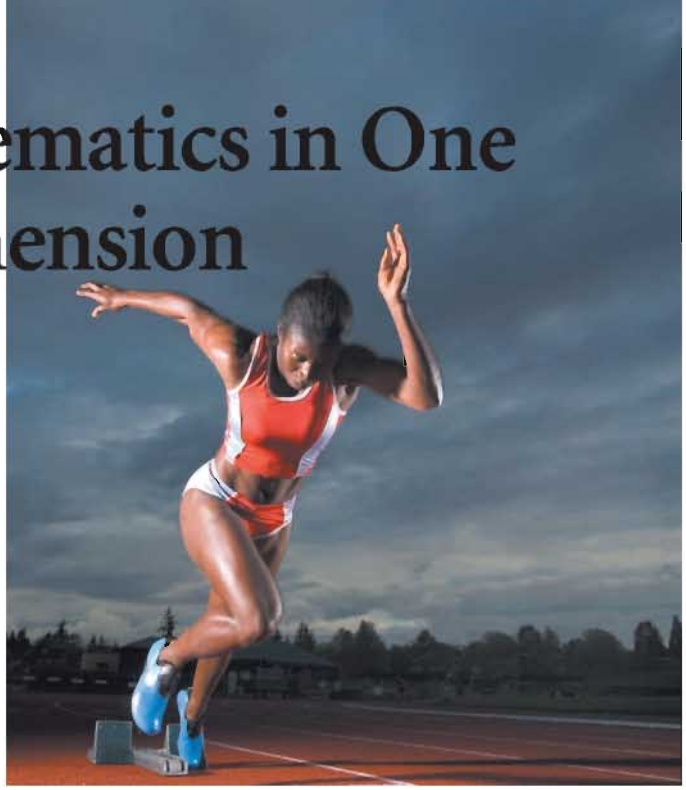
Stop to Think 1.3: e. The average velocity vector is found by connecting one dot in the motion diagram to the next.

Stop to Think 1.4: b. $\vec{v}_2 = \vec{v}_1 + \Delta\vec{v}$, and $\Delta\vec{v}$ points in the direction of $\vec{a}$.



Stop to Think 1.5: d > c > b = a.

2 Kinematics in One Dimension



World-class sprinters have a tremendous acceleration at the start of a race.

▶ Looking Ahead

The goal of Chapter 2 is to learn how to solve problems about motion in a straight line. In this chapter you will learn to:

- Understand the mathematics of position, velocity, and acceleration for motion along a straight line.
- Use a graphical representation of motion.
- Use an explicit problem-solving strategy for kinematics problems.
- Understand free-fall motion and motion along inclined planes.

◀ Looking Back

Each chapter in this textbook builds on ideas and techniques from previous chapters. The Looking Back feature calls your attention to specific sections that are of major significance to the present chapter. A brief review of these sections will improve your study of this chapter. Please review:

- Sections 1.4–1.5 Velocity and acceleration.
- Section 1.6 Motion in one dimension.
- Section 1.7 Problem solving in physics.

A race, whether between runners, bicyclists, or drag racers, exemplifies the idea of motion. Today, we use electronic stopwatches, video recorders, and other sophisticated instruments to analyze motion, but it hasn't always been so. Galileo, who in the early 1600s was the first scientist to study motion experimentally, used his pulse to measure time!

Galileo made a useful distinction between the *cause* of motion and the *description* of motion. **Kinematics** is the modern name for the mathematical description of motion without regard to causes. The term comes from the Greek word *kinema,* meaning "movement." You know this word through its English variation *cinema*—motion pictures! In this chapter on kinematics we'll develop the mathematical tools for describing motion. Then, in Chapter 5, we'll turn our attention to the *cause* of motion.

We will begin our study of kinematics with motion in one dimension; that is, motion along a straight line. Runners, drag racers, and skiers are just a few examples of motion in one dimension. The kinematics of two-dimensional motion—projectile motion and circular motion—will be considered in Chapter 4.

2.1 Uniform Motion

If you drive your car at a perfectly steady 60 miles per hour (mph), you will cover 60 mi during the first hour, another 60 mi during the second hour, yet another 60 mi during the third hour, and so on. This is an example of what we call *uniform motion.* In this case, 60 mi is not your position, but rather the *change* in your position during each hour; that is, your displacement Δx. Similarly, 1 hour is a time interval Δt rather than a specific instant of time. This suggests the following definition: **Straight-line motion in which equal displacements occur during *any* successive equal-time intervals is called uniform motion.**

The qualifier "any" is important. If during each hour you drive 120 mph for 30 minutes and stop for 30 minutes, you will cover 60 mi during each successive 1 hour interval. But you would *not* have equal displacements during successive 30 minute intervals, so this motion is not uniform. Your constant 60 mph driving is uniform motion because you will find equal displacements no matter how you choose your successive time intervals.

FIGURE 2.1 shows how uniform motion appears in motion diagrams and position-versus-time graphs. Notice that the position-versus-time graph for uniform motion is a straight line. This follows from the requirement that all Δx corresponding to the same Δt be equal. In fact, an alternative definition of uniform motion is: **An object's motion is uniform if and only if its position-versus-time graph is a straight line.**

The slope of a straight-line graph is defined as "rise over run." Because position is graphed on the vertical axis, the "rise" of a position-versus-time graph is the object's displacement Δx. The "run" is the time interval Δt. Consequently, the slope is $\Delta x/\Delta t$. The slope of a straight-line graph is constant, so an object in uniform motion has the *same* value of $\Delta x/\Delta t$ during *any* time interval Δt.

Chapter 1 defined the *average velocity* as $\Delta\vec{r}/\Delta t$. For one-dimensional motion this is simply

$$v_{avg} \equiv \frac{\Delta x}{\Delta t} \text{ or } \frac{\Delta y}{\Delta t} = \text{slope of the position-versus-time graph} \qquad (2.1)$$

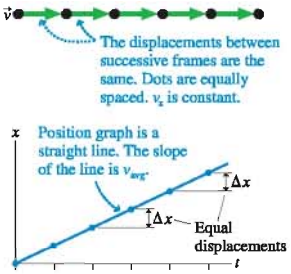
That is, **the average velocity is the slope of the position-versus-time graph.** Velocity has units of "length per time," such as "miles per hour." The SI units of velocity are meters per second, abbreviated m/s.

NOTE ▶ The symbol $\equiv$ in Equation 2.1 stands for "is defined as" or "is equivalent to." This is a stronger statement than the two sides simply being equal. ◀

Equation 2.1 allows us to associate the slope of the position-versus-time graph, a *geometrical* quantity, with the *physical* quantity that we call the average velocity v_{avg}. This is an extremely important idea. In the case of uniform motion, where the slope $\Delta x/\Delta t$ is the same at all times, it appears that the average velocity is constant and unchanging. Consequently, a final definition of uniform motion is: **An object's motion is uniform if and only if its velocity v_x or v_y is constant and unchanging.** There's no real need to specify "average" for a velocity that doesn't change, so we will drop the subscript and refer to the average velocity as v_x or v_y.

Riding steadily over level ground is a good example of uniform motion.

FIGURE 2.1 Motion diagram and position graph for uniform motion.



EXAMPLE 2.1 Skating with constant velocity

The position-versus-time graph of FIGURE 2.2 represents the motion of two students on roller blades. Determine their velocities and describe their motion.

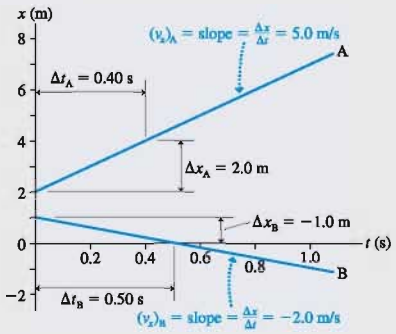
MODEL Represent the two students as particles.

VISUALIZE Figure 2.2 is a graphical representation of the students' motion. Both graphs are straight lines, telling us that both skaters are moving uniformly with constant velocities.

SOLVE We can determine the students' velocities by measuring the slopes of the graphs. Skater A undergoes a displacement $\Delta x_A = 2.0$ m during the time interval $\Delta t_A = 0.40$ s. Thus his velocity is

$$(v_x)_A = \frac{\Delta x_A}{\Delta t_A} = \frac{2.0 \text{ m}}{0.40 \text{ s}}$$
$$= 5.0 \text{ m/s}$$

FIGURE 2.2 Graphical representations of two students on roller blades.



Continued

We need to be more careful with skater B. Although he moves a distance of 1.0 m in 0.50 s, his *displacement* Δx has a very precise definition:

$$\Delta x_B = x_{\text{at } 0.5\text{ s}} - x_{\text{at } 0.0\text{ s}} = 0.0\text{ m} - 1.0\text{ m} = -1.0\text{ m}$$

Careful attention to the signs is very important! This leads to

$$(v_x)_B = \frac{\Delta x_B}{\Delta t_B} = \frac{-1.0\text{ m}}{0.50\text{ s}} = -2.0\text{ m/s}$$

ASSESS The minus sign indicates that skater B is moving to the left. Our interpretation of this graph is that two students on roller blades are moving with constant velocities in opposite directions. Skater A starts at $x = 2.0$ m and moves to the right with a velocity of 5.0 m/s. Skater B starts at $x = 1.0$ m and moves to the left with a velocity of -2.0 m/s. Their speeds, of ≈ 10 mph and ≈ 4 mph, are reasonable for skaters on roller blades.

Example 2.1 brought out several points that are worth emphasizing. These are summarized in Tactics Box 2.1.

TACTICS BOX 2.1 Interpreting position-versus-time graphs MP

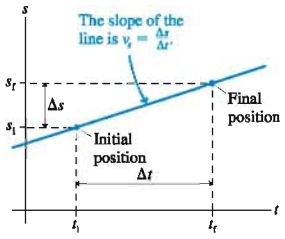
1. Steeper slopes correspond to faster speeds.
2. Negative slopes correspond to negative velocities and, hence, to motion to the left (or down).
3. The slope is a ratio of intervals, $\Delta x/\Delta t$, not a ratio of coordinates. That is, the slope is *not* simply x/t.
4. We are distinguishing between the *actual* slope and the *physically meaningful* slope. If you were to use a ruler to measure the rise and the run of the graph, you could compute the actual slope of the line as drawn on the page. That is not the slope to which we are referring when we equate the velocity with the slope of the line. Instead, we find the *physically meaningful* slope by measuring the rise and run using the scales along the axes. The "rise" Δx is some number of meters; the "run" Δt is some number of seconds. The physically meaningful rise and run include units, and the ratio of these units gives the units of the slope.

Exercises 1–3

An object's **speed** v is how fast it's going, independent of direction. This is simply $v = |v_x|$ or $v = |v_y|$, the magnitude or absolute value of the object's velocity. In Example 2.1, for example, skater B's *velocity* is -2.0 m/s but his *speed* is 2.0 m/s. Speed is a scalar quantity, not a vector.

NOTE ▶ Our mathematical analysis of motion is based on velocity, not speed. The subscript in v_x or v_y is an essential part of the notation, reminding us that, even in one dimension, the velocity is a vector. ◀

FIGURE 2.3 The velocity is found from the slope of the position-versus-time graph.



The Mathematics of Uniform Motion

We need a mathematical analysis of motion that will be valid regardless of whether an object moves along the x-axis, the y-axis, or any other straight line. Consequently, it will be convenient to write equations for a "generic axis" that we will call the s-axis. The position of an object will be represented by the symbol s and its velocity by v_s.

NOTE ▶ Equations written in terms of s are valid for any one-dimensional motion. In a specific problem, however, you should use either x or y, whichever is appropriate, rather than s. ◀

Consider an object in uniform motion along the s-axis with the linear position-versus-time graph shown in FIGURE 2.3. The object's **initial position** is s_i at time t_i. The term *initial position* refers to the starting point of our analysis or the starting point in a

problem; the object may or may not have been in motion prior to t_i. At a later time t_f, the ending point of our analysis or the ending point of a problem, the object's **final position** is s_f.

The object's velocity v_s along the s-axis can be determined by finding the slope of the graph:

$$v_s = \frac{\text{rise}}{\text{run}} = \frac{\Delta s}{\Delta t} = \frac{s_f - s_i}{t_f - t_i} \tag{2.2}$$

Equation 2.2 is easily rearranged to give

$$s_f = s_i + v_s \Delta t \qquad \text{(uniform motion)} \tag{2.3}$$

Equation 2.3 applies to any time interval Δt during which the velocity is constant.

The velocity of a uniformly moving object tells us the amount by which its position changes during each second. A particle with a velocity of 20 m/s *changes* its position by 20 m during every second of motion: by 20 m during the first second of its motion, by another 20 m during the next second, and so on. If the object starts at $s_i = 10$ m, it will be at $s = 30$ m after 1 second of motion and at $s = 50$ m after 2 seconds of motion. Thinking of velocity like this will help you develop an intuitive understanding of the connection between velocity and position.

EXAMPLE 2.2 Lunch in Cleveland?

Bob leaves home in Chicago at 9:00 A.M. and travels east at a steady 60 mph. Susan, 400 miles to the east in Pittsburgh, leaves at the same time and travels west at a steady 40 mph. Where will they meet for lunch?

MODEL Here is a problem where, for the first time, we can really put all four aspects of our problem-solving strategy into play. To begin, represent Bob and Susan as particles.

VISUALIZE FIGURE 2.4 shows the physical representation (the motion diagram) and the pictorial representation. The equal spacings of the dots in the motion diagram indicate that the motion is uniform. In evaluating the given information, we recognize that the starting time of 9:00 A.M. is not relevant to the problem. Consequently, the initial time is chosen as simply $t_0 = 0$ hr. Bob and Susan are traveling in opposite directions, hence one of the velocities must be a negative number. We have chosen a coordinate system in which Bob starts at the origin and moves to the right (east) while Susan is moving to the left (west). Thus Susan has the negative velocity. Notice how we've assigned position, velocity, and time symbols to each point in the motion. Pay special attention to how subscripts are used to distinguish different points in the problem and to distinguish Bob's symbols from Susan's.

One purpose of the pictorial representation is to establish what we need to find. Bob and Susan meet when they have the same position at the same time t_1. Thus we want to find $(x_1)_B$ at the time when $(x_1)_B = (x_1)_S$. Notice that $(x_1)_B$ and $(x_1)_S$ are Bob's and Susan's *positions*, which are equal when they meet, not the distances they have traveled.

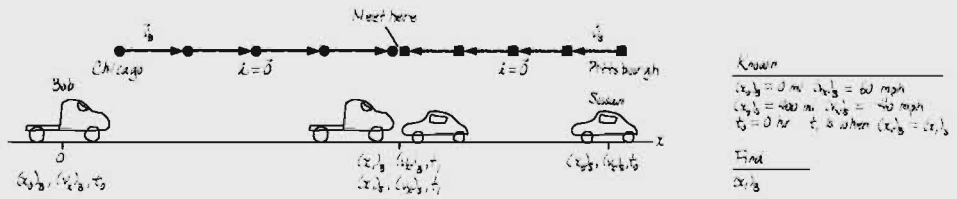
SOLVE The goal of the mathematical representation is to proceed from the pictorial representation to a mathematical solution of the problem. We can begin by using Equation 2.3 to find Bob's and Susan's positions at time t_1 when they meet:

$$(x_1)_B = (x_0)_B + (v_x)_B(t_1 - t_0) = (v_x)_B t_1$$

$$(x_1)_S = (x_0)_S + (v_x)_S(t_1 - t_0) = (x_0)_S + (v_x)_S t_1$$

Notice two things. First, we started by writing the *full* statement of Equation 2.3. Only then did we simplify by dropping those terms known to be zero. You're less likely to make accidental errors if you follow this procedure. Second, we replaced the generic symbol s with the specific horizontal-position symbol x, and we replaced the generic subscripts i and f with the specific symbols 0 and 1 that we defined in the pictorial representation. This is also good problem-solving technique.

FIGURE 2.4 Pictorial representation for Example 2.2.



Continued

The condition that Bob and Susan meet is

$$(x_1)_B = (x_1)_S$$

By equating the right-hand sides of the above equations, we get

$$(v_x)_B t_1 = (x_0)_S + (v_x)_S t_1$$

Solving for t_1, we find that they meet at time

$$t_1 = \frac{(x_0)_S}{(v_x)_B - (v_x)_S} = \frac{400 \text{ miles}}{60 \text{ mph} - (-40) \text{ mph}} = 4.0 \text{ hours}$$

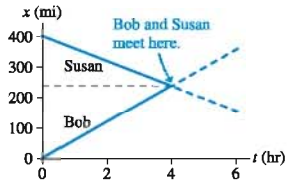
Finally, inserting this time back into the equation for $(x_1)_B$ gives

$$(x_1)_B = \left(60 \frac{\text{miles}}{\text{hour}}\right) \times (4.0 \text{ hours}) = 240 \text{ miles}$$

While this is a number, it is not yet the answer to the question. The phrase "240 miles" by itself does not say anything meaningful. Because this is the value of Bob's *position*, and Bob was driving east, the answer to the question is, "They meet 240 miles east of Chicago."

ASSESS Before stopping, we should check whether or not this answer seems reasonable. We certainly expected an answer between 0 miles and 400 miles. We also know that Bob is driving faster than Susan, so we expect that their meeting point will be *more* than halfway from Chicago to Pittsburgh. Our assessment tells us that 240 miles is a reasonable answer.

FIGURE 2.5 Position-versus-time graphs for Bob and Susan.



It is instructive to look at this example from a graphical perspective. **FIGURE 2.5** shows position-versus-time graphs for Bob and Susan. Notice the negative slope for Susan's graph, indicating her negative velocity. The point of interest is the intersection of the two lines; this is where Bob and Susan have the same position at the same time. Our method of solution, in which we equated $(x_1)_B$ and $(x_1)_S$, is really just solving the mathematical problem of finding the intersection of two lines.

STOP TO THINK 2.1 Which position-versus-time graph represents the motion shown in the motion diagram?

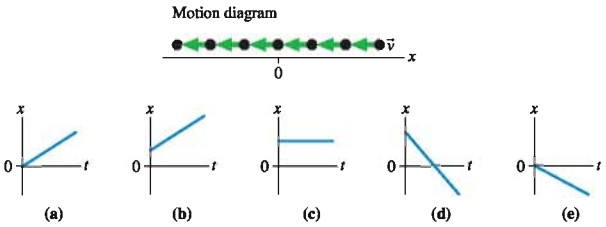
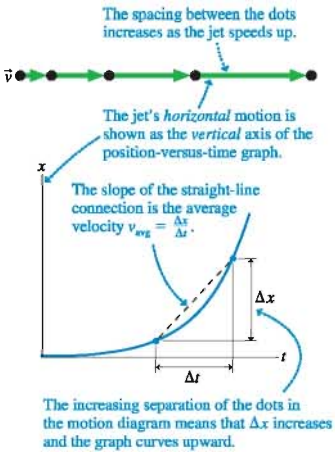


FIGURE 2.6 Motion diagram and position graph of a jet during takeoff.



2.2 Instantaneous Velocity

FIGURE 2.6 shows the motion diagram of a jet as it takes off. The increasing length of the velocity vectors tells us that the jet is speeding up, so this is *not* uniform motion. Consequently, the position-versus-time graph is *not* a straight line. The graph curves upward (increasing Δx) as the spacing between the motion-diagram dots increases. We can determine the jet's average speed v_{avg} between any two times t_i and t_f by selecting those two points on the graph, drawing the straight-line connection between them, measuring Δx and Δt, and using these to compute $v_{avg} = \Delta x/\Delta t$. Graphically, v_{avg} is simply the slope of the straight-line connection between the two points.

However, average velocity has only limited usefulness for an object whose velocity isn't constant. Suppose, for example, you drove your car in a straight line for exactly 1 hr, covering exactly 60 mi. All you can discern from this information is that your average velocity was $v_{avg} = 60$ mph. It's quite possible that you got a slow start but later sped up. If you glanced at your car's speedometer 10 min into the trip, you would have seen a speed less than 60 mph. Similar, the speedometer would have read more than 60 mph 10 min before the end of the trip.

In contrast to a velocity averaged over the entire hour, the speedometer reading tells you how fast you're going *at that instant*. We define an object's **instantaneous velocity** to be its velocity—a speed *and* a direction—at a single *instant* of time t.

Such a definition, though, raises some difficult issues. Just what does it mean to have a velocity "at an instant"? Suppose a police officer pulls you over and says, "I just clocked you going 80 miles per hour." You might respond, "But that's impossible. I've only been driving for 20 minutes, so I can't possibly have gone 80 miles." Unfortunately for you, the police officer was a physics major. He replies, "I mean that at the instant I measured your velocity, you were moving at a rate such that you *would* cover a distance of 80 miles *if* you were to continue at that velocity without change for 1 hour. That will be a $200 fine."

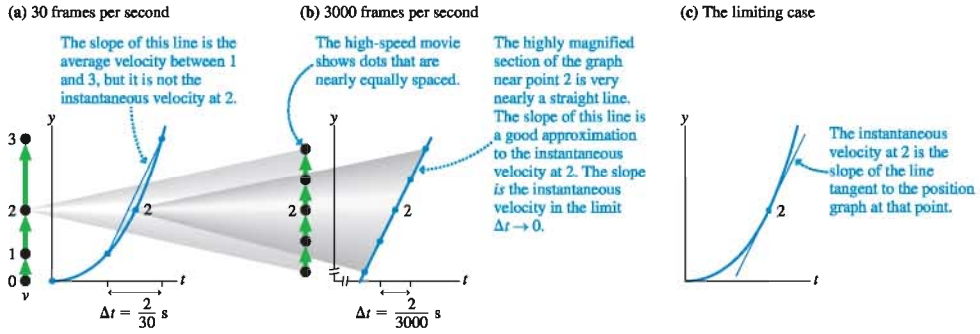
Here, again, is the idea that velocity is the *rate* at which an object changes its position. Rates tell us how quickly or how slowly things change, and that idea is conveyed by the word "per." An instantaneous velocity of 80 miles *per* hour means that the rate at which your car's position is changing—at that exact instant—is such that it would travel 80 miles in 1 hour *if* it continued at that rate without change. Whether or not it actually does travel at that velocity for another hour, or even for another millisecond, is not relevant.

The speedometer reading tells you how fast you're going *at that instant*.

Using Motion Diagrams and Graphs

Let's use motion diagrams and position graphs to analyze a rocket as it takes off. We've chosen an object that moves vertically, so that the motion diagram better matches the graph, but our conclusions will apply to motion along any straight line. FIGURE 2.7a shows a motion diagram made using a normal 30-frames-per-second movie camera. We would like to determine the *instantaneous* velocity v_{2y} at point 2. Because the rocket is accelerating, its velocity at 2 is not the same as the average velocity between 1 and 3. How can we measure v_{2y}?

FIGURE 2.7 Motion diagrams and position graphs of an accelerating rocket.



Suppose we use a high-speed camera, one that takes 3000 frames per second, to film just the segment of motion right around point 2. This "magnified" motion diagram is shown in FIGURE 2.7b. At this level of magnification, each velocity vector is *almost* the same length. Further, the greatly magnified section of the curved position graph is *almost* a straight line. That is, the motion appears very nearly uniform on this

time scale. If the rocket suddenly changed to *constant*-velocity motion at point 2, it would continue to move with a velocity given by the slope of the graph in Figure 2.7b.

The point of Figure 2.7 is that the average velocity $v_{avg} = \Delta s/\Delta t$ becomes a better and better approximation to the instantaneous velocity v_s as the time interval Δt over which the average is taken gets smaller and smaller. By magnifying the motion diagram, we are using smaller and smaller time intervals Δt. But even 3000 frames per second isn't fast enough. We need to let $\Delta t \rightarrow 0$.

We can state this idea mathematically in terms of a limit:

$$v_s \equiv \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt} \qquad \text{(instantaneous velocity)} \qquad (2.4)$$

As Δt continues to get smaller, the average velocity $v_{avg} = \Delta s/\Delta t$ reaches a constant or *limiting* value. That is, **the instantaneous velocity at time t is the average velocity during a time interval Δt, centered on t, as Δt approaches zero.** In calculus, this limit is called *the derivative of s with respect to t*, and it is denoted *ds/dt*. We'll look at derivatives in the next section.

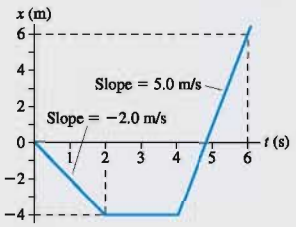
Graphically, $\Delta s/\Delta t$ is the slope of a straight line. As Δt gets smaller (i.e., more and more magnification), the straight line becomes a better and better approximation of the curve *at that one point.* In the limit $\Delta t \rightarrow 0$, the straight line is tangent to the curve. As FIGURE 2.7c shows, **the instantaneous velocity at time t is the slope of the line that is tangent to the position-versus-time graph at time t.**

EXAMPLE 2.3 Relating a velocity graph to a position graph

FIGURE 2.8 is the position-versus-time graph of a car.

a. Draw the car's velocity-versus-time graph.
b. Describe the car's motion.

FIGURE 2.8 Position-versus-time graph.



MODEL Represent the car as a particle, with a well-defined position at each instant of time.

VISUALIZE Figure 2.8 is the graphical representation.

SOLVE

a. The car's position-versus-time graph is a sequence of three straight lines. Each of these straight lines represents uniform motion at a constant velocity. We can determine the car's velocity during each interval of time by measuring the slope of the line. From $t = 0$ s to $t = 2$ s ($\Delta t = 2.0$ s) the car's displacement is $\Delta x = -4.0$ m $- 0.0$ m $= -4.0$ m. The velocity during this interval is

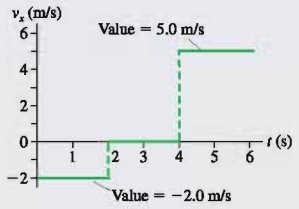
$$v_x = \frac{\Delta x}{\Delta t} = \frac{-4.0 \text{ m}}{2.0 \text{ s}} = -2.0 \text{ m/s}$$

The car's position does not change from $t = 2$ s to $t = 4$ s ($\Delta x = 0$), so $v_x = 0$. Finally, the displacement between $t = 4$ s and $t = 6$ s is $\Delta x = 10.0$ m. Thus the velocity during this interval is

$$v_x = \frac{10.0 \text{ m}}{2.0 \text{ s}} = 5.0 \text{ m/s}$$

These velocities are shown on the velocity-versus-time graph of FIGURE 2.9.

FIGURE 2.9 The corresponding velocity-versus-time graph.



b. The car backs up for 2 s at 2.0 m/s, sits at rest for 2 s, then drives forward at 5.0 m/s for at least 2 s. We can't tell from the graph what happens for $t > 6$ s.

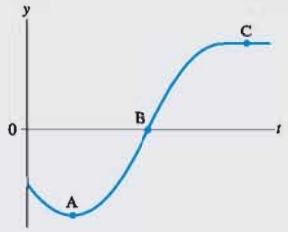
ASSESS The velocity graph and the position graph look completely different. The *value* of the velocity graph at any instant of time equals the *slope* of the position graph.

EXAMPLE 2.4 Finding velocity from position graphically

FIGURE 2.10 shows the position-versus-time graph of an elevator.

a. At which labeled point or points does the elevator have the least speed?
b. At which point or points is the elevator moving the fastest?
c. Sketch an approximate velocity-versus-time graph for the elevator.

FIGURE 2.10 Position-versus-time graph.



MODEL Represent the elevator as a particle.

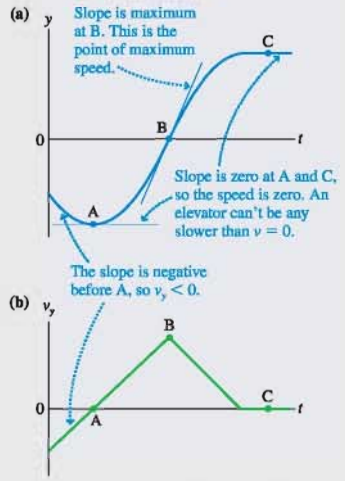
VISUALIZE Figure 2.10 is the graphical representation.

SOLVE a. **FIGURE 2.11a** shows that the elevator has the least speed—no speed at all!—at points A and C. At point A, the speed is only instantaneously zero. At point C, the elevator has actually stopped and remains at rest.

b. The elevator moves the fastest at point B.

c. Although we cannot find an exact velocity-versus-time graph, we can see that the slope, and hence v_y, is initially negative, becomes zero at point A, rises to a maximum value at point B, decreases back to zero a little before point C, then remains at zero thereafter. Thus **FIGURE 2.11b** shows, at least approximately, the elevator's velocity-versus-time graph.

FIGURE 2.11 The velocity-versus-time graph is found from the position graph.



ASSESS Once again, the shape of the velocity graph bears no resemblance to the shape of the position graph. You must transfer *slope* information from the position graph to *value* information on the velocity graph.

A Little Calculus: Derivatives

We have reached the point beyond which Galileo could not proceed because he lacked the mathematical tools. Further progress had to await a new branch of mathematics called *calculus,* invented simultaneously in England by Newton and in Germany by Leibniz. Calculus is designed to deal with instantaneous quantities. In other words, it provides us with the tools for evaluating limits such as the one in Equation 2.4.

The notation ds/dt is called *the derivative of s with respect to t*, and Equation 2.4 defines it as the limiting value of a ratio. As Figure 2.7 showed, ds/dt can be interpreted graphically as the slope of the line that is tangent to the position-versus-time graph at time t.

EXAMPLE 2.5 Finding velocity from position as a derivative

The position of a particle as a function of time is $s = 2t^2$ m, where t is in s. What is the velocity v_s as a function of time?

SOLVE We need to "take the derivative" of s:

$$v_s = \frac{ds}{dt} = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t}$$

During the time interval Δt, the particle moves from position $s_{\text{at } t}$ to the new position $s_{\text{at } t+\Delta t}$. Its displacement is

$$\begin{aligned}\Delta s &= s_{\text{at } t+\Delta t} - s_{\text{at } t} = 2(t + \Delta t)^2 - 2t^2 \\ &= 2(t^2 + 2t\Delta t + (\Delta t)^2) - 2t^2 \\ &= 4t\Delta t + 2(\Delta t)^2\end{aligned}$$

The average velocity during the time interval Δt is

$$v_{\text{avg}} = \frac{\Delta s}{\Delta t} = \frac{4t\Delta t + 2(\Delta t)^2}{\Delta t} = 4t + 2\Delta t$$

Continued

We can finish by taking the limit $\Delta t \to 0$ to find

$$v_s = \frac{ds}{dt} = \lim_{\Delta t \to 0} (4t + 2\Delta t) = 4t \text{ m/s}$$

In other words, the function for calculating the velocity at any instant of time is $v_s = 4t$ m/s, where t is in s. At $t = 3$ s, for example, the particle is located at position $s = 18$ m and its instantaneous velocity, at just that instant, is $v_s = 12$ m/s.

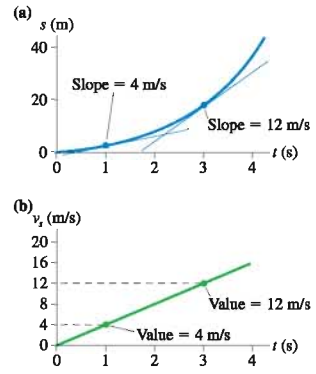


FIGURE 2.12 Position-versus-time and velocity-versus-time graphs for Example 2.5.

Let's look at this example graphically. **FIGURE 2.12a** shows the particle's position-versus-time graph $s = 2t^2$ m. **FIGURE 2.12b** then shows the velocity-versus-time graph, using the velocity function $v = 4t$ m/s that we just calculated. You see that the velocity graph is a straight line.

It is critically important to understand the relationship between these two graphs. The *value* of the velocity graph at any instant of time, which we can read directly off the vertical axis, is the *slope* of the position graph at that same time. This is illustrated at $t = 1$ s and $t = 3$ s.

Example 2.5 showed how the limit of $\Delta s/\Delta t$ can be evaluated to find a derivative, but the procedure is clearly rather tedious. It would hinder us significantly if we had to do this for every new situation. Fortunately, we need only a few basic derivatives in this text. Learn these, and you do not have to go all the way back to the definition in terms of limits.

The only functions we will use in Parts I and II of this book are powers and polynomials. Consider the function $u = ct^n$, where c and n are constants. The following result is proven in calculus:

$$\text{The derivative of } u = ct^n \text{ is } \frac{du}{dt} = nct^{n-1} \tag{2.5}$$

NOTE ▶ The symbol u is a "dummy name." Equation 2.5 can be used to take the derivative of *any* function of the form ct^n. ◀

Example 2.5 needed to find the derivative of the function $s = 2t^2$. Using Equation 2.5 with $c = 2$ and $n = 2$, the derivative of $s = 2t^2$ with respect to t is

$$v_s = \frac{ds}{dt} = 2 \cdot 2t^{2-1} = 4t$$

Scientists and engineers must use calculus to calculate the trajectories of rockets.

Similarly, the derivative of the function $x = 3/t^2 = 3t^{-2}$ is

$$\frac{dx}{dt} = (-2) \cdot 3t^{-2-1} = -6t^{-3} = -\frac{6}{t^3}$$

A value that doesn't change with time, such as the position of an object at rest, can be represented by the function $u = c =$ constant. That is, the exponent of t^n is $n = 0$. You can see from Equation 2.5 that the derivative of a constant is zero. That is,

$$\frac{du}{dt} = 0 \text{ if } u = c = \text{constant} \tag{2.6}$$

This makes sense. The graph of the function $u = c$ is simply a horizontal line at height c. The slope of a horizontal line—which is what the derivative du/dt measures—is zero.

The only other information we need about derivatives for now is how to evaluate the derivative of the sum of two or more functions. Let u and w be two separate functions of time. You will learn in calculus that

$$\frac{d}{dt}(u + w) = \frac{du}{dt} + \frac{dw}{dt} \tag{2.7}$$

That is, the derivative of a sum is the sum of the derivatives.

NOTE ▶ You may have learned in calculus to take the derivative dy/dx, where y is a function of x. The derivatives we use in physics are the same; only the notation is different. We're interested in how quantities change with time, so our derivatives are with respect to t instead of x. ◀

EXAMPLE 2.6 Using calculus to find the velocity

A particle's position is given by the function $x = (-t^3 + 3t)$ m, where t is in s.

a. What are the particle's position and velocity at $t = 2$ s?
b. Draw graphs of x and v_x during the interval $-3 \text{ s} \le t \le 3$ s.
c. Draw a motion diagram to illustrate this motion.

SOLVE

a. We can compute the position at $t = 2$ s directly from the function x:

$$x(\text{at } t = 2 \text{ s}) = -(2)^3 + (3)(2) = -8 + 6 = -2 \text{ m}$$

The velocity is then $v_x = dx/dt$. The function for x is the sum of two polynomials, so

$$v_x = \frac{dx}{dt} = \frac{d}{dt}(-t^3 + 3t) = \frac{d}{dt}(-t^3) + \frac{d}{dt}(3t)$$

The first derivative is a power with $c = -1$ and $n = 3$; the second has $c = 3$ and $n = 1$. Using Equation 2.5,

$$v_x = (-3t^2 + 3) \text{ m/s}$$

where t is in s. Evaluating the velocity at $t = 2$ s gives

$$v_x(\text{at } t = 2 \text{ s}) = -3(2)^2 + 3 = -9 \text{ m/s}$$

The negative sign indicates that the particle, at this instant of time, is moving to the *left* at a speed of 9 m/s.

b. FIGURE 2.13 shows the position graph and the velocity graph. These were created by computing, and then graphing, the values of x and v_x at several points between -3 s and 3 s. The slope of the position-versus-time graph at $t = 2$ s is -9 m/s; this becomes the *value* that is graphed for the velocity at $t = 2$ s. Similar measurements are shown at $t = -1$ s, where the velocity is instantaneously zero.

c. Finally, we can interpret the graphs in Figure 2.13 to draw the motion diagram shown in FIGURE 2.14.

- The particle is initially to the right of the origin ($x > 0$ at $t = -3$ s) but moving to the left ($v_x < 0$). Its *speed* is slowing ($v = |v_x|$ is decreasing), so the velocity vector arrows are getting shorter.
- The particle passes the origin at $t \approx -1.5$ s, but it is still moving to the left.
- The position reaches a minimum at $t = -1$ s; the particle is as far left as it is going. The velocity is *instantaneously* $v_x = 0$ m/s as the particle reverses direction.
- The particle moves back to the right between $t = -1$ s and $t = 1$ s ($v_x > 0$).
- The particle turns around again at $t = 1$ s and begins moving back to the left ($v_x < 0$). It keeps speeding up, then disappears off to the left.

FIGURE 2.13 Position and velocity graphs.

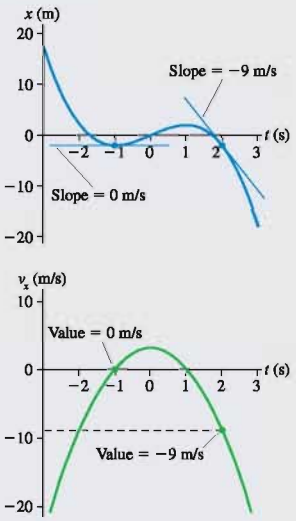
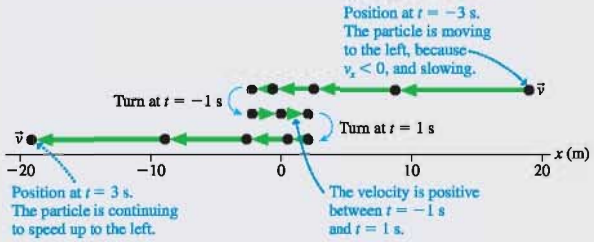
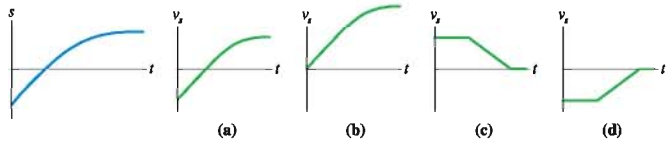


FIGURE 2.14 Motion diagram for Example 2.6.



The particle in this example moved out to $x = -2$ m at $t = -1$ s, then returned. The point in its motion where it reversed direction is called a *turning point*. Because the velocity was negative just before reaching the turning point and positive just after, it had to pass through $v_x = 0$ m/s. Thus, a **turning point** is a point where the velocity is instantaneously zero as the particle reverses direction. A second turning point occurs at $t = 1$ s as the particle reaches $x = 2$ m. We will see many future examples of turning points.

STOP TO THINK 2.2 Which velocity-versus-time graph goes with the position-versus-time graph on the left?



2.3 Finding Position from Velocity

Equation 2.4 provides a means of finding the instantaneous velocity v_s if we know the position s as a function of time. In mathematical terms, the velocity is the derivative of the position function. Graphically, the velocity is the slope of the position-versus-time graph.

But what about the reverse problem? Can we use the object's velocity to calculate its position at some future time t? Equation 2.3, $s_f = s_i + v_s\Delta t$, does this for the case of uniform motion with a constant velocity. We need to find a more general expression that is valid when v_s is not constant.

FIGURE 2.15 Approximating a velocity-versus-time graph with a series of constant-velocity steps.

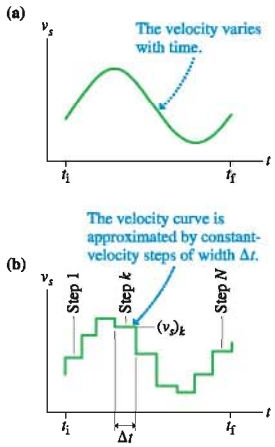


FIGURE 2.15a is a velocity-versus-time graph for a particle whose velocity varies with time. Suppose we know the object's position to be s_i at an initial time t_i. Our goal is to find its position s_f at a later time t_f.

Because we know how to handle constant velocities, using Equation 2.3, let's *approximate* the velocity function of Figure 2.15a as a series of constant-velocity steps of width Δt. This is illustrated in FIGURE 2.15b. During the first step, from time t_i to time $t_i + \Delta t$, the velocity has the constant value $(v_s)_1$. The velocity is a constant $(v_s)_2$ during the second step from $t_i + \Delta t$ to $t_i + 2\Delta t$, and so on. The velocity during step k has the constant value $(v_s)_k$. Altogether the velocity-versus-time curve has been divided into N constant-velocity steps of equal width Δt. Although the approximation shown in the figure is rather rough, with only nine steps, we can easily imagine that it could be made as accurate as desired by having more and more ever-narrower steps.

The velocity during each step is constant (uniform motion), so we can apply Equation 2.3 to each step. The object's displacement Δs_1 during the first step is simply $\Delta s_1 = (v_s)_1\,\Delta t$. The displacement during the second step $\Delta s_2 = (v_s)_2\Delta t$, and during step k the displacement is $\Delta s_k = (v_s)_k\,\Delta t$.

The total displacement of the object between t_i and t_f can be approximated as the sum of all the individual displacements during each of the N constant-velocity steps. That is,

$$\Delta s = s_f - s_i \approx \Delta s_1 + \Delta s_2 + \cdots + \Delta s_N = \sum_{k=1}^{N} (v_s)_k \Delta t \qquad (2.8)$$

where Σ (Greek sigma) is the symbol for summation.

With a simple rearrangement, the particle's final position is

$$s_f \approx s_i + \sum_{k=1}^{N} (v_s)_k \Delta t \tag{2.9}$$

Our goal was to use the velocity to find the final position s_f. Equation 2.9 nearly reaches that goal, but Equation 2.9 is only approximate because the constant-velocity steps are only an approximation of the true velocity graph. But if we now let $\Delta t \rightarrow 0$, each step's width approaches zero while the total number of steps N approaches infinity. In this limit, the series of steps becomes a perfect replica of the velocity-versus-time graph and Equation 2.9 becomes exact. Thus

$$s_f = s_i + \lim_{\Delta t \rightarrow 0} \sum_{k=1}^{N} (v_s)_k \Delta t = s_i + \int_{t_i}^{t_f} v_s \, dt \tag{2.10}$$

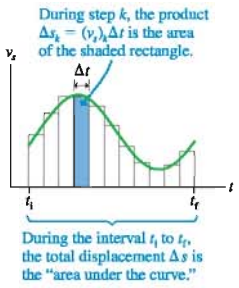
The curlicue symbol is called an *integral*. The expression on the right is read, "the integral of $v_s \, dt$ from t_i to t_f." Equation 2.10 is the result that we were seeking. It allows us to predict an object's position s_f at a future time t_f.

We can give Equation 2.10 an important geometric interpretation. FIGURE 2.16 shows step k in the approximation of the velocity graph as a long, thin rectangle of height $(v_s)_k$ and width Δt. The product $\Delta s_k = (v_s)_k \, \Delta t$ is the area (base × height) of this small rectangle. The sum in Equation 2.10 adds up all of these rectangular areas to give the total area enclosed between the t-axis and the tops of the steps. The limit of this sum as $\Delta t \rightarrow 0$ is the total area enclosed between the t-axis and the velocity curve. This is called the "area under the curve." Thus a graphical interpretation of Equation 2.10 is:

$$s_f = s_i + \text{area under the velocity curve } v_s \text{ between } t_i \text{ and } t_f \tag{2.11}$$

NOTE ▶ Wait a minute! The displacement $\Delta s = s_f - s_i$ is a length. How can a length equal an area? Recall earlier, when we found that the velocity is the slope of the position graph, we made a distinction between the *actual* slope and the *physically meaningful* slope? The same distinction applies here. The velocity graph does indeed bound a certain area on the page. That is the actual area, but it is *not* the area to which we are referring. Once again, we need to measure the quantities we are using, v_s and Δt, by referring to the scales on the axes. Δt is some number of seconds while v_s is some number of meters per second. When these are multiplied together, the *physically meaningful* area has units of meters, appropriate for a displacement. The following examples will help make this clear. ◀

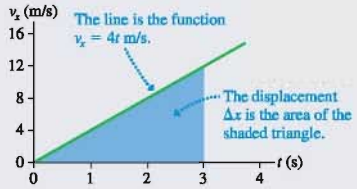
FIGURE 2.16 The total displacement Δs is the "area under the curve."



EXAMPLE 2.7 The displacement during a drag race

FIGURE 2.17 shows the velocity-versus-time graph of a drag racer. How far does the racer move during the first 3.0 s?

FIGURE 2.17 Velocity-versus-time graph for Example 2.7.



MODEL Represent the drag racer as a particle with a well-defined position at all times.

VISUALIZE Figure 2.17 is the graphical representation.

SOLVE The question "how far" indicates that we need to find a displacement Δx rather than a position x. According to Equation 2.11, the car's displacement $\Delta x = x_f - x_i$ between $t = 0$ s and $t = 3$ s is the area under the curve from $t = 0$ s to $t = 3$ s. The curve in this case is an angled line, so the area is that of a triangle:

$$\begin{aligned} \Delta x &= \text{area of triangle between } t = 0 \text{ s and } t = 3 \text{ s} \\ &= \tfrac{1}{2} \times \text{base} \times \text{height} \\ &= \tfrac{1}{2} \times 3 \text{ s} \times 12 \text{ m/s} = 18 \text{ m} \end{aligned}$$

The drag racer moves 18 m during the first 3 seconds.

ASSESS The "area" is a product of s with m/s, so Δx has the proper units of m.

EXAMPLE 2.8 Finding an expression for the racer's position

a. Find an algebraic expression for the position x as a function of time t for the drag racer whose velocity-versus-time graph was shown in Figure 2.17. Assume the car's initial position is $x_i = 0$ m at $t_i = 0$ s.
b. Draw the car's position-versus-time graph.

SOLVE

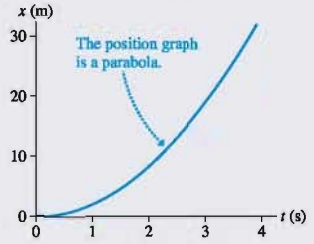
a. Let $x_i = 0$ at $t_i = 0$ and let x be the position at later time t. The straight line for v_x in Figure 2.17 is described by the linear function $v_x = 4t$ m/s, where t is in s. Then

$$x = x_i + \int_0^t v_x\,dt = 0 + \text{area under the triangle between 0 and } t$$

$$= 0 + \tfrac{1}{2}(t - 0)(4t - 0) = 2t^2 \text{ m, where } t \text{ is in s}$$

b. FIGURE 2.18 shows the drag racer's position-versus-time graph. It's simply a graph of the function $x = 2t^2$ m, where t is in s. Notice that the *linear* velocity graph of Figure 2.17 is associated with a *parabolic* position graph. This is a general result that we will see again.

FIGURE 2.18 The position-versus-time graph for the drag racer whose velocity graph was shown in Figure 2.17.



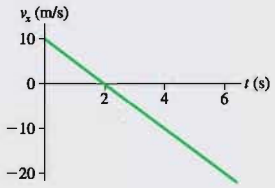
ASSESS This is exactly Example 2.5 in reverse! There we found, by taking the derivative, that a particle whose position is $x = 2t^2$ m has a velocity described by $v_x = 4t$ m/s. Here we have found, by integration, that a drag racer whose velocity is given by $v_x = 4t$ m/s has a position described by $x = 2t^2$ m.

EXAMPLE 2.9 Finding the turning point

FIGURE 2.19 is the velocity graph for a particle that starts at $x_i = 30$ m at time $t_i = 0$ s.

a. Draw a motion diagram for the particle.
b. Where is the particle's turning point?
c. At what time does the particle reach the origin?

FIGURE 2.19 Velocity-versus-time graph for the particle of Example 2.9.



VISUALIZE The particle is initially 30 m to the right of the origin and moving *to the right* ($v_x > 0$) with a speed of 10 m/s. But v_x is decreasing, so the particle is slowing down. At $t = 2$ s the velocity, just for an instant, is zero before becoming negative. This is the turning point. The velocity is negative for $t > 2$ s, so the particle has reversed direction and moves back toward the origin. At some later time, which we want to find, the particle will pass $x = 0$ m.

SOLVE a. FIGURE 2.20 shows the motion diagram. The distance scale will be established in parts b and c but is shown here for convenience.

b. The particle reaches the turning point at $t = 2$ s. To learn *where* it is at that time we need to find the displacement during the first two seconds. We can do this by finding the area under the curve between $t = 0$ s and $t = 2$ s:

$$x(\text{at } t = 2 \text{ s}) = x_i + \int_{0\text{s}}^{2\text{s}} v_x\,dt$$

$$= x_i + \text{area under the curve between 0 s and 2 s}$$

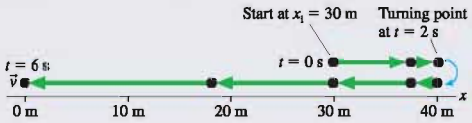
$$= 30 \text{ m} + \tfrac{1}{2}(2 \text{ s} - 0 \text{ s})(10 \text{ m/s} - 0 \text{ m/s})$$

$$= 40 \text{ m}$$

The turning point is at $x = 40$ m.

c. The particle needs to move $\Delta x = -40$ m to get from the turning point to the origin. That is, the area under the curve from $t = 2$ s to the desired time t needs to be -40 m. Because the curve is below the axis, with negative values of v_x, the area to the right of $t = 2$ s is a *negative* area. With a bit of geometry, you will find that the triangle with a base extending from $t = 2$ s to $t = 6$ s has an area of -40 m. Thus the particle reaches the origin at $t = 6$ s.

FIGURE 2.20 Motion diagram for the particle whose velocity graph was shown in Figure 2.19.



A Little More Calculus: Integrals

Taking the derivative of a function is equivalent to finding the slope of a graph of the function. Similarly, evaluating an integral is equivalent to finding the area under a graph of the function. The graphical method is very important for building intuition about motion but is limited in its practical application. Just as derivatives of standard functions can be evaluated and tabulated, so can integrals.

The integral in Equation 2.10 is called a *definite integral* because there are two definite boundaries to the area we want to find. These boundaries are called the lower (t_i) and upper (t_f) *limits of integration*. For the important function $u = ct^n$, the essential result from calculus is that

$$\int_{t_i}^{t_f} u\,dt = \int_{t_i}^{t_f} ct^n\,dt = \frac{ct^{n+1}}{n+1}\bigg|_{t_i}^{t_f} = \frac{ct_f^{n+1}}{n+1} - \frac{ct_i^{n+1}}{n+1} \qquad (n \neq -1) \tag{2.12}$$

The vertical bar in the third step with subscript t_i and superscript t_f is a shorthand notation from calculus that means—as seen in the last step—the integral evaluated at the upper limit t_f *minus* the integral evaluated at the lower limit t_i. You also need to know that for two functions u and w,

$$\int_{t_i}^{t_f} (u + w)\,dt = \int_{t_i}^{t_f} u\,dt + \int_{t_i}^{t_f} w\,dt \tag{2.13}$$

That is, the integral of a sum is equal to the sum of the integrals.

EXAMPLE 2.10 Using calculus to find the position

Use calculus to solve Example 2.9.

SOLVE Figure 2.19 is a linear graph. Its "y-intercept" is seen to be 10 m/s and its slope is −5 (m/s)/s. Thus the velocity graphed here can be described by the equation

$$v_x = (10 - 5t)\ \text{m/s}$$

where t is in s. We can find the position x at time t by using Equation 2.10:

$$x = x_i + \int_0^t v_x\,dt = 30\ \text{m} + \int_0^t (10 - 5t)\,dt$$

$$= 30\ \text{m} + \int_0^t 10\,dt - \int_0^t 5t\,dt$$

We used Equation 2.13 for the integral of a sum to get the final expression. The first integral is a function of the form $u = ct^n$ with $c = 10$ and $n = 0$; the second is of the form $u = ct^n$ with $c = 5$ and $n = 1$. Using Equation 2.12,

$$\int_0^t 10\,dt = 10t\Big|_0^t = 10 \cdot t - 10 \cdot 0 = 10t\ \text{m}$$

and

$$\int_0^t 5t\,dt = \tfrac{5}{2}t^2\Big|_0^t = \tfrac{5}{2} \cdot t^2 - \tfrac{5}{2} \cdot 0^2 = \tfrac{5}{2}t^2\ \text{m}$$

Combining the pieces gives

$$x = (30 + 10t - \tfrac{5}{2}t^2)\ \text{m}$$

where t is in s. The particle's turning point occurs at $t = 2$ s, and its position at that time is

$$x(\text{at } t = 2\ \text{s}) = 30 + (10)(2) - \tfrac{5}{2}(2)^2 = 40\ \text{m}$$

The time at which the particle reaches the origin is found by setting $x = 0$ m:

$$30 + 10t - \tfrac{5}{2}t^2 = 0$$

This quadratic equation has two solutions: $t = -2$ s or $t = 6$ s.

When we solve a quadratic equation, we cannot just arbitrarily select the root we want. Instead, we must decide which is the *meaningful* root. Here the negative root refers to a time before the problem began, so the meaningful one is the positive root, $t = 6$ s.

ASSESS The results agree with the answers we found previously from a graphical solution.

These examples make the point that there are often many ways to solve a problem. The graphical procedures for finding derivatives and integrals are simple, but they work only for a limited range of problems—those where the geometry is simple. The techniques of calculus are more demanding, but these techniques allow us to deal with functions whose graphs are quite complex.

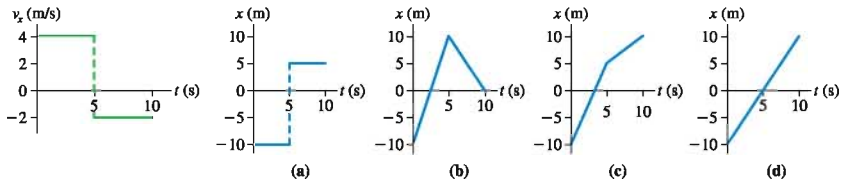
Summing Up

As you work on building intuition about motion, you need to be able to move back and forth between four different representations of the motion:

- The motion diagram;
- The position-versus-time graph;
- The velocity-versus-time graph;
- The description in words.

Given a description of a certain motion, you should be able to sketch the motion diagram and the position and velocity graphs. Given one graph, you should be able to generate the other. And given position and velocity graphs, you should be able to "interpret" them by describing the motion in words or in a motion diagram.

STOP TO THINK 2.3 Which position-versus-time graph goes with the velocity-versus-time graph on the left? The particle's position at $t_i = 0$ s is $x_i = -10$ m.



2.4 Motion with Constant Acceleration

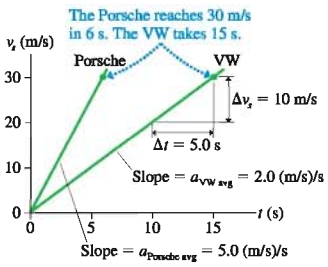
We need one more major concept to describe one-dimensional motion: acceleration. Acceleration, as we noted in Chapter 1, is a rather abstract concept. You cannot "see" the value of acceleration, as you can that of position, nor can you judge it by looking to see if an object is moving quickly or slowly. Nonetheless, acceleration is the linchpin of mechanics. We will see very shortly that Newton's laws relate the acceleration of an object to the forces that are exerted on it.

Let's conduct a race between a Volkswagen Beetle and a Porsche to see which can achieve a velocity of 30 m/s (≈60 mph) in the shortest time. Both cars are equipped with computers that will record the speedometer reading 10 times each second. This gives a nearly continuous record of the *instantaneous* velocity of each car. Table 2.1 shows some of the data. The velocity-versus-time graphs, based on these data, are shown in FIGURE 2.21.

TABLE 2.1 Velocities of a Porsche and a Volkswagen Beetle

t (s)	$v_{Porsche}$ (m/s)	v_{VW} (m/s)
0.0	0.0	0.0
0.1	0.5	0.2
0.2	1.0	0.4
0.3	1.5	0.6
0.4	2.0	0.8
⋮	⋮	⋮

FIGURE 2.21 Velocity-versus-time graphs for the Porsche and the VW Beetle.



How can we describe the difference in performance of the two cars? It is not that one has a different velocity from the other; both achieve every velocity between 0 and 30 m/s. The distinction is how long it took each to *change* its velocity from 0 to 30 m/s. The Porsche changed velocity quickly, in 6.0 s, while the VW needed 15 s to make the same velocity change.

As we compare the two cars, we are looking at the *rate* at which their velocities change. Because the Porsche had a velocity change $\Delta v_x = 30$ m/s during a time interval $\Delta t = 6.0$ s, the *rate* at which its velocity changed was

$$\text{rate of velocity change} = \frac{\Delta v_x}{\Delta t} = \frac{30\ \text{m/s}}{6.0\ \text{s}} = 5.0\ \text{(m/s)/s} \qquad (2.14)$$

Notice the units. They are units of "velocity per second." A rate of velocity change of 5.0 "meters per second per second" means that the velocity increases by 5.0 m/s

during the first second, by another 5.0 m/s during the next second, and so on. In fact, the velocity will increase by 5.0 m/s during any second in which it is changing at the rate of 5.0 (m/s)/s.

Chapter 1 introduced *acceleration* as "the rate of change of velocity." That is, acceleration measures how quickly or slowly an object's velocity changes. The Porsche's velocity changed quickly, so it had a large acceleration. The VW's velocity changed more slowly, so its acceleration was less. In parallel with our treatment of velocity, let's define the **average acceleration** a_{avg} during the time interval Δt to be

$$a_{avg} \equiv \frac{\Delta v_s}{\Delta t} \qquad \text{(average acceleration)} \qquad (2.15)$$

Because Δv_s and Δt are the "rise" and "run" of a velocity-versus-time graph, we see that a_{avg} can be interpreted graphically as the *slope* of a straight-line velocity-versus-time graph. Figure 2.21 uses this idea to show that the VW's average acceleration is

$$a_{VW\ avg} = \frac{\Delta v_s}{\Delta t} = \frac{10\ \text{m/s}}{5.0\ \text{s}} = 2.0\ \text{(m/s)/s} \qquad (2.16)$$

This is less than the acceleration of the Porsche, as expected.

An object whose velocity-versus-time graph is a straight-line graph has a steady and unchanging acceleration. Such a graph represents motion with *constant acceleration,* which we call **uniformly accelerated motion: An object has uniformly accelerated motion if and only if its acceleration a_s is constant and unchanging. The object's velocity-versus-time graph is a straight line, and a_s is the slope of the line.** There's no need to specify "average" if the acceleration is constant, so we'll use the symbol a_s as we discuss motion along the s-axis with constant acceleration.

1.2, 1.3

NOTE ▶ An important aspect of acceleration is its *sign.* Acceleration $\vec{a}$, like position $\vec{r}$ and velocity $\vec{v}$, is a vector. For motion in one dimension the sign of a_x (or a_y) is positive if the vector $\vec{a}$ points to the right (or up), negative if it points to the left (or down). This was illustrated in Figure 1.19 and the very important Tactics Box 1.4, which you may wish to review. It's particularly important to emphasize that positive and negative values of a_s do *not* correspond to "speeding up" and "slowing down." ◀

EXAMPLE 2.11 Relating acceleration to velocity

a. A particle has a velocity of 10 m/s and a constant acceleration of 2 (m/s)/s. What is its velocity 1 s later? 2 s later?

b. A particle has a velocity of −10 m/s and a constant acceleration of 2 (m/s)/s. What is its velocity 1 s later? 2 s later?

SOLVE

a. An acceleration of 2 (m/s)/s *means* that the velocity increases by 2 m/s every 1 s. If the particle's initial velocity is 10 m/s, then 1 s later its velocity will be 12 m/s. After 2 s, which is 1 additional second later, it will increase by another 2 m/s to 14 m/s. After 3 s it will be 16 m/s. Here a positive a_s is causing the particle to speed up.

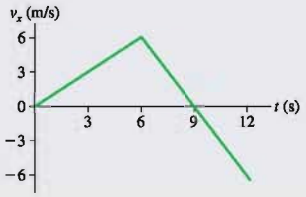
b. If the particle's initial velocity is a *negative* −10 m/s but the acceleration is a positive +2 (m/s)/s, then 1 s later the velocity will be −8 m/s. After 2 s it will be −6 m/s, and so on. In this case, a positive a_s is causing the object to *slow down* (decreasing speed v). This agrees with the rule from Tactics Box 1.4: An object is slowing down if and only if v_s and a_s have opposite signs.

NOTE ▶ It is customary to abbreviate the acceleration units (m/s)/s as m/s^2. For example, the particles in Example 2.11 had an acceleration of 2 m/s^2. We will use this notation, but keep in mind the *meaning* of the notation as "(meters per second) per second." ◀

EXAMPLE 2.12 Running the court

A basketball player starts at the left end of the court and moves with the velocity shown in FIGURE 2.22. Draw a motion diagram and an acceleration-versus-time graph for the basketball player.

FIGURE 2.22 Velocity-versus-time graph for the basketball player of Example 2.12.



VISUALIZE The velocity is positive (motion to the right) and increasing for the first 6 s, so the velocity arrows in the motion diagram are to the right and getting longer. From $t = 6$ s to 9 s the motion is still to the right (v_x is still positive), but the arrows are getting shorter because v_x is decreasing. There's a turning point at $t = 9$ s, when $v_x = 0$, and after that the motion is to the left (v_x is negative) and getting faster. The motion diagram of FIGURE 2.23a shows the velocity and the acceleration vectors.

SOLVE Acceleration is the slope of the velocity graph. For the first 6 s, the slope has the constant value

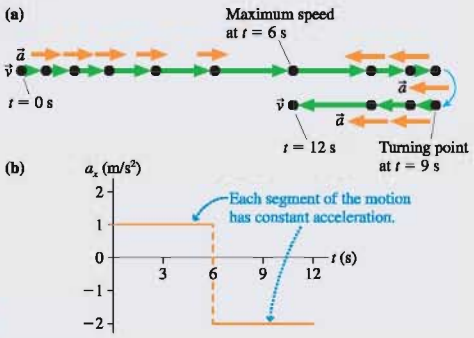
$$a_x = \frac{\Delta v_x}{\Delta t} = \frac{6.0 \text{ m/s}}{6.0 \text{ s}} = 1.0 \text{ m/s}^2$$

The velocity decreases by 12 m/s during the 6 s interval from $t = 6$ s to $t = 12$ s, so

$$a_x = \frac{\Delta v_x}{\Delta t} = \frac{-12 \text{ m/s}}{6.0\ s} = -2.0 \text{ m/s}^2$$

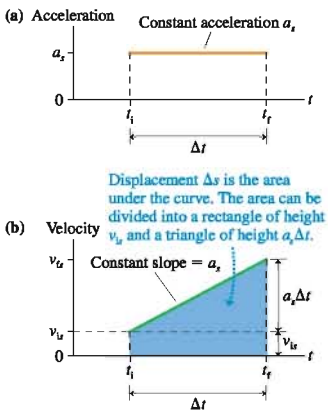
The acceleration graph for these 12 s is shown in FIGURE 2.23b. Notice that there is no change in the acceleration at $t = 9$ s, the turning point.

FIGURE 2.23 Motion diagram and acceleration graph for Example 2.12.



ASSESS The *sign* of a_x does *not* tell us whether the object is speeding up or slowing down. The basketball player is slowing down from $t = 6$ s to $t = 9$ s, then speeding up from $t = 9$ s to $t = 12$ s. Nonetheless, his acceleration is negative during this entire interval because his acceleration vector, as seen in the motion diagram, always points to the left.

FIGURE 2.24 Acceleration and velocity graphs for motion with constant acceleration.



The Kinematic Equations of Constant Acceleration

Consider an object whose acceleration a_s remains constant during the time interval $\Delta t = t_f - t_i$. At the beginning of this interval, at time t_i, the object has initial velocity v_{is} and initial position s_i. Note that t_i is often zero, but it does not have to be. We would like to predict the object's final position s_f and final velocity v_{fs} at time t_f.

The object's velocity is changing because the object is accelerating. FIGURE 2.24a shows the acceleration-versus-time graph, a horizontal line between t_i and t_f. It is not hard to find the object's velocity v_{fs} at a later time t_f. By definition,

$$a_s = \frac{\Delta v_s}{\Delta t} = \frac{v_{fs} - v_{is}}{\Delta t} \tag{2.17}$$

which is easily rearranged to give

$$v_{fs} = v_{is} + a_s \Delta t \tag{2.18}$$

The velocity-versus-time graph, shown in FIGURE 2.24b, is a straight line that starts at v_{is} and has slope a_s.

As you learned in the last section, the object's final position is

$$s_f = s_i + \text{area under the velocity curve } v_s \text{ between } t_i \text{ and } t_f \tag{2.19}$$

The shaded area in Figure 2.24b can be subdivided into a rectangle of area $v_{is}\,\Delta t$ and a triangle of area $\frac{1}{2}(a_s\Delta t)(\Delta t) = \frac{1}{2}a_s(\Delta t)^2$. Adding these gives

$$s_f = s_i + v_{is}\Delta t + \tfrac{1}{2}a_s(\Delta t)^2 \tag{2.20}$$

where $\Delta t = t_f - t_i$ is the elapsed time. The quadratic dependence on Δt causes the position-versus-time graph for constant-acceleration motion to have a parabolic shape. You saw this earlier in Figure 2.18, and it will appear below in Figure 2.25.

Equations 2.18 and 2.20 are two of the basic kinematic equations for motion with *constant* acceleration. They allow us to predict an object's position and velocity at a future instant of time. We need one more equation to complete our set, a direct relation between position and velocity. First use Equation 2.18 to write $\Delta t = (v_{fs} - v_{is})/a_s$. Substitute this into Equation 2.20, giving

$$\begin{aligned} s_f &= s_i + v_{is}\left(\frac{v_{fs} - v_{is}}{a_s}\right) + \tfrac{1}{2}a_s\left(\frac{v_{fs} - v_{is}}{a_s}\right)^2 \\ &= s_i + \left(\frac{v_{is}v_{fs}}{a_s} - \frac{v_{is}^2}{a_s}\right) + \left(\frac{v_{fs}^2}{2a_s} - \frac{v_{is}v_{fs}}{a_s} + \frac{v_{is}^2}{2a_s}\right) \\ &= s_i + \frac{v_{fs}^2 - v_{is}^2}{2a_s} \end{aligned} \tag{2.21}$$

This is easily rearranged to read

$$v_{fs}^2 = v_{is}^2 + 2a_s\Delta s \tag{2.22}$$

where $\Delta s = s_f - s_i$ is the *displacement* (not the distance!).

Equations 2.18, 2.20, and 2.22, which are summarized in Table 2.2, are the key results for motion with constant acceleration.

FIGURE 2.25 is a comparison of motion with constant velocity (uniform motion) and motion with constant acceleration (uniformly accelerated motion). Notice that uniform motion is really a special case of uniformly accelerated motion in which the constant acceleration happens to be zero. The graphs for a negative acceleration are left as an exercise.

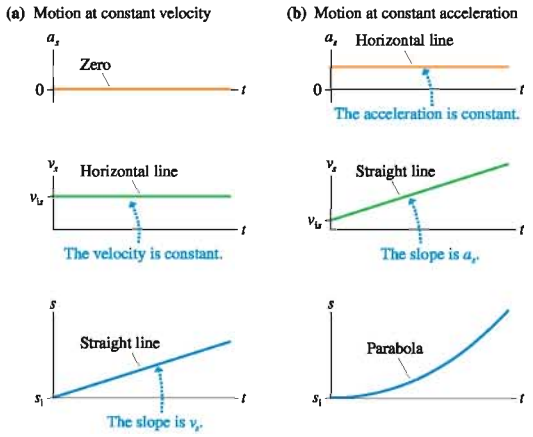
TABLE 2.2 The kinematic equations for motion with constant acceleration

$v_{fs} = v_{is} + a_s\,\Delta t$

$s_f = s_i + v_{is}\,\Delta t + \frac{1}{2}a_s(\Delta t)^2$

$v_{fs}^2 = v_{is}^2 + 2a_s\,\Delta s$

FIGURE 2.25 Motion with constant velocity and constant acceleration. These graphs assume $s_i = 0$, $v_{is} > 0$, and (for constant acceleration) $a_s > 0$.



A Problem-Solving Strategy

1.4, 1.5, 1.6, 1.8, 1.9, 1.11, 1.12, 1.13, 1.14 Activ ONLINE Physics

This information can be assembled into a problem-solving strategy for kinematics with constant acceleration.

PROBLEM-SOLVING STRATEGY 2.1 **Kinematics with constant acceleration** MP

MODEL Use the particle model. Make simplifying assumptions.

VISUALIZE Use different representations of the information in the problem.

- Draw a *pictorial representation*. This helps you assess the information you are given and starts the process of translating the problem into symbols.
- Use a *graphical representation* if it is appropriate for the problem.
- Go back and forth between these two representations as needed.

SOLVE The mathematical representation is based on the three kinematic equations

$$v_{fs} = v_{is} + a_s\Delta t$$

$$s_f = s_i + v_{is}\Delta t + \tfrac{1}{2}a_s(\Delta t)^2$$

$$v_{fs}^2 = v_{is}^2 + 2a_s\Delta s$$

- Use x or y, as appropriate to the problem, rather than the generic s.
- Replace i and f with numerical subscripts defined in the pictorial representation.
- Uniform motion with constant velocity has $a_s = 0$.

ASSESS Is your result believable? Does it have proper units? Does it make sense?

NOTE ▶ You are strongly encouraged to solve problems on the Dynamics Worksheets found at the back of the *Student Workbook*. These worksheets will help you use the Problem-Solving Strategy and develop good problem-solving skills. End-of-chapter Exercises and Problems suitable for solution on a worksheet are marked with the icon . ◀

EXAMPLE 2.13 **The motion of a rocket sled**

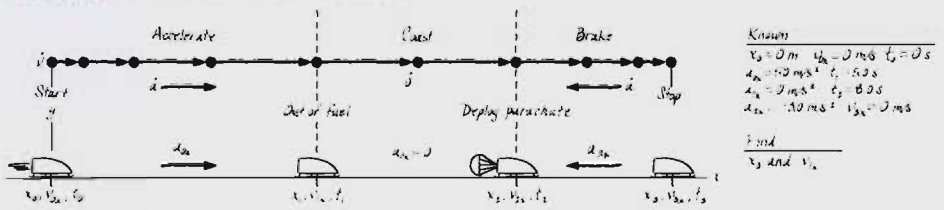
A rocket sled accelerates at 50 m/s² for 5.0 s, coasts for 3.0 s, then deploys a braking parachute and decelerates at 3.0 m/s² until coming to a halt.

a. What is the maximum velocity of the rocket sled?
b. What is the total distance traveled?

MODEL Represent the rocket sled as a particle.

VISUALIZE FIGURE 2.26 shows the pictorial representation. Recall that we discussed the first two-thirds of this problem as Example 1.8 in Chapter 1.

FIGURE 2.26 Pictorial representation of the rocket sled.



SOLVE a. The maximum velocity is identified in the pictorial representation as v_{1x}, the velocity at time t_1 when the acceleration phase ends. The first kinematic equation in Table 2.2 gives

$$v_{1x} = v_{0x} + a_{0x}(t_1 - t_0) = a_{0x}t_1$$

$$= (50\ \text{m/s}^2)(5.0\ \text{s}) = 250\ \text{m/s}$$

We started with the complete equation, then simplified by noting which terms were zero.

b. Finding the total distance requires several steps. First, the sled's position when the acceleration ends at t_1 is found from the second equation in Table 2.2:

$$x_1 = x_0 + v_{0x}(t_1 - t_0) + \tfrac{1}{2}a_{0x}(t_1 - t_0)^2 = \tfrac{1}{2}a_{0x}t_1^2$$
$$= \tfrac{1}{2}(50\ \text{m/s}^2)(5.0\ \text{s})^2 = 625\ \text{m}$$

During the coasting phase, which is uniform motion with no acceleration ($a_{1x} = 0$),

$$x_2 = x_1 + v_{1x}\Delta t = x_1 + v_{1x}(t_2 - t_1)$$
$$= 625\ \text{m} + (250\ \text{m/s})(3.0\ \text{s}) = 1375\ \text{m}$$

Notice that, in this case, Δt is not simply t. The braking phase is a little different because we don't know how long it lasts. But we do know that the sled ends with $v_{3x} = 0$ m/s, so we can use the third equation in Table 2.2:

$$v_{3x}^2 = v_{2x}^2 + 2a_{2x}\Delta x = v_{2x}^2 + 2a_{2x}(x_3 - x_2)$$

This can be solved for x_3:

$$x_3 = x_2 + \frac{v_{3x}^2 - v_{2x}^2}{2a_{2x}}$$
$$= 1375\ \text{m} + \frac{0 - (250\ \text{m/s})^2}{2(-3.0\ \text{m/s}^2)} = 11{,}800\ \text{m}$$

ASSESS Using the approximate conversion factor 1 m/s ≈ 2 mph from Table 1.5, we see that the top speed is ≈ 500 mph. The total distance traveled is ≈ 12 km ≈ 7 mi. This is reasonable because it takes a very long distance to stop from a top speed of 500 mph!

NOTE ▶ We used explicit numerical subscripts throughout the mathematical representation, each referring to a symbol that was defined in the pictorial representation. The subscripts i and f in the Table 2.2 equations are just generic "place holders" and don't have unique values. During the acceleration phase we had i = 0 and f = 1. Later, during the coasting phase, these became i = 1 and f = 2. The numerical subscripts have a clear meaning and are less likely to lead to confusion. ◀

EXAMPLE 2.14 Friday night football

Fred catches the football while standing directly on the goal line. He immediately starts running forward with an acceleration of 6 ft/s². At the moment the catch is made, Tommy is 20 yards away and heading directly toward Fred with a steady speed of 15 ft/s. If neither deviates from a straight-ahead path, where will Tommy tackle Fred?

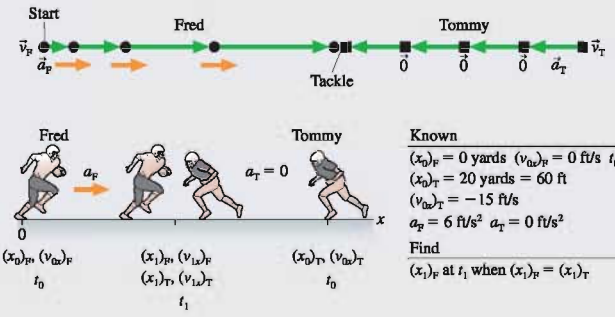
MODEL Represent Fred and Tommy as particles.

VISUALIZE The pictorial representation is shown again in FIGURE 2.27. With two moving objects we need the additional subscripts F and T to distinguish Fred's symbols and Tommy's symbols.

SOLVE We want to find *where* Fred and Tommy have the same position. The pictorial representation designates time t_1 as *when* they meet. The axes have been chosen so that Fred starts at $(x_0)_F$ = 0 ft and moves to the right while Tommy starts at $(x_0)_T$ = 60 ft and runs to the left with a *negative* velocity. The second equation of Table 2.2 allows us to find their positions at time t_1. These are:

$$(x_1)_F = (x_0)_F + (v_{0x})_F(t_1 - t_0) + \tfrac{1}{2}(a_x)_F(t_1 - t_0)^2$$
$$= \tfrac{1}{2}(a_x)_F t_1^2$$
$$(x_1)_T = (x_0)_T + (v_{0x})_T(t_1 - t_0) + \tfrac{1}{2}(a_x)_T(t_1 - t_0)^2$$
$$= (x_0)_T + (v_{0x})_T t_1$$

FIGURE 2.27 Pictorial representation for Example 2.14.



Known
$(x_0)_F = 0$ yards $(v_{0x})_F = 0$ ft/s $t_0 = 0$ s
$(x_0)_T = 20$ yards $= 60$ ft
$(v_{0x})_T = -15$ ft/s
$a_F = 6$ ft/s² $a_T = 0$ ft/s²

Find
$(x_1)_F$ at t_1 when $(x_1)_F = (x_1)_T$

Continued

Notice that Tommy's position equation contains the term $(v_{0x})_T t_1$, not $-(v_{0x})_T t_1$. The fact that he is moving to the left has already been considered in assigning a *negative value* to $(v_{0x})_T$, hence we don't want to add any additional negative signs in the equation. If we now set $(x_1)_F$ and $(x_1)_T$ equal to each other, indicating the point of the tackle, we can solve for t_1:

$$\tfrac{1}{2}(a_x)_F t_1^2 = (x_0)_T + (v_{0x})_T t_1$$
$$\tfrac{1}{2}(a_x)_F t_1^2 - (v_{0x})_T t_1 - (x_0)_T = 0$$
$$3t_1^2 + 15t_1 - 60 = 0$$

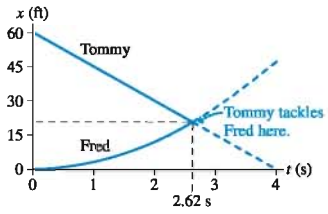
The solutions of this quadratic equation for t_1 are $t_1 = (-7.62\text{ s}, +2.62\text{ s})$. The negative time is not meaningful in this problem, so the time of the tackle is $t_1 = 2.62$ s. We've kept an extra significant digit in the solution to minimize round-off error in the next step. Using this value to compute $(x_1)_F$ gives

$$(x_1)_F = \tfrac{1}{2}(a_x)_F t_1^2 = 20.6 \text{ feet} = 6.9 \text{ yards}$$

Tommy makes the tackle at just about the 7-yard line!

ASSESS The answer had to be between 0 yards and 20 yards. Because Tommy was already running, whereas Fred started from rest, it is reasonable that Fred will cover less than half the 20-yard separation before meeting Tommy. Thus 6.9 yards is a reasonable answer.

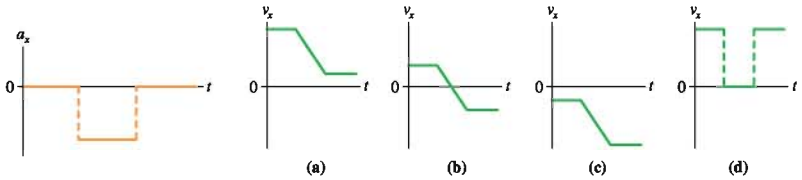
FIGURE 2.28 Position-versus-time graphs for Fred and Tommy.



NOTE ▶ The purpose of the assessment step is not to prove that an answer must be right but to rule out answers that, with a little thought, are clearly wrong. ◀

It is worth exploring Example 2.14 graphically. FIGURE 2.28 shows position-versus-time graphs for Fred and Tommy. The curves intersect at $t = 2.62$ s, and that is where the tackle occurs. You should compare this problem to Example 2.2 and Figure 2.5 for Bob and Susan to notice the similarities and the differences.

STOP TO THINK 2.4 Which velocity-versus-time graph or graphs go with this acceleration-versus-time graph? The particle is initially moving to the right.



2.5 Free Fall

The motion of an object moving under the influence of gravity only, and no other forces, is called **free fall.** Strictly speaking, free fall occurs only in a vacuum, where there is no air resistance. Fortunately, the effect of air resistance is small for "heavy objects," so we'll make only a very slight error in treating these objects *as if* they were in free fall. For very light objects, such as a feather, or for objects that fall through very large distances and gain very high speeds, the effect of air resistance is *not* negligible. Motion with air resistance is a problem we will study in Chapter 6. Until then, we will restrict our attention to "heavy objects" and will make the reasonable assumption that falling objects are in free fall.

The motion of falling objects has interested scientists since antiquity, but Galileo, in the 17th century, was the first to make detailed measurements. The story of Galileo dropping different weights from the leaning bell tower at the cathedral in Pisa is well

known, although historians cannot confirm its truth. But bell towers were common in the Italy of Galileo's day, so he had ample opportunity to make the measurements and observations that he describes in his writings.

Careful observations show that falling objects *don't* "hit the ground" at the same time. There are slight differences in the arrival times, but Galileo correctly identified these differences as due to air resistance. He then imagined an idealized situation of motion in a vacuum. In doing so, Galileo developed a *model* of motion—motion in the absence of air resistance—that could only be approximated by any real object. It was Galileo's innovative use of experiments, models, and mathematics that made him the first "modern" scientist.

Galileo's discovery can be summarized as follows:

- Two objects dropped from the same height will, if air resistance can be neglected, hit the ground at the same time and with the same speed.
- Consequently, **any two objects in free fall, regardless of their mass, have the same acceleration $\vec{a}_{\text{free fall}}$.** This is an especially important conclusion.

FIGURE 2.29a shows the motion diagram of an object that was released from rest and falls freely. FIGURE 2.29b shows the object's velocity graph. The motion diagram and graph are identical for a falling pebble and a falling boulder. The fact that the velocity graph is a straight line tells us the motion is one of uniform acceleration, and $a_{\text{free fall}}$ is easily found from the slope of the graph. Careful measurements show that the value of $\vec{a}_{\text{free fall}}$ varies ever so slightly at different places on the earth, due to the slightly nonspherical shape of the earth and to the fact that the earth is rotating. A global average, at sea level, is

$$\vec{a}_{\text{free fall}} = (9.80 \text{ m/s}^2, \text{ vertically downward}) \qquad (2.23)$$

For practical purposes, *vertically downward* means along a line toward the center of the earth. However, we'll learn in Chapter 13 that the rotation of the earth has a small effect on both the size and direction of $\vec{a}_{\text{free fall}}$.

The length, or magnitude, of $\vec{a}_{\text{free fall}}$ is known as the **free-fall acceleration,** and it has the special symbol g:

$$g = 9.80 \text{ m/s}^2 \text{ (free-fall acceleration)}$$

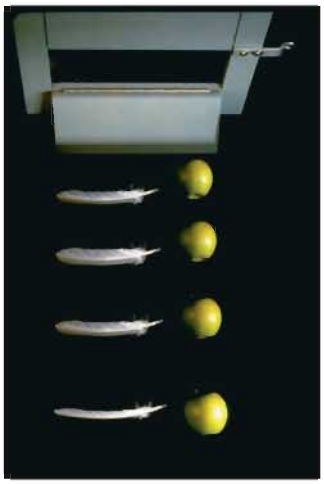
Several points about free fall are worthy of note:

- g, by definition, is *always* positive. **There will never be a problem that will use a negative value for g.** But, you say, objects fall when you release them rather than rise, so how can g be positive?
- g is *not* the acceleration $a_{\text{free fall}}$, but simply its magnitude. Because we've chosen the y-axis to point vertically up, the downward acceleration vector $\vec{a}_{\text{free fall}}$ has the one-dimensional acceleration

$$a_y = a_{\text{free fall}} = -g \qquad (2.24)$$

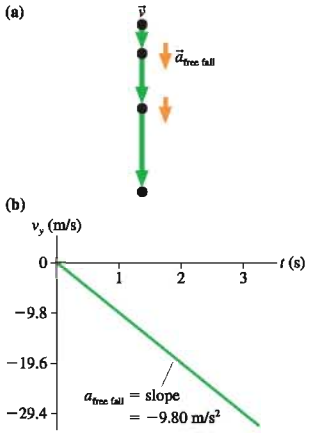
  It is a_y that is negative, not g.
- Because free fall is motion with constant acceleration, we can use the kinematic equations of Table 2.2 with the acceleration being that of free fall, $a_y = -g$.
- g is not called "gravity." Gravity is a force, not an acceleration. The symbol g recognizes the influence of gravity, but g is *the free-fall acceleration.*
- $g = 9.80 \text{ m/s}^2$ only on earth. Other planets have different values of g. You will learn in Chapter 13 how to determine g for other planets.

NOTE ▶ Despite the name, free fall is not restricted to objects that are literally falling. Any object moving under the influence of gravity only, and no other forces, is in free fall. This includes objects falling straight down, objects that have been tossed or shot straight up, and projectile motion. This chapter considers only objects that move up and down along a vertical line; projectile motion will be studied in Chapter 4. ◀



In the absence of air resistance, any two objects fall at the same rate and hit the ground at the same time. The apple and feather seen here are falling in a vacuum.

FIGURE 2.29 Motion of an object in free fall.



Activ Physics ONLINE 1.7, 1.10

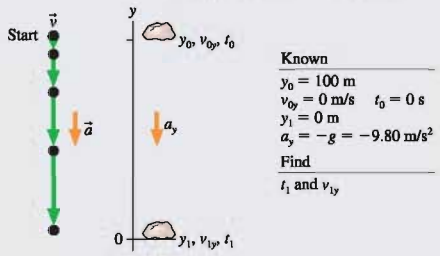
EXAMPLE 2.15 A falling rock

A rock is released from rest at the top of a 100-m-tall building. How long does the rock take to fall to the ground, and what is its impact velocity?

MODEL Represent the rock as a particle. Assume air resistance is negligible.

VISUALIZE FIGURE 2.30 shows the pictorial representation. We have placed the origin at the ground, which makes $y_0 = 100$ m. Although the rock falls 100 m, it is important to notice that the *displacement* is $\Delta y = y_1 - y_0 = -100$ m.

FIGURE 2.30 Pictorial representation of a falling rock.



SOLVE Free fall is motion with the specific constant acceleration $a_y = -g$. The first question involves a relation between time and distance, so only the second equation in Table 2.2 is relevant. Using $v_{0y} = 0$ m/s and $t_0 = 0$ s, we find

$$y_1 = y_0 + v_{0y}\Delta t + \tfrac{1}{2}a_y\Delta t^2 = y_0 + v_{0y}\Delta t - \tfrac{1}{2}g\Delta t^2 = y_0 - \tfrac{1}{2}gt_1^2$$

We can now solve for t_1, finding:

$$t_1 = \sqrt{\frac{2(y_0 - y_1)}{g}} = \sqrt{\frac{2(100\text{ m} - 0\text{ m})}{9.80\text{ m/s}^2}} = \pm 4.52\text{ s}$$

The ± sign indicates that there are two mathematical solutions; therefore we have to use physical reasoning to choose between them. A negative t_1 would refer to a time before we dropped the rock, so we select the positive root: $t_1 = 4.52$ s.

Now that we know the fall time, we can use the first kinematic equation to find v_{1y}:

$$v_{1y} = v_{0y} - g\,\Delta t = -gt_1 = -(9.80\text{ m/s}^2)(4.52\text{ s})$$
$$= -44.3\text{ m/s}$$

Alternatively, we could work directly from the third kinematic equation:

$$v_{1y} = \sqrt{v_{0y}^2 - 2g\,\Delta y} = \sqrt{-2g(y_1 - y_0)}$$
$$= \sqrt{-2(9.80\text{ m/s}^2)(0\text{ m} - 100\text{ m})} = \pm 44.3\text{ m/s}$$

This method is useful if you don't know Δt. However, we must again choose the correct sign of the square root. Because the velocity vector points downward, the sign of v_y has to be negative. Thus $v_{1y} = -44.3$ m/s. The importance of careful attention to the signs cannot be overemphasized!

A common error would be to say "The rock fell 100 m, so $\Delta y = 100$ m." This would have you trying to take the square root of a negative number. As noted above, Δy is not a distance. It is a *displacement*, with a carefully defined meaning of $y_f - y_i$. In this case, $\Delta y = y_1 - y_0 = -100$ m.

ASSESS Are the answers reasonable? Well, 100 m is about 300 feet, which is about the height of a 30-floor building. How long does it take something to fall 30 floors? Four or five seconds seems pretty reasonable. How fast would it be going at the bottom? Using 1 m/s ≈ 2 mph, we find that 44.3 m/s ≈ 90 mph. That also seems pretty reasonable after falling 30 floors. Had we misplaced a decimal point, though, and found 443 m/s, we would be suspicious when we converted this to ≈ 900 mph! The answers all seem reasonable.

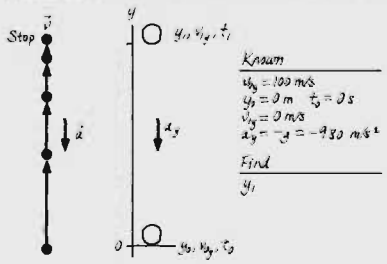
EXAMPLE 2.16 A vertical cannonball

A cannonball is shot straight up with an initial speed of 100 m/s. How high does it go?

MODEL Represent the cannonball as a particle. Assume air resistance is negligible.

VISUALIZE FIGURE 2.31 shows the pictorial representation for the cannonball's motion. Even though the ball was shot upward, this is a free-fall problem because the ball (after being launched) is moving under the influence of gravity *only*. A critical aspect of the problem is knowing where it ends. How do we put "how high" into symbols? The clue is that the very top point of the trajectory is a *turning point*. Recall that the instantaneous velocity at a turning point is $v = 0$. Thus we can characterize the "top" of the trajectory as the point where $v_{1y} = 0$ m/s. This was not explicitly stated but is part of our interpretation of the problem.

FIGURE 2.31 Pictorial representation for Example 2.16.



SOLVE We are looking for a relationship between distance and velocity, without knowing the time interval. This relationship is described mathematically by the third kinematic equation in Table 2.2. Using $y_0 = 0$ m and $v_{1y} = 0$ m/s, we have

$$v_{1y}^2 = 0 = v_{0y}^2 - 2g\Delta y = v_{0y}^2 - 2gy_1$$

Solving for y_1, we find that the cannonball reaches a height

$$y_1 = \frac{v_{0y}^2}{2g} = \frac{(100 \text{ m/s})^2}{2(9.80 \text{ m/s}^2)} = 510 \text{ m}$$

ASSESS Is this answer reasonable? A speed of 100 m/s is ≈200 mph—that's pretty fast! The calculated height is 510 m ≈ 1500 ft. In Example 2.15 we found that an object dropped from 100 m is going 44 m/s when it hits the ground, so it seems reasonable that an object shot upward at 100 m/s will go significantly higher than 100 m. While we cannot say that 510 m is necessarily better than 400 m or 600 m, we can say that it is not unreasonable. The point of the assessment is not to prove that the answer *has* to be right, but to find answers that are obviously wrong.

2.6 Motion on an Inclined Plane

A problem closely related to free fall is that of motion down a straight, but frictionless, inclined plane, such as a skier going down a slope on frictionless snow. **FIGURE 2.32a** shows an object sliding down a frictionless, inclined plane tilted at angle θ. The object's motion is constrained to be parallel to the surface. What is the object's acceleration? Although we're not yet prepared to give a rigorous derivation, we can deduce the acceleration with a plausibility argument.

FIGURE 2.32b shows the free-fall acceleration $\vec{a}_{\text{free fall}}$ the ball would have if the incline suddenly vanished. The free-fall acceleration points straight down. This vector can be broken into two pieces: a vector $\vec{a}_{\parallel}$ that is parallel to the incline and a vector $\vec{a}_{\perp}$ that is perpendicular to the incline. The vector addition rules of Chapter 1 tell us that $\vec{a}_{\text{free fall}} = \vec{a}_{\parallel} + \vec{a}_{\perp}$.

The motion diagram shows that the object's actual acceleration is parallel to the incline. The surface of the incline somehow "blocks" $\vec{a}_{\perp}$, through a process we will examine in Chapter 6, but $\vec{a}_{\parallel}$ is unhindered. It is this piece of $\vec{a}_{\text{free fall}}$, parallel to the incline, that accelerates the object.

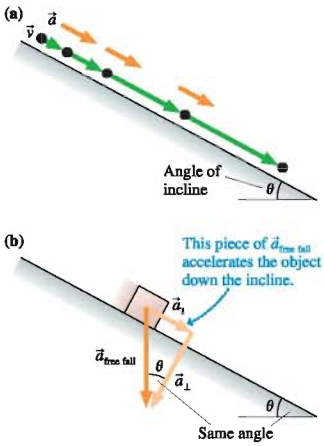
Figure 2.32b shows that the three vectors form a right triangle with angle θ at the bottom. By definition, the length, or magnitude, of $\vec{a}_{\text{free fall}}$ is g. Vector $\vec{a}_{\parallel}$ is opposite angle θ, so the length, or magnitude, of $\vec{a}_{\parallel}$ must be $g \sin\theta$. Consequently, the one-dimensional acceleration along the incline is

$$a_s = \pm g \sin\theta \qquad (2.25)$$

The correct sign depends on the direction in which the ramp is tilted, as the following examples will illustrate. We'll use Newton's laws of motion in Chapter 6 to verify Equation 2.25.

Equation 2.25 makes sense. Suppose the plane is perfectly horizontal. If you place an object on a horizontal surface, you expect it to stay at rest with no acceleration. Equation 2.25 gives $a_s = 0$ when $\theta = 0°$, in agreement with our expectations. Now suppose you tilt the plane until it becomes vertical, at $\theta = 90°$. Without friction, an object would simply fall, in free fall, parallel to the vertical surface. Equation 2.25 gives $a_s = -g = a_{\text{free fall}}$ when $\theta = 90°$, again in agreement with our expectations. We see that Equation 2.25 gives the correct result in these *limiting cases.*

FIGURE 2.32 Acceleration on an inclined plane.



Skiing is an example of motion on an inclined plane.

EXAMPLE 2.17 Skiing down an incline

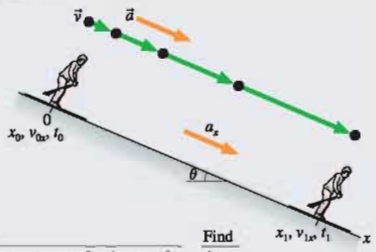
A skier's speed at the bottom of a 100-m-long, frictionless, snow-covered slope is 20 m/s. What is the angle of the slope?

MODEL Represent the skier as a particle. Assume that air resistance is negligible. Assume that the slope is a straight line.

VISUALIZE **FIGURE 2.33** on the next page shows the pictorial representation of the skier. Notice that we've chosen the x-axis to be parallel to the motion. Straight-line motion is almost always easier to analyze if the motion is parallel to a coordinate axis.

Continued

FIGURE 2.33 Pictorial representation for the skier of Example 2.17.



Known			Find
$x_0 = 0$ m	$v_{0x} = 0$ m/s	$t_0 = 0$ s	θ
$x_1 = 100$ m	$v_{1x} = 20$ m/s		
$a_x = g\sin\theta$			

SOLVE The motion diagram shows that the acceleration vector points in the positive x-direction. Thus the one-dimensional acceleration is $a_x = +g\sin\theta$. This is constant-acceleration motion. The third kinematic equation from Table 2.2 is

$$v_{1x}^2 = v_{0x}^2 + 2a_x\Delta x = 2g\sin\theta\Delta x$$

where we used $v_{0x} = 0$ m/s. Solving for $\sin\theta$, we find

$$\sin\theta = \frac{v_{1x}^2}{2g\Delta x} = \frac{(20\text{ m/s})^2}{2(9.80\text{ m/s}^2)(100\text{ m})} = 0.204$$

Thus

$$\theta = \sin^{-1}(0.204) = 12°$$

ASSESS A 100-m-long slope and a speed of 20 m/s ≈ 40 mph are fairly typical parameters for skiing. A 1° angle or an 80° angle would be unrealistic, but 12° seems plausible.

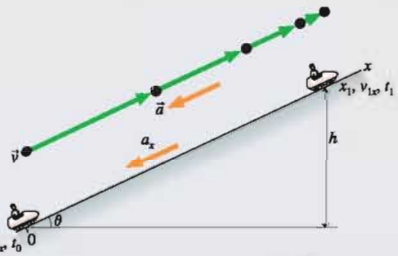
EXAMPLE 2.18 At the amusement park

An amusement park ride shoots a car up a frictionless track inclined at 30°. The car rolls up, then rolls back down. If the height of the track is 20 m, what is the maximum allowable speed with which the car can start?

MODEL Represent the car as a particle. Assume air resistance is negligible.

VISUALIZE FIGURE 2.34 shows the pictorial representation of the car. We've chosen the x-axis to be parallel to the motion. The problem starts as the car is shot up the incline, and it ends when the car reaches its highest point. The highest point is a turning point, so $v_{1x} = 0$ m/s. The motion diagram shows that the acceleration vector points in the negative x-direction, so we need the minus sign: $a_x = -g\sin\theta$. The *maximum* starting speed is that at which the car goes to the very top of the ramp, a height of 20 m.

FIGURE 2.34 Pictorial representation for the car of Example 2.18.



Known				Find
$x_0 = 0$ m	$t_0 = 0$ s	$\theta = 30°$	$h = 20$ m	v_{0x}
$v_{1x} = 0$ m/s	$a_x = -g\sin\theta$		$\Delta x = h/\sin\theta$	

SOLVE The maximum possible displacement Δx_{max} is related to the height h by

$$\Delta x_{max} = x_1 - x_0 = \frac{h}{\sin 30°} = \frac{20\text{ m}}{\sin 30°} = 40\text{ m}$$

The initial speed v_{0x} that allows the car to travel this distance is found from

$$v_{1x}^2 = 0 = v_{0x}^2 + 2a_x\Delta x = v_{0x}^2 - 2g\sin\theta\Delta x$$

$$v_{0x} = \sqrt{2g\sin 30°\Delta x} = \sqrt{2(9.80\text{ m/s}^2)(0.500)(40\text{ m})}$$

$$= 20\text{ m/s}$$

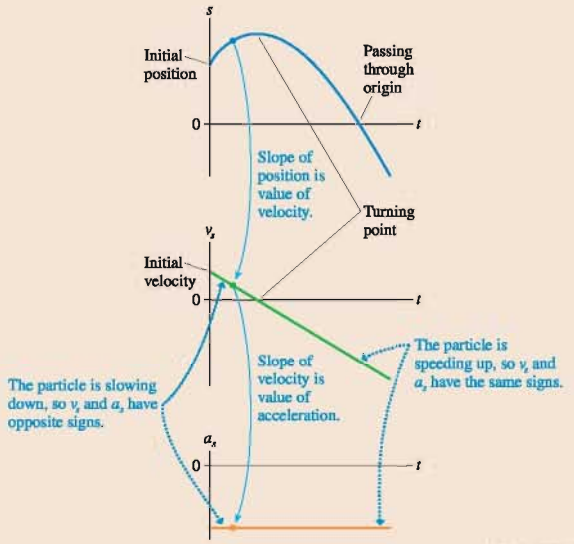
This is the maximum speed, because a car starting any faster will run off the top.

ASSESS 20 m ≈ 60 feet and 20 m/s ≈ 40 mph. It seems plausible that a car would need to be going this fast to gain 60 feet of elevation rolling up a ramp. Be sure you understand why the sign of a_x is negative here but positive in Example 2.17.

Thinking Graphically

Kinematics is the language of motion. We will spend the entire rest of this course studying moving objects, from baseballs to electrons, and the concepts we have developed in this chapter will be used extensively. One of the most important ideas, summarized in Tactics Box 2.2, has been that the relationships between position, velocity, and acceleration can be expressed graphically.

TACTICS BOX 2.2 **Interpreting graphical representations of motion**



Exercises 15, 16, 22

A good way to solidify your understanding of motion graphs is to consider the problem of a hard, smooth ball rolling on a smooth track. The track is made up of several straight segments connected together. Each segment may be either horizontal or inclined. Your task is to analyze the ball's motion graphically. This will require you to reason about, rather than calculate, the relationships between s, v_s, and a_s.

There are two variations to this type of problem. In the first, you are given a picture of a track and the initial condition of the ball. The problem is then to draw graphs of s, v_s, and a_s. In the second, you are given the graphs, and the problem is to deduce the shape of the track on which the ball is rolling.

There are a small number of rules to follow in each of these problems:

1. Assume that the ball passes smoothly from one segment of the track to the next, with no loss of speed and without ever leaving the track.
2. The position, velocity, and acceleration graphs should be stacked vertically. They should each have the same horizontal scale so that a vertical line drawn through all three connects points describing the same instant of time.
3. The graphs have no numbers, but they should show the correct *relationships*. For example, if the velocity is greater during the first part of the motion than during the second part, then the position graph should be steeper in the first part than in the second.
4. The position s is the position measured *along* the track. Similarly, v_s and a_s are the velocity and acceleration parallel to the track.

EXAMPLE 2.19 **From track to graphs**

Draw position, velocity, and acceleration graphs for the ball on the frictionless track of FIGURE 2.35.

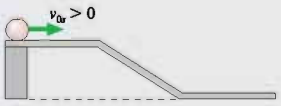


FIGURE 2.35 A ball rolling along a track.

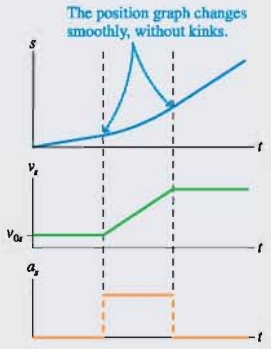
Continued

VISUALIZE It is often easiest to begin with the velocity. Here the ball starts with an initial velocity v_{0x}. There is no acceleration on the horizontal surface ($a_x = 0$ if $\theta = 0°$), so the velocity remains constant until the ball reaches the slope. The slope is an inclined plane that, as we have learned, has constant acceleration. The velocity increases linearly with time during constant-acceleration motion. The ball returns to constant-velocity motion after reaching the bottom horizontal segment. The middle graph of FIGURE 2.36 shows the velocity.

We have enough information to draw the acceleration graph. We noted that the acceleration is zero while the ball is on the horizontal segments, and a_x has a constant positive value on the slope. These accelerations are consistent with the slope of the velocity graph: zero slope, then positive slope, then a return to zero slope. The acceleration cannot *really* change instantly from zero to a nonzero value, but the change can be so quick that we do not see it on the time scale of the graph. That is what the vertical dotted lines imply.

Finally, we need to find the position-versus-time graph. You might want to refer back to Figure 2.35 to review how the position graph looks for constant-velocity and constant-acceleration motion. The position increases linearly with time during the first segment at constant velocity. It also does so during the third segment of motion, but with a steeper slope to indicate a faster velocity. In between, while the acceleration is nonzero but constant, the position graph has a *parabolic* shape.

FIGURE 2.36 Motion graphs for the ball in Example 2.19.



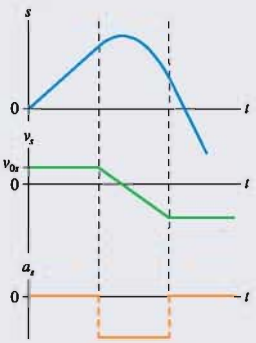
Two points are worth noting:

1. The dotted vertical lines through the graphs show the instants when the ball moves from one segment of the track to the next. Because of Rule 1, the speed does not change abruptly at these points; it changes gradually.
2. The parabolic section of the position-versus-time graph blends *smoothly* into the straight lines on either side. This is a consequence of Rule 1. An abrupt change of slope (a "kink") would indicate an abrupt change in velocity and would violate Rule 1.

EXAMPLE 2.20 From graphs to track

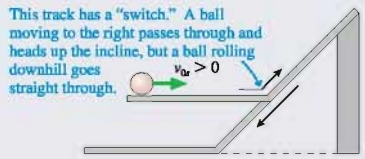
FIGURE 2.37 shows a set of motion graphs for a ball moving on a track. Draw a picture of the track and describe the ball's initial condition. Each segment of the track is *straight*, but the segments may be tilted.

FIGURE 2.37 Motion graphs of a ball rolling on a track of unknown shape.

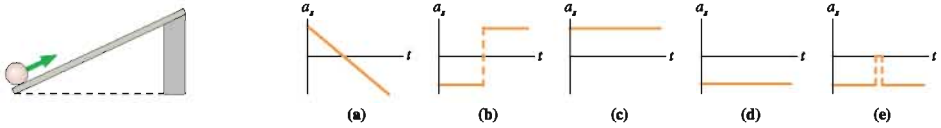


VISUALIZE Let's begin by examining the velocity graph. The ball starts with initial velocity $v_{0x} > 0$ and maintains this velocity for awhile; there's no acceleration. Thus the ball must start out rolling to the right on a horizontal track. At the end of the motion, the ball is again rolling on a horizontal track (no acceleration, constant velocity), but it's rolling to the *left* because v_x is negative. Further, the final speed ($|v_x|$) is greater than the initial speed. The middle section of the graph shows us what happens. The ball starts slowing with constant acceleration (rolling uphill), reaches a turning point (s is maximum, $v_x = 0$), then speeds up in the opposite direction (rolling downhill). This is still a negative acceleration because the ball is speeding up in the negative s-direction. It must roll farther downhill than it had rolled uphill before reaching a horizontal section of track. **FIGURE 2.38** shows the track and the initial conditions that are responsible for the graphs of Figure 2.37.

FIGURE 2.38 Track responsible for the motion graphs of Figure 2.37.



STOP TO THINK 2.5 The ball rolls up the ramp, then back down. Which is the correct acceleration graph?



2.7 Instantaneous Acceleration

FIGURE 2.39 shows a velocity that increases with time, reaches a maximum, then decreases. This is *not* uniformly accelerated motion. Instead, the acceleration is changing with time.

We can define an instantaneous acceleration in much the same way that we defined the instantaneous velocity. The instantaneous velocity was found to be the limit of the average velocity as the time interval $\Delta t \rightarrow 0$. Graphically, the instantaneous velocity at time t is the slope of the position-versus-time graph at that time. By analogy: **The instantaneous acceleration a_s at a specific instant of time t is the slope of the line that is tangent to the velocity-versus-time curve at time t.** Mathematically, this is

$$a_s \equiv \lim_{\Delta t \rightarrow 0} \frac{\Delta v_s}{\Delta t} = \frac{dv_s}{dt} \qquad \text{(instantaneous acceleration)} \qquad (2.26)$$

The instantaneous acceleration is the derivative (i.e., the rate of change) of the velocity.

The reverse problem—to find the velocity v_s if we know the acceleration a_s at all instants of time—is also important. When we wanted to find the position from the velocity, we took a velocity curve, divided it into N steps, found that the displacement Δs_k during step k was the area $(v_s)_k \, \Delta t$ of a small rectangle, then added all the steps (i.e., integrated) to find s_f.

We can do the same with acceleration. An acceleration curve can be divided into N very narrow steps so that during each step the acceleration is essentially constant. During step k, the velocity changes by $\Delta(v_s)_k = (a_s)_k \, \Delta t$. This is the area of the small rectangle under the step. The total velocity change between t_i and t_f is found by adding all the small $\Delta(v_s)_k$. In the limit $\Delta t \rightarrow 0$, we have

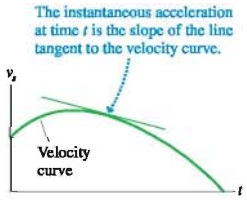
$$v_{fs} = v_{is} + \lim_{\Delta t \rightarrow 0} \sum_{k=1}^{N} (a_s)_k \Delta t = v_{is} + \int_{t_i}^{t_f} a_s \, dt \qquad (2.27)$$

This mathematical statement has a graphical interpretation analogous to Equation 2.11. In this case:

$$v_{fs} = v_{is} + \text{area under the acceleration curve } a_s \text{ between } t_i \text{ and } t_f \qquad (2.28)$$

The constant-acceleration equation $v_{fs} = v_{is} + a_s \Delta t$ is a special example of Equation 2.28. If you look back at Figure 2.24a you will see that the quantity $a_s \Delta t$ is the rectangular area under the horizontal acceleration curve.

FIGURE 2.39 Motion with nonuniform acceleration.



EXAMPLE 2.21 Finding velocity from acceleration

FIGURE 2.40 shows the acceleration graph for a particle with an initial velocity of 10 m/s. What is the particle's velocity at $t = 8$ s?

MODEL We're told this is the motion of a particle.

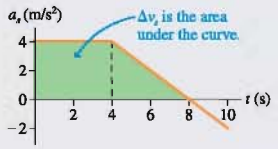


FIGURE 2.40 Acceleration graph for Example 2.21.

Continued

VISUALIZE Figure 2.40 is a graphical representation of the motion.

SOLVE The change in velocity is found as the area under the acceleration curve:

$$v_{1x} = v_{0x} + \text{area under the acceleration curve } a_x \text{ between } t_i \text{ and } t_f$$

The area under the curve between $t_i = 0$ s and $t_f = 8$ s can be subdivided into a rectangle ($0\text{ s} \le t \le 4\text{ s}$) and a triangle ($4\text{ s} \le t \le 8\text{ s}$). These areas are easily computed. Thus

$$\begin{aligned} v_x(\text{at } t = 8\text{ s}) &= 10\text{ m/s} + (4\text{ (m/s)/s})(4\text{ s}) \\ &\quad + \tfrac{1}{2}(4\text{ (m/s)/s})(4\text{ s}) \\ &= 34\text{ m/s} \end{aligned}$$

EXAMPLE 2.22 A nonuniform acceleration

FIGURE 2.41a shows the velocity-versus-time graph for a particle whose velocity is given by $v_x = [10 - (t - 5)^2]$ m/s, where t is in s.

a. Find an expression for the particle's acceleration a_x and draw the acceleration-versus-time graph.
b. Describe the motion.

MODEL We're told that this is a particle.

VISUALIZE The figure shows the velocity graph. It is a parabola centered at $t = 5$ s with an apex $v_{max} = 10$ m/s. The slope of v_x is positive but decreasing in magnitude for $t < 5$ s. The slope is zero at $t = 5$ s, and it is negative and increasing in magnitude for $t > 5$ s. Thus the acceleration graph should start positive, decrease steadily, pass through zero at $t = 5$ s, then become increasingly negative.

SOLVE a. We can find an expression for a_x by taking the derivative of v_x. First, expand the square to give

$$v_x = (-t^2 + 10t - 15)\text{ m/s}$$

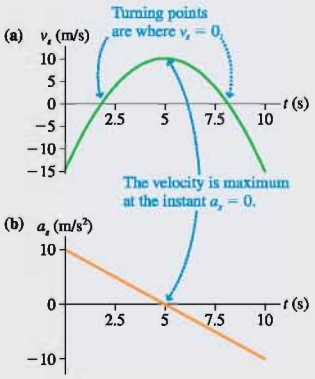
Then use the derivative rule (Equation 2.5) to find

$$a_x = \frac{dv_x}{dt} = (-2t + 10)\text{ m/s}^2$$

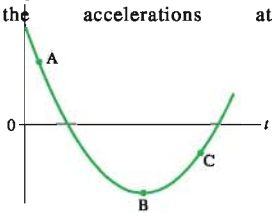
where t is in s. This is a linear equation that is graphed in **FIGURE 2.41b**. The graph meets our expectations.

b. This is a complex motion. The particle starts out moving to the left ($v_x < 0$) at 15 m/s. The positive acceleration causes the speed to decrease (slowing down because v_x and a_x have opposite signs) until the particle reaches a turning point ($v_x = 0$) just before $t = 2$ s. The particle then moves to the right ($v_x > 0$) and speeds up until reaching maximum speed at $t = 5$ s. From $t = 5$ s to just after $t = 8$ s, the particle is still moving to the right ($v_x > 0$) but slowing down. Another turning point occurs just after $t = 8$ s. Then the particle moves back to the left and gains speed as the negative a_x makes the velocity ever more negative.

FIGURE 2.41 Velocity and acceleration graphs for Example 2.22.



STOP TO THINK 2.6 Rank in order, v_x from most positive to least positive, the accelerations at points A to C.



a. $a_A > a_B > a_C$
b. $a_C > a_A > a_B$
c. $a_C > a_B > a_A$
d. $a_B > a_A > a_C$

SUMMARY

The goal of Chapter 2 has been to learn how to solve problems about motion in a straight line.

General Principles

Kinematics describes motion in terms of position, velocity, and acceleration.

General kinematic relationships are given **mathematically** by:

Instantaneous velocity $v_s = ds/dt$ = slope of position graph

Instantaneous acceleration $a_s = dv_s/dt$ = slope of velocity graph

Final position
$$s_f = s_i + \int_{t_i}^{t_f} v_s\,dt = s_i + \begin{cases}\text{area under the velocity}\\ \text{curve from } t_i \text{ to } t_f\end{cases}$$

Final velocity
$$v_{fs} = v_{is} + \int_{t_i}^{t_f} a_s\,dt = v_{is} + \begin{cases}\text{area under the acceleration}\\ \text{curve from } t_i \text{ to } t_f\end{cases}$$

Motion with constant acceleration is **uniformly accelerated motion**. The kinematic equations are:

$$v_{fs} = v_{is} + a_s\Delta t$$
$$s_f = s_i + v_{is}\Delta t + \tfrac{1}{2}a_s(\Delta t)^2$$
$$v_{fs}^2 = v_{is}^2 + 2a_s\Delta s$$

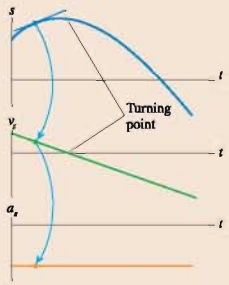
Uniform motion is motion with constant velocity and zero acceleration:

$$s_f = s_i + v_s\Delta t$$

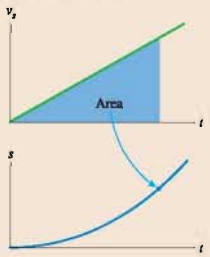
Important Concepts

Position, velocity, and acceleration are related **graphically.**

- The slope of the position-versus-time graph is the value on the velocity graph.
- The slope of the velocity graph is the value on the acceleration graph.
- s is a maximum or minimum at a turning point, and $v_s = 0$.



- Displacement is the area under the velocity curve.



Applications

The **sign of v_s** indicates the direction of motion.

- $v_s > 0$ is motion to the right or up.
- $v_s < 0$ is motion to the left or down.

The **sign of a_s** indicates which way $\vec{a}$ points, *not* whether the object is speeding up or slowing down.

- $a_s > 0$ if $\vec{a}$ points to the right or up.
- $a_s < 0$ if $\vec{a}$ points to the left or down.
- The direction of $\vec{a}$ is found with a motion diagram.

An object is **speeding up** if and only if v_s and a_s have the same sign. An object is **slowing down** if and only if v_s and a_s have opposite signs.

Free fall is constant-acceleration motion with

$$a_y = -g = -9.80\ \text{m/s}^2$$

Motion on an inclined plane has $a_s = \pm g\sin\theta$. The sign depends on the direction of the tilt.

Terms and Notation

kinematics	initial position, s_i	turning point	free fall
uniform motion	final position, s_f	average acceleration, a_{avg}	free-fall acceleration, g
speed, v	instantaneous velocity, v_s	uniformly accelerated motion	instantaneous acceleration, a_s

For homework assigned on MasteringPhysics, go to www.masteringphysics.com

Problem difficulty is labeled as | (straightforward) to ||| (challenging).

Problems labeled can be done on a Dynamics Worksheet.

Problems labeled integrate significant material from earlier chapters.

CONCEPTUAL QUESTIONS

For Questions 1 through 3, interpret the position graph given in each figure by writing a very short "story" of what is happening. Be creative! Have characters and situations! Simply saying that "a car moves 100 meters to the right" doesn't qualify as a story. Your stories should make *specific reference* to information you obtain from the graph, such as distance moved or time elapsed.

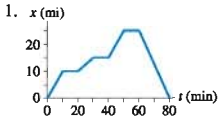
1. 

FIGURE Q2.1

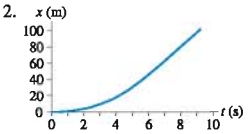
2. 

FIGURE Q2.2

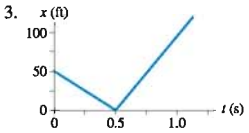
3. 

FIGURE Q2.3

4. FIGURE Q2.4 shows a position-versus-time graph for the motion of objects A and B as they move along the same axis.
   a. At the instant $t = 1$ s, is the speed of A greater than, less than, or equal to the speed of B? Explain.
   b. Do objects A and B ever have the *same* speed? If so, at what time or times? Explain.

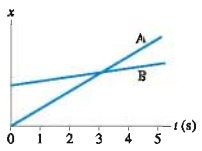


FIGURE Q2.4

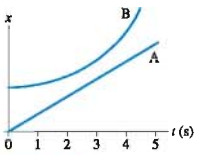


FIGURE Q2.5

5. FIGURE Q2.5 shows a position-versus-time graph for the motion of objects A and B as they move along the same axis.
   a. At the instant $t = 1$ s, is the speed of A greater than, less than, or equal to the speed of B? Explain.
   b. Do objects A and B ever have the *same* speed? If so, at what time or times? Explain.

6. FIGURE Q2.6 shows the position-versus-time graph for a moving object. At which lettered point or points:
   a. Is the object *moving* the slowest?
   b. Is the object moving the fastest?
   c. Is the object at rest?
   d. Is the object moving to the left?

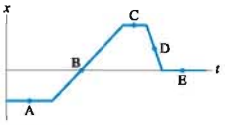


FIGURE Q2.6

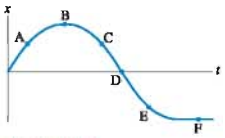


FIGURE Q2.7

7. FIGURE Q2.7 shows the position-versus-time graph for a moving object. At which lettered point or points:
   a. Is the object moving the fastest?
   b. Is the object moving to the left?
   c. Is the object speeding up?
   d. Is the object turning around?
8. FIGURE Q2.8 shows six frames from the motion diagrams of two moving cars, A and B.
   a. Do the two cars ever have the same position at one instant of time? If so, in which frame number (or numbers)?
   b. Do the two cars ever have the same velocity at one instant of time? If so, between which two frames?

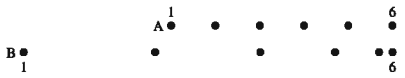


FIGURE Q2.8

9. You're driving along the highway at a steady speed of 60 mph when another car decides to pass you. At the moment when the front of his car is exactly even with the front of your car, and you turn your head to smile at him, do the two cars have equal velocities? Explain.
10. A car is traveling north. Can its acceleration vector ever point south? Explain.
11. (a) Give an example of a vertical motion with a positive velocity and a negative acceleration. (b) Give an example of a vertical motion with a negative velocity and a negative acceleration.
12. A ball is thrown straight up into the air. At each of the following instants, is the magnitude of the ball's acceleration greater than g, equal to g, less than g, or 0? Explain.
    a. Just after leaving your hand.
    b. At the very top (maximum height).
    c. Just before hitting the ground.

13. A rock is *thrown* (not dropped) straight down from a bridge into the river below. At each of the following instants, is the magnitude of the rock's acceleration greater than g, equal to g, less than g, or 0? Explain.
    a. Immediately after being released.
    b. Immediately before hitting the water.

14. Drop a rubber ball or a tennis ball from a height of about 25 cm (≈1 ft) and watch carefully as it bounces. Draw a position graph, a velocity graph, and an acceleration graph showing the ball's motion from the instant you drop it until it returns to its maximum height. Stack your three graphs vertically so that the time axes are aligned with each other. Pay particular attention to the time when the ball is in contact with the ground. This is a short interval of time, but it's not zero.

EXERCISES AND PROBLEMS

Exercises

Section 2.1 Uniform Motion

1. | A car starts at the origin and moves with velocity $\vec{v}$ = (10 m/s, northeast). How far from the origin will the car be after traveling for 45 s?
2. || Larry leaves home at 9:05 and runs at constant speed to the lamppost. He reaches the lamppost at 9:07, immediately turns, and runs to the tree. Larry arrives at the tree at 9:10.
    a. What is Larry's average velocity, in yards/min, during each of these two intervals.
    b. What is Larry's average velocity for the entire run?

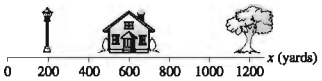


FIGURE EX2.2

3. || Alan leaves Los Angeles at 8:00 a.m. to drive to San Francisco, 400 mi away. He travels at a steady 50 mph. Beth leaves Los Angeles at 9:00 a.m. and drives a steady 60 mph.
    a. Who gets to San Francisco first?
    b. How long does the first to arrive have to wait for the second?
4. || Julie drives 100 mi to Grandmother's house. On the way to Grandmother's, Julie drives half the distance at 40 mph and half the distance at 60 mph. On her return trip, she drives half the time at 40 mph and half the time at 60 mph.
    a. What is Julie's average speed on the way to Grandmother's house?
    b. What is her average speed on the return trip?
5. | A bicyclist has the position-versus-time graph shown. What is the bicyclist's velocity at t = 10 s, at t = 25 s, and at t = 35 s?

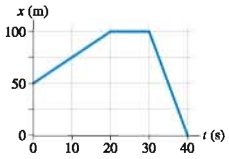


FIGURE EX2.5

Section 2.2 Instantaneous Velocity

Section 2.3 Finding Position from Velocity

6. | FIGURE EX2.6 shows the position graph of a particle.
    a. Draw the particle's velocity graph for the interval 0 s ≤ t ≤ 4 s.
    b. Does this particle have a turning point or points? If so, at what time or times?

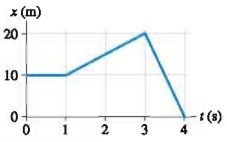


FIGURE EX2.6

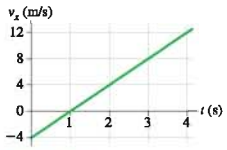


FIGURE EX2.7

7. | A particle starts from x_0 = 10 at t_0 = 0 and moves with the velocity graph shown in FIGURE EX2.7.
    a. Does this particle have a turning point? If so, at what time?
    b. What is the object's position at t = 2 s, 3 s, and 4 s?

Section 2.4 Motion with Constant Acceleration

8. | FIGURE EX2.8 shows the velocity graph of a particle. Draw the particle's acceleration graph for the interval 0 s ≤ t ≤ 4 s. Give both axes an appropriate numerical scale.

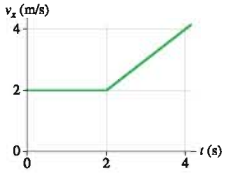


FIGURE EX2.8

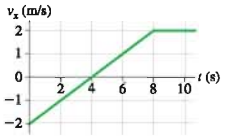


FIGURE EX2.9

9. | FIGURE EX2.9 shows the velocity graph of a train that starts from the origin at t = 0 s.
    a. Find the acceleration of the train at t = 3.0 s.
    b. Draw position and acceleration graphs for the train.
10. | FIGURE EX2.10 shows the velocity graph of a particle moving along the x-axis. Its initial position is x_0 = 2.0 m at t_0 = 0 s. At t = 2.0 s, what are the particle's (a) position, (b) velocity, and (c) acceleration?

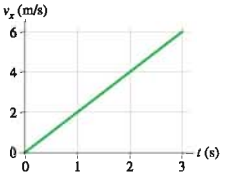


FIGURE EX2.10

11. | FIGURE EX2.11 shows the velocity-versus-time graph is shown for a particle moving along the x-axis. Its initial position is $x_0 = 2.0$ m at $t_0 = 0$ s.
    a. What are the particle's position, velocity, and acceleration at $t_0 = 1.0$ s?
    b. What are the particle's position, velocity, and acceleration at $t_0 = 3.0$ s?

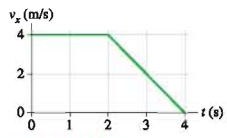


FIGURE EX2.11

12. || A jet plane is cruising at 300 m/s when suddenly the pilot turns the engines up to full throttle. After traveling 4.0 km, the jet is moving with a speed of 400 m/s.
    a. What is the jet's acceleration, assuming it to be a constant acceleration?
    b. Is your answer reasonable? Explain.
13. || A speed skater moving across frictionless ice at 8.0 m/s hits a 5.0-m-wide patch of rough ice. She slows steadily, then continues on at 6.0 m/s. What is her acceleration on the rough ice?
14. || A Porsche challenges a Honda to a 400 m race. Because the Porsche's acceleration of 3.5 m/s^2 is larger than the Honda's 3.0 m/s^2, the Honda gets a 50 m head start. Both cars start accelerating at the same instant. Who wins?

Section 2.5 Free Fall

15. | Ball bearings are made by letting spherical drops of molten metal fall inside a tall tower—called a *shot tower*—and solidify as they fall.
    a. If a bearing needs 4.0 s to solidify enough for impact, how high must the tower be?
    b. What is the bearing's impact velocity?
16. | A ball is thrown vertically upward with a speed of 19.6 m/s.
    a. What is the ball's velocity and its height after 1.0, 2.0, 3.0, and 4.0 s?
    b. Draw the ball's velocity-versus-time graph. Give both axes an appropriate numerical scale.
17. || A student standing on the ground throws a ball straight up. The ball leaves the student's hand with a speed of 15 m/s when the hand is 2.0 m above the ground. How long is the ball in the air before it hits the ground? (The student moves her hand out of the way.)
18. || A rock is tossed straight up with a speed of 20 m/s. When it returns, it falls into a hole 10 m deep.
    a. What is the rock's velocity as it hits the bottom of the hole?
    b. How long is the rock in the air, from the instant it is released until it hits the bottom of the hole?

Section 2.6 Motion on an Inclined Plane

19. || A skier is gliding along at 3.0 m/s on horizontal, frictionless snow. He suddenly starts down a 10° incline. His speed at the bottom is 15 m/s.
    a. What is the length of the incline?
    b. How long does it take him to reach the bottom?
20. || A car traveling at 30 m/s runs out of gas while traveling up a 20° slope. How far up the hill will it coast before starting to roll back down?

Section 2.7 Instantaneous Acceleration

21. | A particle moving along the x-axis has its position described by the function $x = (2t^2 - t + 1)$ m, where t is in s. At $t = 2$ s what are the particle's (a) position, (b) velocity, and (c) acceleration?
22. || A particle moving along the x-axis has its velocity described by the function $v_x = 2t^2$ m/s, where t is in s. Its initial position is $x_0 = 1$ m at $t_0 = 0$ s. At $t = 1$ s what are the particle's (a) position, (b) velocity, and (c) acceleration?
23. || FIGURE EX2.23 shows the acceleration-versus-time graph of a particle moving along the x-axis. Its initial velocity is $v_{0x} = 8.0$ m/s at $t_0 = 0$ s. What is the particle's velocity at $t = 4.0$ s?

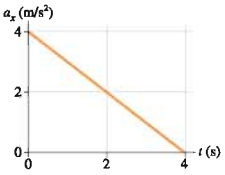


FIGURE EX2.23

Problems

24. | FIGURE P2.24 shows the motion diagram, made at two frames of film per second, of a ball rolling along a track. The track has a 3.0-m-long sticky section.
    a. Use the meter stick to measure the positions of the center of the ball. Place your data in a table, similar to Table 1.1, showing each position and the instant of time at which it occurred.
    b. Make a position-versus-time graph for the ball. Because you have data only at certain instants of time, your graph should consist of dots that are not connected together.
    c. What is the *change* in the ball's position from $t = 0$ s to $t = 1.0$ s?
    d. What is the *change* in the ball's position from $t = 2.0$ s to $t = 4.0$ s?
    e. What is the ball's velocity before reaching the sticky section?
    f. What is the ball's velocity after passing the sticky section?
    g. Determine the ball's acceleration on the sticky section of the track.

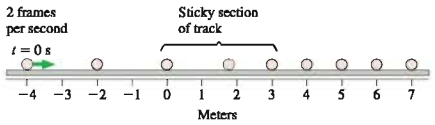


FIGURE P2.24

25. || A particle's position on the x-axis is given by the function $x = (t^2 - 4t + 2)$ m where t is in s.
    a. Make a position-versus-time graph for the interval $0 \text{ s} \le t \le 5$ s. Do this by calculating and plotting x every 0.5 s from 0 s to 5 s, then drawing a smooth curve through the points.

b. Determine the particle's velocity at $t = 1.0$ s by drawing the tangent line on your graph and measuring its slope.
c. Determine the particle's velocity at $t = 1.0$ s by evaluating the derivative at that instant. Compare this to your result from part b.
d. Are there any turning points in the particle's motion? If so, at what position or positions?
e. Where is the particle when $v_x = 4.0$ m/s?
f. Draw a motion diagram for the particle.

26. || Three particles move along the x-axis, each starting with $v_{0x} = 10$ m/s at $t_0 = 0$ s. The graph for A is a position-versus-time graph; the graph for B is a velocity-versus-time graph; the graph for C is an acceleration-versus-time graph. Find each particle's velocity at $t = 7.0$ s. Work with the geometry of the graphs, not with kinematic equations.

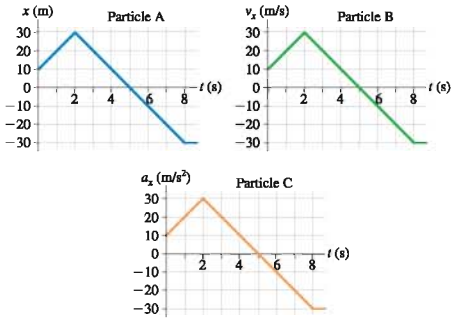


FIGURE P2.26

27. || FIGURE P2.27 shows the velocity graph for a particle having initial position $x_0 = 0$ m at $t_0 = 0$ s.
a. At what time or times is the particle found at $x = 35$ m? Work with the geometry of the graph, not with kinematic equations.
b. Draw a motion diagram for the particle.

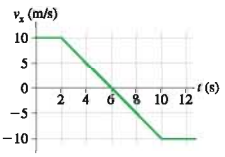


FIGURE P2.27

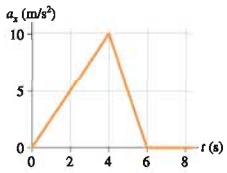


FIGURE P2.28

28. || FIGURE P2.28 shows the acceleration graph for a particle that starts from rest at $t = 0$ s. Determine the object's velocity at times $t = 0$ s, 2 s, 4 s, 6 s, and 8 s.
29. || A block is suspended from a spring, pulled down, and released. The block's position-versus-time graph is shown in FIGURE P2.29.
a. At what times is the velocity zero? At what times is the velocity most positive? Most negative?
b. Draw a reasonable velocity-versus-time graph.

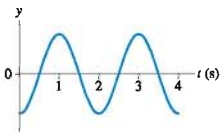


FIGURE P2.29

30. || FIGURE P2.30 shows the acceleration graph for a particle that starts from rest at $t = 0$ s.
a. Draw the particle's velocity graph over the interval $0\text{ s} \le t \le 10$ s. Include an appropriate numerical scale on both axes.
b. Describe, in words, how the velocity graph would differ if the particle had an initial velocity of 2.0 m/s.

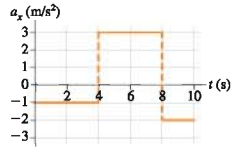


FIGURE P2.30

31. | The position of a particle is given by the function $x = (2t^3 - 9t^2 + 12)$ m, where t is in s.
a. At what time or times is $v_x = 0$ m/s?
b. What are the particle's position and its acceleration at this time(s)?
32. | An object starts from rest at $x = 0$ m at time $t = 0$ s. Five seconds later, at $t = 5.0$ s, the object is observed to be at $x = 40.0$ m and to have velocity $v_x = 11$ m/s.
a. Was the object's acceleration uniform or nonuniform? Explain your reasoning.
b. Sketch the velocity-versus-time graph implied by these data. Is the graph a straight line or curved? If curved, is it concave upward or downward?
33. || A particle's velocity is described by the function $v_x = kt^2$ m/s, where k is a constant and t is in s. The particle's position at $t_0 = 0$ s is $x_0 = -9.0$ m. At $t_1 = 3.0$ s, the particle is at $x_1 = 9.0$ m. Determine the value of the constant k. Be sure to include the proper units.
34. || A particle's acceleration is described by the function $a_x = (10 - t)$ m/s^2, where t is in s. Its initial conditions are $x_0 = 0$ m and $v_{0x} = 0$ m/s at $t = 0$ s.
a. At what time is the velocity again zero?
b. What is the particle's position at that time?
35. || A ball rolls along the frictionless track shown in FIGURE P2.35. Each segment of the track is straight, and the ball passes smoothly from one segment to the next without changing speed or leaving the track. Draw three vertically stacked graphs showing position, velocity, and acceleration versus time. Each graph should have the same time axis, and the proportions of the graph should be qualitatively correct. Assume that the ball has enough speed to reach the top.

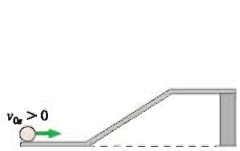


FIGURE P2.35

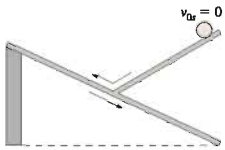


FIGURE P2.36

36. || Draw position, velocity, and acceleration graphs for the ball shown in FIGURE P2.36. See Problem 35 for more information.

37. || Draw position, velocity, and acceleration graphs for the ball shown in FIGURE P2.37. See Problem 35 for more information. The ball changes direction but not speed as it bounces from the reflecting wall.

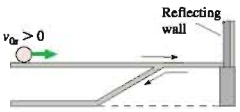


FIGURE P2.37

38. || FIGURE P2.38 shows a set of kinematic graphs for a ball rolling on a track. All segments of the track are straight lines, but some may be tilted. Draw a picture of the track and also indicate the ball's initial condition.

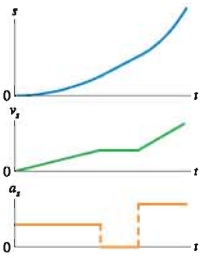


FIGURE P2.38

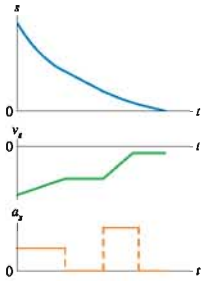


FIGURE P2.39

39. || FIGURE P2.39 shows a set of kinematic graphs for a ball rolling on a track. All segments of the track are straight lines, but some may be tilted. Draw a picture of the track and also indicate the ball's initial condition.

40. || FIGURE P2.40 shows a set of kinematic graphs for a ball rolling on a track. All segments of the track are straight lines, but some may be tilted. Draw a picture of the track and also indicate the ball's initial condition.

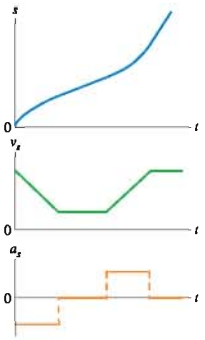


FIGURE P2.40

41. || The takeoff speed for an Airbus A320 jetliner is 80 m/s. Velocity data measured during takeoff are as shown.

t (s)	v_x (m/s)
0	0
10	23
20	46
30	69

a. What is the takeoff speed in miles per hour?
b. Is the jetliner's acceleration constant during takeoff? Explain.
c. At what time do the wheels leave the ground?
d. For safety reasons, in case of an aborted takeoff, the runway must be three times the takeoff distance. Can an A320 take off safely on a 2.5-mi-long runway?

42. || Does a real automobile have constant acceleration? Measured data for a Porsche 944 Turbo at maximum acceleration are as shown.

t (s)	v_x (mph)
0	0
2	28
4	46
6	60
8	70
10	78

a. Make a graph of velocity versus time. Based on your graph, is the acceleration constant? Explain.
b. Draw a smooth curve through the points on your graph, then use your graph to estimate the car's acceleration at 2.0 s and 8.0 s. Give your answer in SI units.
c. Use your graph to estimate the distance traveled in the first 10 s.

43. || a. What constant acceleration, in SI units, must a car have to go from zero to 60 mph in 10 s?
b. What fraction of g is this?
c. How far has the car traveled when it reaches 60 mph? Give your answer both in SI units and in feet.

44. || a. How many days will it take a spaceship to accelerate to the speed of light (3.0×10^8 m/s) with the acceleration g?
b. How far will it travel during this interval?
c. What fraction of a light year is your answer to part b? A *light year* is the distance light travels in one year.

NOTE ▶ We know, from Einstein's theory of relativity, that no object can travel at the speed of light. So this problem, while interesting and instructive, is not realistic. ◀

45. || A driver has a reaction time of 0.50 s, and the maximum deceleration of her car is 6.0 m/s^2. She is driving at 20 m/s when suddenly she sees an obstacle in the road 50 m in front of her. Can she stop the car in time to avoid a collision?

46. || You are driving to the grocery store at 20 m/s. You are 110 m from an intersection when the traffic light turns red. Assume that your reaction time is 0.50 s and that your car brakes with constant acceleration.
a. How far are you from the intersection when you begin to apply the brakes?
b. What acceleration will bring you to rest right at the intersection?
c. How long does it take you to stop after the light turns red?

47. || You're driving down the highway late one night at 20 m/s when a deer steps onto the road 35 m in front of you. Your reaction time before stepping on the brakes is 0.50 s, and the maximum deceleration of your car is 10 m/s^2.
a. How much distance is between you and the deer when you come to a stop?
b. What is the maximum speed you could have and still not hit the deer?

48. III The minimum stopping distance for a car traveling at a speed of 30 m/s is 60 m, including the distance traveled during the driver's reaction time of 0.50 s.
    a. What is the minimum stopping distance for the same car traveling at a speed of 40 m/s?
    b. Draw a position-versus-time graph for the motion of the car in part a. Assume the car is at $x_0 = 0$ m when the driver first sees the emergency situation ahead that calls for a rapid halt.
49. III A 200 kg weather rocket is loaded with 100 kg of fuel and fired straight up. It accelerates upward at 30 m/s^2 for 30 s, then runs out of fuel. Ignore any air resistance effects.
    a. What is the rocket's maximum altitude?
    b. How long is the rocket in the air before hitting the ground?
    c. Draw a velocity-versus-time graph for the rocket from liftoff until it hits the ground.
50. II A 1000 kg weather rocket is launched straight up. The rocket motor provides a constant acceleration for 16 s, then the motor stops. The rocket altitude 20 s after launch is 5100 m. You can ignore any effects of air resistance.
    a. What was the rocket's acceleration during the first 16 s?
    b. What is the rocket's speed as it passes through a cloud 5100 m above the ground?
51. II A lead ball is dropped into a lake from a diving board 5.0 m above the water. After entering the water, it sinks to the bottom with a constant velocity equal to the velocity with which it hit the water. The ball reaches the bottom 3.0 s after it is released. How deep is the lake?
52. II A hotel elevator ascends 200 m with a maximum speed of 5.0 m/s. Its acceleration and deceleration both have a magnitude of 1.0 m/s^2.
    a. How far does the elevator move while accelerating to full speed from rest?
    b. How long does it take to make the complete trip from bottom to top?
53. II A car starts from rest at a stop sign. It accelerates at 4.0 m/s^2 for 6.0 s, coasts for 2.0 s, and then slows down at a rate of 3.0 m/s^2 for the next stop sign. How far apart are the stop signs?
54. II A car accelerates at 2.0 m/s^2 along a straight road. It passes two marks that are 30 m apart at times $t = 4.0$ s and $t = 5.0$ s. What was the car's velocity at $t = 0$ s?
55. II Santa loses his footing and slides down a frictionless, snowy roof that is tilted at an angle of 30°. If Santa slides 10 m before reaching the edge, what is his speed as he leaves the roof?
56. II Ann and Carol are driving their cars along the same straight road. Carol is located at $x = 2.4$ mi at $t = 0$ hours and drives at a steady 36 mph. Ann, who is traveling in the same direction, is located at $x = 0.0$ mi at $t = 0.50$ hours and drives at a steady 50 mph.
    a. At what time does Ann overtake Carol?
    b. What is their position at this instant?
    c. Draw a position-versus-time graph showing the motion of both Ann and Carol.
57. II A puck slides along the frictionless track shown in FIGURE P2.57 with an initial speed of 5.0 m/s. Assume the puck turns all the corners smoothly, with no loss of speed.
    a. What is the puck's speed as it goes over the top?
    b. What is its speed when it reaches the level track on the right side?
    c. By what percentage does the puck's final speed differ from its initial speed?

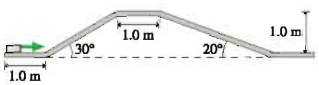


FIGURE P2.57

58. II A toy train is pushed forward and released at $x_0 = 2.0$ m with a speed of 2.0 m/s. It rolls at a steady speed for 2.0 s, then one wheel begins to stick. The train comes to a stop 6.0 m from the point at which it was released. What is the magnitude of the train's acceleration after its wheel begins to stick?
59. II Bob is driving the getaway car after the big bank robbery. He's going 50 m/s when his headlights suddenly reveal a nail strip that the cops have placed across the road 150 m in front of him. If Bob can stop in time, he can throw the car into reverse and escape. But if he crosses the nail strip, all his tires will go flat and he will be caught. Bob's reaction time before he can hit the brakes is 0.60 s, and his car's maximum deceleration is 10 m/s^2. Is Bob in jail?
60. II One game at the amusement park has you push a puck up a long, frictionless ramp. You win a stuffed animal if the puck, at its highest point, comes to within 10 cm of the end of the ramp without going off. You give the puck a push, releasing it with a speed of 5.0 m/s when it is 8.5 m from the end of the ramp. The puck's speed after traveling 3.0 m is 4.0 m/s. Are you a winner?
61. II A professional skier's *initial* acceleration on fresh snow is 90% of the acceleration expected on a frictionless, inclined plane, the loss being due to friction. Due to air resistance, his acceleration slowly decreases as he picks up speed. The speed record on a mountain in Oregon is 180 kilometers per hour at the bottom of a 25° slope that drops 200 m.
    a. What exit speed could a skier reach in the absence of air resistance?
    b. What percentage of this ideal speed is lost to air resistance?
62. II Heather and Jerry are standing on a bridge 50 m above a river. Heather throws a rock straight down with a speed of 20 m/s. Jerry, at exactly the same instant of time, throws a rock straight up with the same speed. Ignore air resistance.
    a. How much time elapses between the first splash and the second splash?
    b. Which rock has the faster speed as it hits the water?
63. II Nicole throws a ball straight up. Chad watches the ball from a window 5.0 m above the point where Nicole released it. The ball passes Chad on the way up, and it has a speed of 10 m/s as it passes him on the way back down. How fast did Nicole throw the ball?
64. II A motorist is driving at 20 m/s when she sees that a traffic light 200 m ahead has just turned red. She knows that this light stays red for 15 s, and she wants to reach the light just as it turns green again. It takes her 1.0 s to step on the brakes and begin slowing. What is her speed as she reaches the light at the instant it turns green?
65. II When a 1984 Alfa Romeo Spider sports car accelerates at the maximum possible rate, its motion during the first 20 s is extremely well modeled by the simple equation

$$v_x^2 = \frac{2P}{m}t$$

where $P = 3.6 \times 10^4$ watts is the car's power output, $m = 1200$ kg is its mass, and v_x is in m/s. That is, the square of the car's velocity increases linearly with time.
    a. What is the car's speed at $t = 10$ s and at $t = 20$ s?

b. Find a *symbolic expression*, in terms of P, m, and t, for the car's acceleration at time t.
c. Evaluate the acceleration at $t = 1$ s and $t = 10$ s.
d. This simple model fails for t less than about 0.5 s. Explain how you can recognize the failure.
e. Find a *symbolic expression* for the distance the car has traveled at time t.
f. One-quarter mile is 402 m. What is the Spider's best time in a quarter-mile race? (The model's failure in the first 0.5 s has very little effect on your answer because the car travels almost no distance during that time.)

66. || David is driving a steady 30 m/s when he passes Tina, who is sitting in her car at rest. Tina begins to accelerate at a steady 2.0 m/s^2 at the instant when David passes.
a. How far does Tina drive before passing David?
b. What is her speed as she passes him?

67. || A cat is sleeping on the floor in the middle of a 3.0-m-wide room when a barking dog enters with a speed of 1.50 m/s. As the dog enters, the cat (as only cats can do) immediately accelerates at 0.85 m/s^2 toward an open window on the opposite side of the room. The dog (all bark and no bite) is a bit startled by the cat and begins to slow down at 0.10 m/s^2 as soon as it enters the room. Does the dog catch the cat before the cat is able to leap through the window?

68. || You want to visit your friend in Seattle during spring break. To save money, you decide to travel there by train. Unfortunately, your physics final exam took the full 3 hours, so you are late in arriving at the train station. You run as fast as you can, but just as you reach the platform you see your train, 30 m ahead of you down the platform, begin to accelerate at 1.0 m/s^2. You chase after the train at your maximum speed of 8.0 m/s, but there's a barrier 50 m ahead. Will you be able to leap onto the back step of the train before you crash into the barrier?

69. || Jill has just gotten out of her car in the grocery store parking lot. The parking lot is on a hill and is tilted 3°. Fifty meters downhill from Jill, a little old lady lets go of a fully loaded shopping cart. The cart, with frictionless wheels, starts to roll straight downhill. Jill immediately starts to sprint after the cart with her top acceleration of 2.0 m/s^2. How far has the cart rolled before Jill catches it?

70. || As a science project, you drop a watermelon off the top of the Empire State Building, 320 m above the sidewalk. It so happens that Superman flies by at the instant you release the watermelon. Superman is headed straight down with a speed of 35 m/s. How fast is the watermelon going when it passes Superman?

71. || I was driving along at 20 m/s, trying to change a CD and not watching where I was going. When I looked up, I found myself 45 m from a railroad crossing. And wouldn't you know it, a train moving at 30 m/s was only 60 m from the crossing. In a split second, I realized that the train was going to beat me to the crossing and that I didn't have enough distance to stop. My only hope was to accelerate enough to cross the tracks before the train arrived. If my reaction time before starting to accelerate was 0.50 s, what minimum acceleration did my car need for me to be here today writing these words?

In Problems 72 through 75, you are given the kinematic equation or equations that are used to solve a problem. For each of these, you are to:
a. Write a *realistic* problem for which this is the correct equation(s). Be sure that the answer your problem requests is consistent with the equation(s) given.
b. Draw the pictorial representation for your problem.
c. Finish the solution of the problem.

72. $64\text{ m} = 0\text{ m} + (32\text{ m/s})(4\text{ s} - 0\text{ s}) + \frac{1}{2}a_x(4\text{ s} - 0\text{ s})^2$

73. $(10\text{ m/s})^2 = v_{0y}{}^2 - 2(9.8\text{ m/s}^2)(10\text{ m} - 0\text{ m})$

74. $(0\text{ m/s})^2 = (5\text{ m/s})^2 - 2(9.8\text{ m/s}^2)(\sin 10°)(x_1 - 0\text{ m})$

75. $v_{1x} = 0\text{ m/s} + (20\text{ m/s}^2)(5\text{ s} - 0\text{ s})$
$x_1 = 0\text{ m} + (0\text{ m/s})(5\text{ s} - 0\text{ s}) + \frac{1}{2}(20\text{ m/s}^2)(5\text{ s} - 0\text{ s})^2$
$x_2 = x_1 + v_{1x}(10\text{ s} - 5\text{ s})$

Challenge Problems

76. The two masses in the figure slide on frictionless wires. They are connected by a pivoting rigid rod of length L. Prove that $v_{2x} = -v_{1y}\tan\theta$.

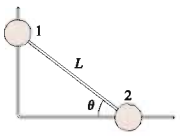


FIGURE CP2.76

77. A rocket is launched straight up with constant acceleration. Four seconds after liftoff, a bolt falls off the side of the rocket. The bolt hits the ground 6.0 s later. What was the rocket's acceleration?

78. Your school science club has devised a special event for homecoming. You've attached a rocket to the rear of a small car that has been decorated in the blue-and-gold school colors. The rocket provides a constant acceleration for 9.0 s. As the rocket shuts off, a parachute opens and slows the car at a rate of 5.0 m/s^2. The car passes the judges' box in the center of the grandstand, 990 m from the starting line, exactly 12 s after you fire the rocket. What is the car's speed as it passes the judges?

79. Careful measurements have been made of Olympic sprinters in the 100-meter dash. A simple but reasonably accurate model is that a sprinter accelerates at 3.6 m/s^2 for $3\frac{1}{3}$ s, then runs at constant velocity to the finish line.
a. What is the race time for a sprinter who follows this model?
b. A sprinter could run a faster race by accelerating faster at the beginning, thus reaching top speed sooner. If a sprinter's top speed is the same as in part a, what acceleration would he need to run the 100-meter dash in 9.9 s?
c. By what percent did the sprinter need to increase his acceleration in order to decrease his time by 1%?

80. Careful measurements have been made of Olympic sprinters in the 100-meter dash. A quite realistic model is that the sprinter's velocity is given by

$$v_x = a(1 - e^{-bt})$$

where t is in s, v_x is in m/s, and the constants a and b are characteristic of the sprinter. Sprinter Carl Lewis's run at the 1987 World Championships is modeled with $a = 11.81$ m/s and $b = 0.6887\ \text{s}^{-1}$.

a. What was Lewis's acceleration at $t = 0$ s, 2.00 s, and 4.00 s?
b. Find an expression for the distance traveled at time t.
c. Your expression from part b is a transcendental equation, meaning that you can't solve it for t. However, it's not hard to use trial and error to find the time needed to travel a specific distance. To the nearest 0.01 s, find the time Lewis needed to sprint 100.0 m. His official time was 0.01 s more than your answer, showing that this model is very good, but not perfect.

81. A sprinter can accelerate with constant acceleration for 4.0 s before reaching top speed. He can run the 100-meter dash in 10 s. What is his speed as he crosses the finish line?

82. A rubber ball is shot straight up from the ground with speed v_0. Simultaneously, a second rubber ball at height h directly above the first ball is dropped from rest.

a. At what height above the ground do the balls collide? Your answer will be a *symbolic expression* in terms of v_0 and g.
b. What is the maximum value of h for which a collision occurs before the first ball falls back to the ground?
c. For what value of h does the collision occur at the instant when the first ball is at its highest point?

83. The Starship Enterprise returns from warp drive to ordinary space with a forward speed of 50 km/s. To the crew's great surprise, a Klingon ship is 100 km directly ahead, traveling in the same direction at a mere 20 km/s. Without evasive action, the Enterprise will overtake and collide with the Klingons in just slightly over 3.0 s. The Enterprise's computers react instantly to brake the ship. What magnitude acceleration does the Enterprise need to just barely avoid a collision with the Klingon ship? Assume the acceleration is constant.

Hint: Draw a position-versus-time graph showing the motions of both the Enterprise and the Klingon ship. Let $x_0 = 0$ km be the location of the Enterprise as it returns from warp drive. How do you show graphically the situation in which the collision is "barely avoided"? Once you decide what it looks like graphically, express that situation mathematically.

STOP TO THINK ANSWERS

Stop to Think 2.1: d. The particle starts with positive x and moves to negative x.

Stop to Think 2.2: c. The velocity is the slope of the position graph. The slope is positive and constant until the position graph crosses the axis, then positive but decreasing, and finally zero when the position graph is horizontal.

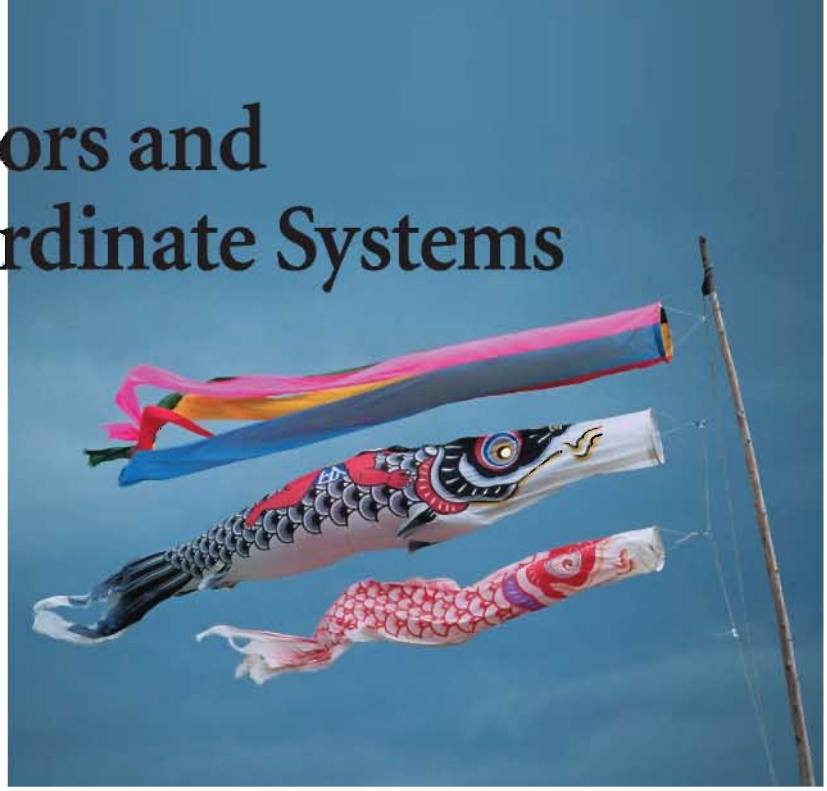
Stop to Think 2.3: b. A constant positive v_x corresponds to a linearly increasing x, starting from $x_i = -10$ m. The constant negative v_x then corresponds to a linearly decreasing x.

Stop to Think 2.4: a and **b.** The velocity is constant while $a = 0$, it decreases linearly while a is negative. Graphs a, b, and c all have the same acceleration, but only graphs a and b have a positive initial velocity that represents a particle moving to the right.

Stop to Think 2.5: d. The acceleration vector points downhill (negative s-direction) and has the constant value $-g\sin\theta$ throughout the motion.

Stop to Think 2.6: c. Acceleration is the slope of the graph. The slope is zero at B. Although the graph is steepest at A, the slope at that point is negative, and so $a_A < a_B$. Only C has a positive slope, so $a_C > a_B$.

3 Vectors and Coordinate Systems



Wind has both a speed and a direction; hence the motion of the wind is described by a vector.

▶ Looking Ahead

The goals of Chapter 3 are to learn how vectors are represented and used. In this chapter you will learn to:

- Understand and use the basic properties of vectors.
- Decompose a vector into its components and reassemble vector components into a magnitude and direction.
- Add and subtract vectors both graphically and using components.

◀ Looking Back

This chapter continues the development of vectors that was begun in Chapter 1. Please review:

- Section 1.3 Vector addition and subtraction.

Many of the quantities that we use to describe the physical world are simply numbers. For example, the mass of an object is 2 kg, its temperature is 21°C, and it occupies a volume of 250 cm^3. A quantity that is fully described by a single number (with units) is called a **scalar quantity.** Mass, temperature, and volume are all scalars. Other scalar quantities include pressure, density, energy, charge, and voltage. We will often use an algebraic symbol to represent a scalar quantity. Thus m will represent mass, T temperature, V volume, E energy, and so on. Notice that scalars, in printed text, are shown in italics.

Our universe has three dimensions, so some quantities also need a direction for a full description. If you ask someone for directions to the post office, the reply "Go three blocks" will not be very helpful. A full description might be, "Go three blocks south." A quantity having both a size and a direction is called a **vector quantity.**

You met examples of vector quantities in Chapter 1: position, displacement, velocity, and acceleration. You will soon make the acquaintance of others, such as force, momentum, and the electric field. Now, before we begin a study of forces, it's worth spending a little time to look more closely at vectors.

3.1 Vectors

Suppose you are assigned the task of measuring the temperature at various points throughout a building and then showing the information on a building floor plan. To do this, you could put little dots on the floor plan, to show the points at which you made measurements, then write the temperature at that point beside the dot. In other words, as FIGURE 3.1a shows, you can represent the temperature at each point with a simple number (with units). Temperature is a scalar quantity.

Having done such a good job on your first assignment, you are next assigned the task of measuring the velocities of several employees as they move about in their work. Recall from Chapter 1 that velocity is a vector; it has both a size and a direction. Simply writing each employee's speed is not sufficient because speed doesn't take into account the direction in which the person moved. After some thought, you conclude that a good way to represent the velocity is by drawing an arrow whose length is proportional to the speed and that points in the direction of motion. Further, as FIGURE 3.1b shows, you decide to place the *tail* of an arrow at the point where you measured the velocity.

As this example illustrates, the *geometric representation* of a vector is an arrow, with the tail of the arrow (not its tip!) placed at the point where the measurement is made. The vector then seems to radiate outward from the point to which it is attached. An arrow makes a natural representation of a vector because it inherently has both a length and a direction. As you've already seen, we label vectors by drawing a small arrow over the letter that represents the vector: $\vec{r}$ for position, $\vec{v}$ for velocity, $\vec{a}$ for acceleration, and so on.

The mathematical term for the length, or size, of a vector is **magnitude,** so we can say that **a vector is a quantity having a magnitude and a direction.** As an example, FIGURE 3.2 shows the geometric representation of a particle's velocity vector $\vec{v}$. The particle's speed at this point is 5 m/s, *and* it is moving in the direction indicated by the arrow. The arrow is drawn with its tail at the point where the velocity was measured.

NOTE ▶ Although the vector arrow is drawn across the page, from its tail to its tip, this does *not* indicate that the vector "stretches" across this distance. Instead, the vector arrow tells us the value of the vector quantity only at the one point where the tail of the vector is placed. ◀

The *magnitude* of a vector is sometimes shown using absolute value signs, but more frequently indicated by the letter without the arrow. For example, the magnitude of the velocity vector in Figure 3.2 is $v = |\vec{v}| = 5$ m/s. This is the object's *speed.* The magnitude of the acceleration vector $\vec{a}$ is written a. **The magnitude of a vector is a scalar quantity.**

NOTE ▶ The magnitude of a vector cannot be a negative number; it must be positive or zero, with appropriate units. ◀

It is important to get in the habit of using the arrow symbol for vectors. If you omit the vector arrow from the velocity vector $\vec{v}$ and write only v, then you're referring only to the object's speed, not its velocity. The symbols $\vec{r}$ and r, or $\vec{v}$ and v, do *not* represent the same thing, so if you omit the vector arrow from vector symbols you will soon have confusion and mistakes.

FIGURE 3.1 Measurements of scalar and vector quantities.

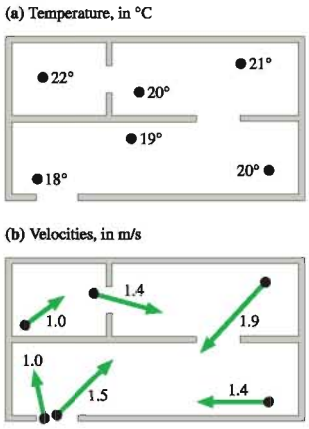
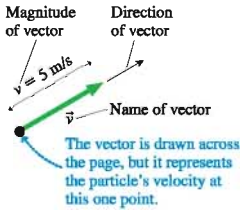


FIGURE 3.2 The velocity vector $\vec{v}$ has both a magnitude and a direction.



3.2 Properties of Vectors

Recall from Chapter 1 that the *displacement* is a vector drawn from an object's initial position to its position at some later time. Because displacement is an easy concept to think about, we can use it to introduce some of the properties of vectors. However, these properties apply to *all* vectors, not just to displacement.

Suppose Sam starts from his front door, walks across the street, and ends up 200 ft to the northeast of where he started. Sam's displacement, which we will label $\vec{S}$, is shown in FIGURE 3.3a on the next page. The displacement vector is a *straight-line connection* from his initial to his final position, not necessarily his actual path. The dashed line indicates a possible route Sam might have taken, but his displacement is the vector $\vec{S}$.

To describe a vector we must specify both its magnitude and its direction. We can write Sam's displacement as

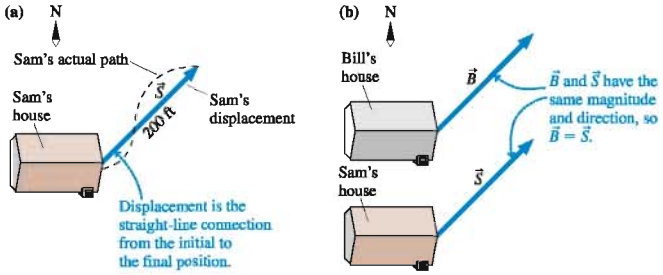
$$\vec{S} = (200 \text{ ft, northeast})$$

where the first piece of information specifies the magnitude and the second is the direction. The magnitude of Sam's displacement is $S = |\vec{S}| = 200$ ft, the distance between his initial and final points.

The boat's displacement is the straight-line connection from its initial to its final position.

Sam's next-door neighbor Bill also walks 200 ft to the northeast, starting from his own front door. Bill's displacement $\vec{B}$ = (200 ft, northeast) has the same magnitude and direction as Sam's displacement $\vec{S}$. Because vectors are defined by their magnitude and direction, **two vectors are equal if they have the same magnitude and direction.** This is true regardless of the starting points of the vectors. Thus the two displacements in FIGURE 3.3b are equal to each other, and we can write $\vec{B} = \vec{S}$.

FIGURE 3.3 Displacement vectors.



NOTE ▶ A vector is unchanged if you move it to a different point on the page as long as you don't change its length or the direction it points. We used this idea in Chapter 1 when we moved velocity vectors around in order to find the average acceleration vector $\vec{a}$. ◀

Vector Addition

FIGURE 3.4 The net displacement $\vec{C}$ resulting from two displacements $\vec{A}$ and $\vec{B}$.

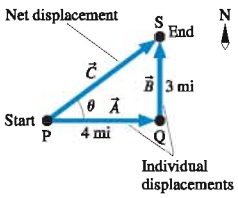


FIGURE 3.4 shows the displacement of a hiker who starts at point P and ends at point S. She first hikes 4 miles to the east, then 3 miles to the north. The first leg of the hike is described by the displacement $\vec{A}$ = (4 mi, east). The second leg of the hike has displacement $\vec{B}$ = (3 mi, north). Now, by definition, a vector from the initial position P to the final position S is also a displacement. This is vector $\vec{C}$ on the figure. $\vec{C}$ is the *net displacement* because it describes the net result of the hiker's first having displacement $\vec{A}$, then displacement $\vec{B}$.

If you earn \$50 on Saturday and \$60 on Sunday, your *net* income for the weekend is the sum of \$50 and \$60. With scalars, the word *net* implies addition. The same is true with vectors. The net displacement $\vec{C}$ is an initial displacement $\vec{A}$ *plus* a second displacement $\vec{B}$, or

$$\vec{C} = \vec{A} + \vec{B} \tag{3.1}$$

The sum of two vectors is called the **resultant vector.** It's not hard to show that vector addition is commutative: $\vec{A} + \vec{B} = \vec{B} + \vec{A}$. That is, you can add vectors in any order you wish.

Look back at Tactics Box 1.1 on page 8 to see the three-step procedure for adding two vectors. This tip-to-tail method for adding vectors, which is used to find $\vec{C} = \vec{A} + \vec{B}$ in Figure 3.4, is called **graphical addition.** Any two vectors of the same type—two velocity vectors or two force vectors—can be added in exactly the same way.

The graphical method for adding vectors is straightforward, but we need to do a little geometry to come up with a complete description of the resultant vector $\vec{C}$. Vector $\vec{C}$ of Figure 3.4 is defined by its magnitude C and by its direction. Because the three vectors $\vec{A}$, $\vec{B}$, and $\vec{C}$ form a right triangle, the magnitude, or length, of $\vec{C}$ is given by the Pythagorean theorem:

$$C = \sqrt{A^2 + B^2} = \sqrt{(4\text{ mi})^2 + (3\text{ mi})^2} = 5\text{ mi} \tag{3.2}$$

Notice that Equation 3.2 uses the magnitudes A and B of the vectors $\vec{A}$ and $\vec{B}$. The angle θ, which is used in Figure 3.4 to describe the direction of $\vec{C}$, is easily found for a right triangle:

$$\theta = \tan^{-1}\left(\frac{B}{A}\right) = \tan^{-1}\left(\frac{3\text{ mi}}{4\text{ mi}}\right) = 37^\circ \qquad (3.3)$$

Altogether, the hiker's net displacement is

$$\vec{C} = \vec{A} + \vec{B} = (5\text{ mi}, 37^\circ \text{ north of east}) \qquad (3.4)$$

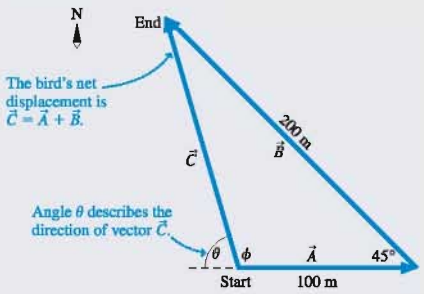
NOTE ▶ Vector mathematics makes extensive use of geometry and trigonometry. Appendix A, at the end of this book, contains a brief review of these topics. ◀

EXAMPLE 3.1 Using graphical addition to find a displacement

A bird flies 100 m due east from a tree, then 200 m northwest (that is, 45° north of west). What is the bird's net displacement?

VISUALIZE FIGURE 3.5 shows the two individual displacements, which we've called $\vec{A}$ and $\vec{B}$. The net displacement is the vector sum $\vec{C} = \vec{A} + \vec{B}$, which is found graphically.

FIGURE 3.5 The bird's net displacement is $\vec{C} = \vec{A} + \vec{B}$.



SOLVE The two displacements are $\vec{A} = (100\text{ m, east})$ and $\vec{B} = (200\text{ m, northwest})$. The net displacement $\vec{C} = \vec{A} + \vec{B}$ is found by drawing a vector from the initial to the final position. But describing $\vec{C}$ is a bit trickier than the example of the hiker because $\vec{A}$ and $\vec{B}$ are not at right angles. First, we can find the magnitude of $\vec{C}$ by using the law of cosines from trigonometry:

$$\begin{aligned} C^2 &= A^2 + B^2 - 2AB\cos(45^\circ) \\ &= (100\text{ m})^2 + (200\text{ m})^2 - 2(100\text{ m})(200\text{ m})\cos(45^\circ) \\ &= 21{,}720\text{ m}^2 \end{aligned}$$

Thus $C = \sqrt{21{,}720\text{ m}^2} = 147$ m. Then a second use of the law of cosines can determine angle ϕ (the Greek letter phi):

$$B^2 = A^2 + C^2 - 2AC\cos\phi$$

$$\phi = \cos^{-1}\left[\frac{A^2 + C^2 - B^2}{2AC}\right] = 106^\circ$$

It is easier to describe $\vec{C}$ with the angle $\theta = 180^\circ - \phi = 74^\circ$. The bird's net displacement is

$$\vec{C} = (147\text{ m}, 74^\circ \text{ north of west})$$

When two vectors are to be added, it is often convenient to draw them with their tails together, as shown in **FIGURE 3.6a**. To evaluate $\vec{D} + \vec{E}$, you could move vector $\vec{E}$ over to where its tail is on the tip of $\vec{D}$, then use the tip-to-tail rule of graphical addition. That gives vector $\vec{F} = \vec{D} + \vec{E}$ in **FIGURE 3.6b**. Alternatively, **FIGURE 3.6c** shows that the vector sum $\vec{D} + \vec{E}$ can be found as the diagonal of the parallelogram defined by $\vec{D}$ and $\vec{E}$. This method for vector addition, which some of you may have learned, is called the *parallelogram rule* of vector addition.

FIGURE 3.6 Two vectors can be added using the tip-to-tail rule or the parallelogram rule.

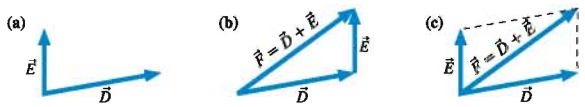
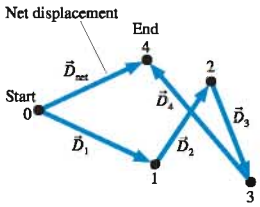


FIGURE 3.7 The net displacement after four individual displacements.

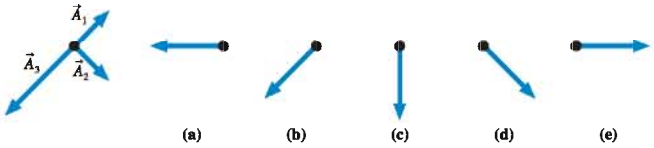


Vector addition is easily extended to more than two vectors. FIGURE 3.7 shows a hiker moving from initial position 0 to position 1, then position 2, then position 3, and finally arriving at position 4. These four segments are described by displacement vectors $\vec{D}_1$, $\vec{D}_2$, $\vec{D}_3$, and $\vec{D}_4$. The hiker's *net* displacement, an arrow from position 0 to position 4, is the vector $\vec{D}_{net}$. In this case,

$$\vec{D}_{net} = \vec{D}_1 + \vec{D}_2 + \vec{D}_3 + \vec{D}_4 \tag{3.5}$$

The vector sum is found by using the tip-to-tail method three times in succession.

STOP TO THINK 3.1 Which figure shows $\vec{A}_1 + \vec{A}_2 + \vec{A}_3$?



Multiplication by a Scalar

Suppose a second bird flies twice as far to the east as the bird in Example 3.1. The first bird's displacement was $\vec{A}_1$ = (100 m, east), where a subscript has been added to denote the first bird. The second bird's displacement will then certainly be $\vec{A}_2$ = (200 m, east). The words "twice as" indicate a multiplication, so we can say

$$\vec{A}_2 = 2\vec{A}_1$$

Multiplying a vector by a positive scalar gives another vector of *different magnitude* but pointing in the *same direction*.

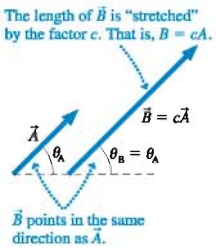
Let the vector $\vec{A}$ be

$$\vec{A} = (A, \theta_A) \tag{3.6}$$

Now let $\vec{B} = c\vec{A}$, where c is a positive scalar constant. We define the multiplication of a vector by a scalar such that

$$\vec{B} = c\vec{A} \text{ means that } (B, \theta_B) = (cA, \theta_A) \tag{3.7}$$

FIGURE 3.8 Multiplication of a vector by a scalar.



In other words, the vector is stretched or compressed by the factor c (i.e., vector $\vec{B}$ has magnitude $B = cA$), but $\vec{B}$ points in the same direction as $\vec{A}$. This is illustrated in FIGURE 3.8.

We used this property of vectors in Chapter 1 when we asserted that vector $\vec{a}$ points in the same direction as $\Delta\vec{v}$. From the definition

$$\vec{a} = \frac{\Delta\vec{v}}{\Delta t} = \left(\frac{1}{\Delta t}\right)\Delta\vec{v} \tag{3.8}$$

where $(1/\Delta t)$ is a scalar constant, we see that $\vec{a}$ points in the same direction as $\Delta\vec{v}$ but differs in length by the factor $(1/\Delta t)$.

Suppose we multiply $\vec{A}$ by zero. Using Equation 3.7,

$$0 \cdot \vec{A} = \vec{0} = (0 \text{ m, direction undefined}) \tag{3.9}$$

The product is a vector having zero length or magnitude. This vector is known as the **zero vector,** denoted $\vec{0}$. The direction of the zero vector is irrelevant; you cannot describe the direction of an arrow of zero length!

What happens if we multiply a vector by a negative number? Equation 3.7 does not apply if $c < 0$ because vector $\vec{B}$ cannot have a negative magnitude. Consider the vector $-\vec{A}$, which is equivalent to multiplying $\vec{A}$ by -1. Because

$$\vec{A} + (-\vec{A}) = \vec{0} \tag{3.10}$$

the vector $-\vec{A}$ must be such that, when it is added to $\vec{A}$, the resultant is the zero vector $\vec{0}$. In other words, the *tip* of $-\vec{A}$ must return to the *tail* of $\vec{A}$, as shown in FIGURE 3.9. This will be true only if $-\vec{A}$ is equal in magnitude to $\vec{A}$, but opposite in direction. Thus we can conclude that

$$-\vec{A} = (A, \text{direction opposite } \vec{A}) \tag{3.11}$$

That is, **multiplying a vector by −1 reverses its direction without changing its length.**

As an example, FIGURE 3.10 shows vectors $\vec{A}$, $2\vec{A}$, and $-3\vec{A}$. Multiplication by 2 doubles the length of the vector but does not change its direction. Multiplication by -3 stretches the length by a factor of 3 *and* reverses the direction.

FIGURE 3.9 Vector $-\vec{A}$.

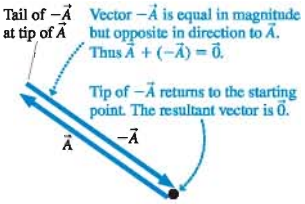
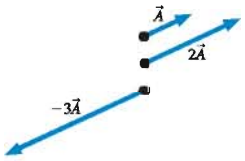


FIGURE 3.10 Vectors $\vec{A}$, $2\vec{A}$, and $-3\vec{A}$.



EXAMPLE 3.2 Velocity and displacement

Carolyn drives her car north at 30 km/hr for 1 hour, east at 60 km/hr for 2 hours, then north at 50 km/hr for 1 hour. What is Carolyn's net displacement?

SOLVE Chapter 1 defined velocity as

$$\vec{v} = \frac{\Delta\vec{r}}{\Delta t}$$

so the displacement $\Delta\vec{r}$ during the time interval Δt is $\Delta\vec{r} = (\Delta t)\vec{v}$. This is multiplication of the vector $\vec{v}$ by the scalar Δt. Carolyn's velocity during the first hour is $\vec{v}_1 = (30 \text{ km/hr, north})$, so her displacement during this interval is

$$\Delta\vec{r}_1 = (1 \text{ hour})(30 \text{ km/hr, north}) = (30 \text{ km, north})$$

Similarly,

$$\Delta\vec{r}_2 = (2 \text{ hours})(60 \text{ km/hr, east}) = (120 \text{ km, east})$$

$$\Delta\vec{r}_3 = (1 \text{ hour})(50 \text{ km/hr, north}) = (50 \text{ km, north})$$

In this case, multiplication by a scalar changes not only the length of the vector but also its units, from km/hr to km. The direction, however, is unchanged. Carolyn's net displacement is

$$\Delta\vec{r}_{net} = \Delta\vec{r}_1 + \Delta\vec{r}_2 + \Delta\vec{r}_3$$

This addition of the three vectors is shown in FIGURE 3.11, using the tip-to-tail method. $\Delta\vec{r}_{net}$ stretches from Carolyn's initial position to her final position. The magnitude of her net displacement is found using the Pythagorean theorem:

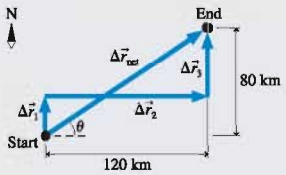
$$r_{net} = \sqrt{(120 \text{ km})^2 + (80 \text{ km})^2} = 144 \text{ km}$$

The direction of $\Delta\vec{r}_{net}$ is described by angle θ, which is

$$\theta = \tan^{-1}\left(\frac{80 \text{ km}}{120 \text{ km}}\right) = 34°$$

Thus Carolyn's net displacement is $\Delta\vec{r}_{net} = (144 \text{ km}, 34° \text{ north of east})$.

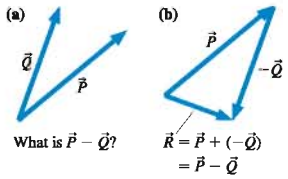
FIGURE 3.11 The net displacement is the vector sum $\Delta\vec{r}_{net} = \Delta\vec{r}_1 + \Delta\vec{r}_2 + \Delta\vec{r}_3$.



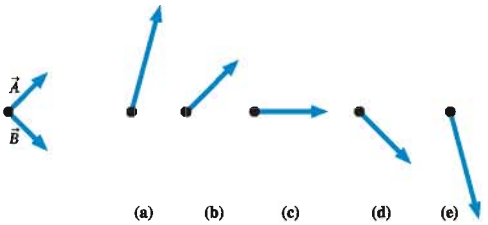
Vector Subtraction

FIGURE 3.12a shows two vectors, $\vec{P}$ and $\vec{Q}$. What is $\vec{R} = \vec{P} - \vec{Q}$? Look back at Tactics Box 1.2 on page 9, which showed how to perform vector subtraction graphically. FIGURE 3.12b finds $\vec{P} - \vec{Q}$ by writing $\vec{R} = \vec{P} + (-\vec{Q})$, then using the rules of vector addition.

FIGURE 3.12 Vector subtraction.



STOP TO THINK 3.2 Which figure shows $2\vec{A} - \vec{B}$?



The navigator had better know which way to go, and how far, if she and the crew are to make landfall at the expected location.

3.3 Coordinate Systems and Vector Components

Vectors do not require a coordinate system. We can add and subtract vectors graphically, and we will do so frequently to clarify our understanding of a situation. But the graphical addition of vectors is not an especially good way to find quantitative results. In this section we will introduce a *coordinate representation* of vectors that will be the basis of an easier method for doing vector calculations.

Coordinate Systems

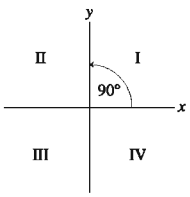
As we noted in the first chapter, the world does not come with a coordinate system attached to it. A coordinate system is an artificially imposed grid that you place on a problem in order to make quantitative measurements. It may be helpful to think of drawing a grid on a piece of transparent plastic that you can then overlay on top of the problem. This conveys the idea that *you* choose:

- Where to place the origin, and
- How to orient the axes.

Different problem solvers may choose to use different coordinate systems; that is perfectly acceptable. However, some coordinate systems will make a problem easier to solve. Part of our goal is to learn how to choose an appropriate coordinate system for each problem.

We will generally use **Cartesian coordinates.** This is a coordinate system with the axes perpendicular to each other, forming a rectangular grid. The standard *xy*-coordinate system with which you are familiar is a Cartesian coordinate system. An *xyz*-coordinate system is a Cartesian coordinate system in three dimensions. There are other possible coordinate systems, such as polar coordinates, but we will not be concerned with those for now.

FIGURE 3.13 A conventional Cartesian coordinate system and the quadrants of the *xy*-plane.



The placement of the axes is not entirely arbitrary. By convention, the positive *y*-axis is located 90° *counterclockwise* (ccw) from the positive *x*-axis, as illustrated in FIGURE 3.13. Figure 3.13 also identifies the four **quadrants** of the coordinate system, I through IV. Notice that the quadrants are counted ccw from the positive *x*-axis.

Coordinate axes have a positive end and a negative end, separated by zero at the origin where the two axes cross. When you draw a coordinate system, it is important to label the axes. This is done by placing *x* and *y* labels at the *positive* ends of the axes, as in Figure 3.13. The purpose of the labels is twofold:

- To identify which axis is which, and
- To identify the positive ends of the axes.

This will be important when you need to determine whether the quantities in a problem should be assigned positive or negative values.

Component Vectors

FIGURE 3.14 shows a vector $\vec{A}$ and an xy-coordinate system that we've chosen. Once the directions of the axes are known, we can define two new vectors *parallel to the axes* that we call the **component vectors** of $\vec{A}$. Vector $\vec{A}_x$, called the *x-component vector*, is the projection of $\vec{A}$ along the x-axis. Vector $\vec{A}_y$, the *y-component vector*, is the projection of $\vec{A}$ along the y-axis. Notice that the component vectors are perpendicular to each other.

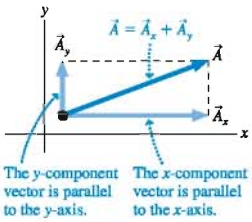
You can see, using the parallelogram rule, that $\vec{A}$ is the vector sum of the two component vectors:

$$\vec{A} = \vec{A}_x + \vec{A}_y \tag{3.12}$$

In essence, we have broken vector $\vec{A}$ into two perpendicular vectors that are parallel to the coordinate axes. This process is called the **decomposition** of vector $\vec{A}$ into its component vectors.

NOTE ▶ It is not necessary for the tail of $\vec{A}$ to be at the origin. All we need to know is the *orientation* of the coordinate system so that we can draw $\vec{A}_x$ and $\vec{A}_y$ parallel to the axes. ◀

FIGURE 3.14 Component vectors $\vec{A}_x$ and $\vec{A}_y$ are drawn parallel to the coordinate axes such that $\vec{A} = \vec{A}_x + \vec{A}_y$.



Components

You learned in Chapter 1 to give the one-dimensional kinematic variable v_x a positive sign if the velocity vector $\vec{v}$ points toward the positive end of the x-axis, a negative sign if $\vec{v}$ points in the negative x-direction. The basis of that rule is that v_x is what we call the *x-component* of the velocity vector. We need to extend this idea to vectors in general.

Suppose vector $\vec{A}$ has been decomposed into component vectors $\vec{A}_x$ and $\vec{A}_y$ parallel to the coordinate axes. We can describe each component vector with a single number called the **component.** The *x-component* and *y-component* of vector $\vec{A}$, denoted A_x and A_y, are determined as follows:

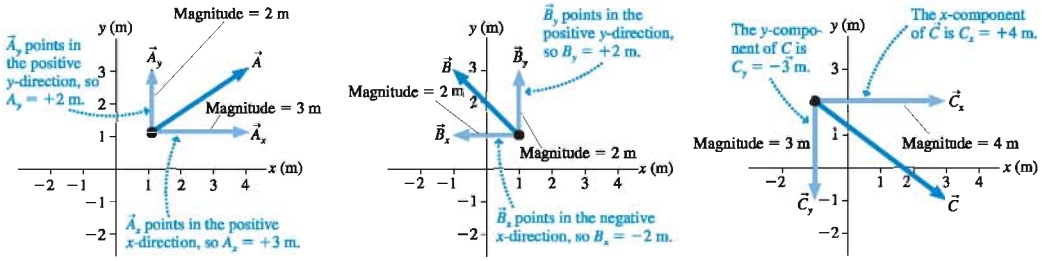
TACTICS BOX 3.1 Determining the components of a vector MP

1. The absolute value $|A_x|$ of the x-component A_x is the magnitude of the component vector $\vec{A}_x$.
2. The *sign* of A_x is positive if $\vec{A}_x$ points in the positive x-direction, negative if $\vec{A}_x$ points in the negative x-direction.
3. The y-component A_y is determined similarly.

Exercises 10–18

In other words, the component A_x tells us two things: how big $\vec{A}_x$ is and, with its sign, which end of the axis $\vec{A}_x$ points toward. FIGURE 3.15 shows three examples of determining the components of a vector.

FIGURE 3.15 Determining the components of a vector.



NOTE ▶ Beware of the somewhat confusing terminology. $\vec{A}_x$ and $\vec{A}_y$ are called *component vectors*, whereas A_x and A_y are simply called *components*. The components A_x and A_y are just numbers (with units), so make sure you do *not* put arrow symbols over the components. ◀

Much of physics is expressed in the language of vectors. We will frequently need to decompose a vector into its components. We will also need to "reassemble" a vector from its components. In other words, we need to move back and forth between the geometric and the component representations of a vector.

Consider first the problem of decomposing a vector into its x- and y-components. FIGURE 3.16a shows a vector $\vec{A}$ at angle θ from the x-axis. It is *essential* to use a picture or diagram such as this to *define the angle* you are using to describe the vector's direction.

$\vec{A}$ points to the right and up, so Tactics Box 3.1 tells us that the components A_x and A_y are both positive. We can use trigonometry to find

$$\begin{aligned} A_x &= A\cos\theta \\ A_y &= A\sin\theta \end{aligned} \tag{3.13}$$

where A is the magnitude, or length, of $\vec{A}$. These equations convert the length and angle description of vector $\vec{A}$ into the vector's components, but they are correct *only* if angle θ is measured from the positive x-axis.

FIGURE 3.16b shows a vector $\vec{C}$ whose direction is specified by the angle ϕ, measured from the negative y-axis. In this case, the components of $\vec{C}$ are

$$\begin{aligned} C_x &= C\sin\phi \\ C_y &= -C\cos\phi \end{aligned} \tag{3.14}$$

The role of sine and cosine is reversed from that in Equations 3.13 because we are using a different angle.

NOTE ▶ Each decomposition requires that you pay close attention to the direction in which the vector points and the angles that are defined. The minus sign, when needed, must be inserted manually. ◀

We can also go in the opposite direction and determine the length and angle of a vector from its x- and y-components. Because A in Figure 3.16a is the hypotenuse of a right triangle, its length is given by the Pythagorean theorem:

$$A = \sqrt{A_x^2 + A_y^2} \tag{3.15}$$

Similarly, the tangent of angle θ is the ratio of the far side to the adjacent side, so

$$\theta = \tan^{-1}\left(\frac{A_y}{A_x}\right) \tag{3.16}$$

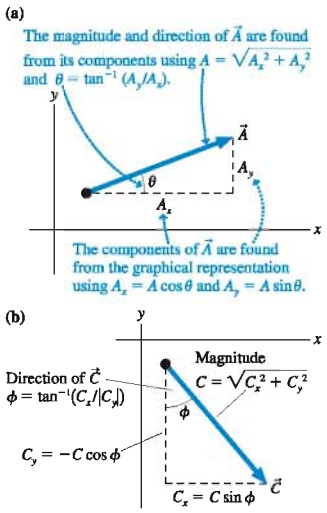
where $\tan^{-1}$ is the inverse tangent function. Equations 3.15 and 3.16 can be thought of as the "reverse" of Equations 3.13.

Equation 3.15 always works for finding the length or magnitude of a vector because the squares eliminate any concerns over the signs of the components. But finding the angle, just like finding the components, requires close attention to how the angle is defined and to the signs of the components. For example, finding the angle of vector $\vec{C}$ in Figure 3.16b requires the length of C_y *without* the minus sign. Thus vector $\vec{C}$ has magnitude and direction

$$\begin{aligned} C &= \sqrt{C_x^2 + C_y^2} \\ \phi &= \tan^{-1}\left(\frac{C_x}{|C_y|}\right) \end{aligned} \tag{3.17}$$

Notice that the roles of x and y differ from those in Equation 3.16.

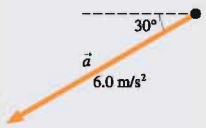
FIGURE 3.16 Moving between the geometric representation and the component representation.



EXAMPLE 3.3 Finding the components of an acceleration vector

Find the x- and y-components of the acceleration vector $\vec{a}$ shown in FIGURE 3.17.

FIGURE 3.17 The acceleration vector $\vec{a}$ of Example 3.3.



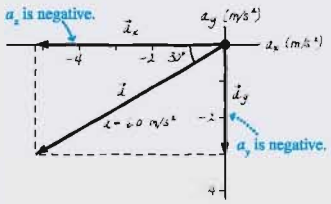
VISUALIZE It's important to *draw* vectors. FIGURE 3.18 shows the original vector $\vec{a}$ decomposed into components parallel to the axes. Notice that the axes are "acceleration axes" because we're measuring an acceleration vector.

SOLVE The acceleration vector $\vec{a}$ = (6.0 m/s², 30° below the negative x-axis) points to the left (negative x-direction) and down (negative y-direction), so the components a_x and a_y are both negative:

$$a_x = -a\cos 30° = -(6.0\ \text{m/s}^2)\cos 30° = -5.2\ \text{m/s}^2$$

$$a_y = -a\sin 30° = -(6.0\ \text{m/s}^2)\sin 30° = -3.0\ \text{m/s}^2$$

FIGURE 3.18 Decomposition of $\vec{a}$.

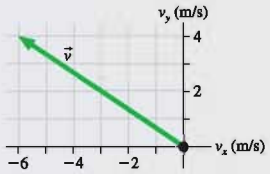


ASSESS The units of a_x and a_y are the same as the units of vector $\vec{a}$. Notice that we had to insert the minus signs manually by observing that the vector points left and down.

EXAMPLE 3.4 Finding the direction of motion

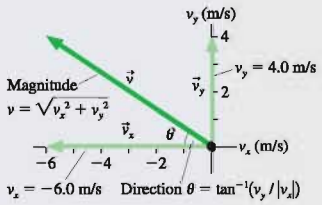
FIGURE 3.19 shows a car's velocity vector $\vec{v}$. Determine the car's speed and direction of motion.

FIGURE 3.19 The velocity vector $\vec{v}$ of Example 3.4.



VISUALIZE FIGURE 3.20 shows the components v_x and v_y and defines an angle θ with which we can specify the direction of motion.

FIGURE 3.20 Decomposition of $\vec{v}$.



SOLVE We can read the components of $\vec{v}$ directly from the axes: $v_x = -6.0$ m/s and $v_y = 4.0$ m/s. Notice that v_x is negative. This is enough information to find the car's speed v, which is the magnitude of $\vec{v}$:

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(-6.0\ \text{m/s})^2 + (4.0\ \text{m/s})^2} = 7.2\ \text{m/s}$$

From trigonometry, angle θ is

$$\theta = \tan^{-1}\left(\frac{v_y}{|v_x|}\right) = \tan^{-1}\left(\frac{4.0\ \text{m/s}}{6.0\ \text{m/s}}\right) = 34°$$

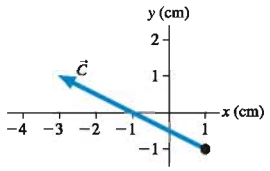
The absolute value signs are necessary because v_x is a negative number. The velocity vector $\vec{v}$ can be written in terms of the speed and the direction of motion as

$$\vec{v} = (7.2\ \text{m/s},\ 34°\ \text{above the negative } x\text{-axis})$$

or, if the axes are aligned to north,

$$\vec{v} = (7.2\ \text{m/s},\ 34°\ \text{north of west})$$

STOP TO THINK 3.3 What are the x- and y-components C_x and C_y of vector $\vec{C}$?

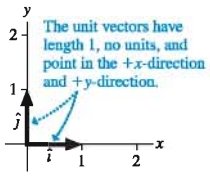


3.4 Vector Algebra

Vector components are a powerful tool for doing mathematics with vectors. In this section you'll learn how to use components to add and subtract vectors. First, we'll introduce an efficient way to write a vector in terms of its components.

Unit Vectors

FIGURE 3.21 The unit vectors $\hat{\imath}$ and $\hat{\jmath}$.



The vectors (1, $+x$-direction) and (1, $+y$-direction), shown in FIGURE 3.21, have some interesting and useful properties. Each has a magnitude of 1, no units, and is parallel to a coordinate axis. A vector with these properties is called a **unit vector.** These unit vectors have the special symbols

$$\hat{\imath} \equiv (1, \text{positive } x\text{-direction})$$

$$\hat{\jmath} \equiv (1, \text{positive } y\text{-direction})$$

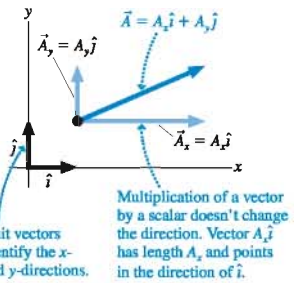
The notation $\hat{\imath}$ (read "i hat") and $\hat{\jmath}$ (read "j hat") indicates a unit vector with a magnitude of 1.

Unit vectors establish the directions of the positive axes of the coordinate system. Our choice of a coordinate system may be arbitrary, but once we decide to place a coordinate system on a problem we need something to tell us "That direction is the positive x-direction." This is what the unit vectors do.

The unit vectors provide a useful way to write component vectors. The component vector $\vec{A}_x$ is the piece of vector $\vec{A}$ that is parallel to the x-axis. Similarly, $\vec{A}_y$ is parallel to the y-axis. Because, by definition, the vector $\hat{\imath}$ points along the x-axis and $\hat{\jmath}$ points along the y-axis, we can write

$$\vec{A}_x = A_x\hat{\imath}$$
$$\vec{A}_y = A_y\hat{\jmath} \qquad (3.18)$$

FIGURE 3.22 The decomposition of vector $\vec{A}$ is $A_x\hat{\imath} + A_y\hat{\jmath}$.



Equations 3.18 separate each component vector into a length and a direction. The full decomposition of vector $\vec{A}$ can then be written

$$\vec{A} = \vec{A}_x + \vec{A}_y = A_x\hat{\imath} + A_y\hat{\jmath} \qquad (3.19)$$

FIGURE 3.22 shows how the unit vectors and the components fit together to form vector $\vec{A}$.

NOTE ▶ In three dimensions, the unit vector along the $+z$-direction is called $\hat{k}$, and to describe vector $\vec{A}$ we would include an additional component vector $\vec{A}_z = A_z\hat{k}$. ◀

You may have learned in a math class to think of vectors as pairs or triplets of numbers, such as $(4, -2, 5)$. This is another, and completely equivalent, way to write the components of a vector. Thus we could write

$$\vec{B} = 4\hat{\imath} - 2\hat{\jmath} + 5\hat{k} = (4, -2, 5)$$

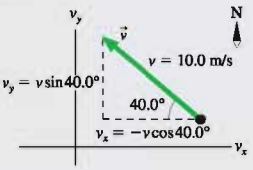
You will find the notation using unit vectors to be more convenient for the equations we will use in physics, but rest assured that you already know a lot about vectors if you learned about them as pairs or triplets of numbers.

EXAMPLE 3.5 Run rabbit run!

A rabbit, escaping a fox, runs 40.0° north of west at 10.0 m/s. A coordinate system is established with the positive x-axis to the east and the positive y-axis to the north. Write the rabbit's velocity in terms of components and unit vectors.

VISUALIZE FIGURE 3.23 shows the rabbit's velocity vector and the coordinate axes. We're showing a velocity vector, so the axes are labeled v_x and v_y rather than x and y.

FIGURE 3.23 The velocity vector $\vec{v}$ is decomposed into components v_x and v_y.



SOLVE 10.0 m/s is the rabbit's *speed*, not its velocity. The velocity, which includes directional information, is

$$\vec{v} = (10.0 \text{ m/s}, 40.0° \text{ north of west})$$

Vector $\vec{v}$ points to the left and up, so the components v_x and v_y are negative and positive, respectively. The components are

$$v_x = -(10.0 \text{ m/s})\cos 40.0° = -7.66 \text{ m/s}$$

$$v_y = +(10.0 \text{ m/s})\sin 40.0° = 6.43 \text{ m/s}$$

With v_x and v_y now known, the rabbit's velocity vector is

$$\vec{v} = v_x\hat{\imath} + v_y\hat{\jmath} = (-7.66\hat{\imath} + 6.43\hat{\jmath}) \text{ m/s}$$

Notice that we've pulled the units to the end, rather than writing them with each component.

ASSESS Notice that the minus sign for v_x was inserted manually. **Signs don't occur automatically; you have to set them after checking the vector's direction.**

Working with Vectors

You learned in Section 3.2 how to add vectors graphically, but it is a tedious problem in geometry and trigonometry to find precise values for the magnitude and direction of the resultant. The addition and subtraction of vectors become much easier if we use components and unit vectors.

To see this, let's evaluate the vector sum $\vec{D} = \vec{A} + \vec{B} + \vec{C}$. To begin, write this sum in terms of the components of each vector:

$$\begin{aligned}\vec{D} &= D_x\hat{\imath} + D_y\hat{\jmath} = \vec{A} + \vec{B} + \vec{C} \\ &= (A_x\hat{\imath} + A_y\hat{\jmath}) + (B_x\hat{\imath} + B_y\hat{\jmath}) + (C_x\hat{\imath} + C_y\hat{\jmath})\end{aligned} \tag{3.20}$$

We can group together all the x-components and all the y-components on the right side, in which case Equation 3.20 is

$$(D_x)\hat{\imath} + (D_y)\hat{\jmath} = (A_x + B_x + C_x)\hat{\imath} + (A_y + B_y + C_y)\hat{\jmath} \tag{3.21}$$

Comparing the x- and y-components on the left and right sides of Equation 3.21, we find:

$$\begin{aligned}D_x &= A_x + B_x + C_x \\ D_y &= A_y + B_y + C_y\end{aligned} \tag{3.22}$$

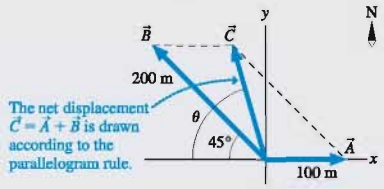
Stated in words, Equation 3.22 says that we can perform vector addition by adding the x-components of the individual vectors to give the x-component of the resultant and by adding the y-components of the individual vectors to give the y-component of the resultant. This method of vector addition is called **algebraic addition.**

EXAMPLE 3.6 Using algebraic addition to find a displacement

Example 3.1 was about a bird that flew 100 m to the east, then 200 m to the northwest. Use the algebraic addition of vectors to find the bird's net displacement.

VISUALIZE FIGURE 3.24 shows displacement vectors $\vec{A}$ = (100 m, east) and $\vec{B}$ = (200 m, northwest). We draw vectors tip-to-tail to add them graphically, but it's usually easier to draw them all from the origin if we are going to use algebraic addition.

FIGURE 3.24 The net displacement is $\vec{C} = \vec{A} + \vec{B}$.



SOLVE To add the vectors algebraically we must know their components. From the figure these are seen to be

$$\vec{A} = 100\,\hat{\imath}\,\text{m}$$

$$\vec{B} = (-200\cos 45°\,\hat{\imath} + 200\sin 45°\,\hat{\jmath})\ \text{m}$$

$$= (-141\hat{\imath} + 141\hat{\jmath})\ \text{m}$$

Notice that vector quantities must include units. Also notice, as you would expect from the figure, that $\vec{B}$ has a negative x-component. Adding $\vec{A}$ and $\vec{B}$ by components gives

$$\vec{C} = \vec{A} + \vec{B} = 100\hat{\imath}\ \text{m} + (-141\hat{\imath} + 141\hat{\jmath})\ \text{m}$$

$$= (100\ \text{m} - 141\ \text{m})\hat{\imath} + (141\ \text{m})\hat{\jmath} = (-41\hat{\imath} + 141\hat{\jmath})\ \text{m}$$

This would be a perfectly acceptable answer for many purposes. However, we need to calculate the magnitude and direction of $\vec{C}$ if we want to compare this result to our earlier answer. The magnitude of $\vec{C}$ is

$$C = \sqrt{C_x^2 + C_y^2} = \sqrt{(-41\ \text{m})^2 + (141\ \text{m})^2} = 147\ \text{m}$$

The angle θ, as defined in Figure 3.24, is

$$\theta = \tan^{-1}\left(\frac{C_y}{|C_x|}\right) = \tan^{-1}\left(\frac{141\ \text{m}}{41\ \text{m}}\right) = 74°$$

Thus $\vec{C}$ = (147 m, 74° north of west), in perfect agreement with Example 3.1.

Vector subtraction and the multiplication of a vector by a scalar, using components, are very much like vector addition. To find $\vec{R} = \vec{P} - \vec{Q}$ we would compute

$$\begin{aligned} R_x &= P_x - Q_x \\ R_y &= P_y - Q_y \end{aligned} \tag{3.23}$$

Similarly, $\vec{T} = c\vec{S}$ would be

$$\begin{aligned} T_x &= cS_x \\ T_y &= cS_y \end{aligned} \tag{3.24}$$

The next few chapters will make frequent use of *vector equations*. For example, you will learn that the equation to calculate the force on a car skidding to a stop is

$$\vec{F} = \vec{n} + \vec{w} + \mu\vec{f} \tag{3.25}$$

The following general rule is used to evaluate such an equation:

The x-component of the left-hand side of a vector equation is found by doing arithmetic calculations (addition, subtraction, multiplication) with just the x-components of all the vectors on the right-hand side. A separate set of calculations uses just the y-components and, if needed, the z-components.

Thus Equation 3.25 is really just a shorthand way of writing three simultaneous equations:

$$\begin{aligned} F_x &= n_x + w_x + \mu f_x \\ F_y &= n_y + w_y + \mu f_y \\ F_z &= n_z + w_z + \mu f_z \end{aligned} \tag{3.26}$$

In other words, a vector equation is interpreted as meaning: Equate the x-components on both sides of the equals sign, then equate the y-components, and then the z-components. Vector notation allows us to write these three equations in a much more compact form.

Tilted Axes and Arbitrary Directions

As we've noted, the coordinate system is entirely your choice. It is a grid that you impose on the problem in a manner that will make the problem easiest to solve. We will soon meet problems where it will be convenient to tilt the axes of the coordinate system, such as those shown in FIGURE 3.25. The axes are perpendicular, and the y-axis is oriented correctly with respect to the x-axis, so this is a legitimate coordinate system. There is no requirement that the x-axis has to be horizontal.

Finding components with tilted axes is no harder than what we have done so far. Vector $\vec{C}$ in Figure 3.25 can be decomposed into $\vec{C} = C_x\hat{\imath} + C_y\hat{\jmath}$, where $C_x = C\cos\theta$ and $C_y = C\sin\theta$. Note that the unit vectors $\hat{\imath}$ and $\hat{\jmath}$ correspond to the *axes*, not to "horizontal" and "vertical," so they are also tilted.

Tilted axes are useful if you need to determine component vectors "parallel to" and "perpendicular to" an arbitrary line or surface. For example, FIGURE 3.26a shows a vector $\vec{A}$ and a tilted line. To find the component vectors of $\vec{A}$ parallel and perpendicular to the line, establish a tilted coordinate system with the x-axis parallel to the line and the y-axis perpendicular to the line, as shown in FIGURE 3.26b. Then $\vec{A}_x$ is equivalent to vector $\vec{A}_\parallel$, the component of $\vec{A}$ parallel to the line, and $\vec{A}_y$ is equivalent to the perpendicular component vector $\vec{A}_\perp$. Notice that $\vec{A} = \vec{A}_\parallel + \vec{A}_\perp$.

If ϕ is the angle between $\vec{A}$ and the line, we can easily calculate the parallel and perpendicular components of $\vec{A}$:

$$\begin{aligned} A_\parallel &= A_x = A\cos\phi \\ A_\perp &= A_y = A\sin\phi \end{aligned} \qquad (3.27)$$

It was not necessary to have the tail of $\vec{A}$ on the line in order to find a component of $\vec{A}$ parallel to the line. The line simply indicates a direction, and the component vector $\vec{A}_\parallel$ points in that direction.

FIGURE 3.25 A coordinate system with tilted axes.

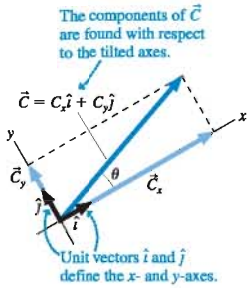
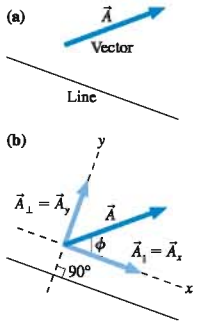


FIGURE 3.26 Finding the components of $\vec{A}$ parallel to and perpendicular to the line.

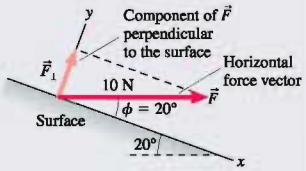


EXAMPLE 3.7 Finding the force perpendicular to a surface

A horizontal force $\vec{F}$ with a strength of 10 N is applied to a surface. (You'll learn in Chapter 5 that force is a vector quantity measured in units of *newtons*, abbreviated N.) The surface is tilted at a 20° angle. Find the component of the force vector perpendicular to the surface.

VISUALIZE FIGURE 3.27 shows a horizontal force $\vec{F}$ applied to the surface. A tilted coordinate system has its y-axis perpendicular to the surface, so the perpendicular component is $F_\perp = F_y$.

FIGURE 3.27 Finding the component of a force vector perpendicular to a surface.

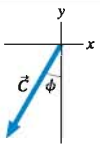


SOLVE From geometry, the force vector $\vec{F}$ makes an angle $\phi = 20°$ with the tilted x-axis. The perpendicular component of $\vec{F}$ is thus

$$F_\perp = F\sin 20° = (10\text{ N})\sin 20° = 3.4\text{ N}$$

STOP TO THINK 3.4 Angle ϕ that specifies the direction of $\vec{C}$ is given by

a. $\tan^{-1}(C_x/C_y)$.
b. $\tan^{-1}(C_x/|C_y|)$.
c. $\tan^{-1}(|C_x|/|C_y|)$.
d. $\tan^{-1}(C_y/C_x)$.
e. $\tan^{-1}(C_y/|C_x|)$.
f. $\tan^{-1}(|C_y|/|C_x|)$.

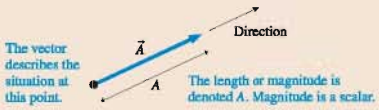


SUMMARY

The goals of Chapter 3 have been to learn how vectors are represented and used.

Important Concepts

A vector is a quantity described by both a magnitude and a direction.



Unit Vectors

Unit vectors have magnitude 1 and no units. Unit vectors $\hat{\imath}$ and $\hat{\jmath}$ define the directions of the x- and y-axes.

Using Vectors

Components

The component vectors are parallel to the x- and y-axes:

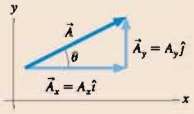
$$\vec{A} = \vec{A}_x + \vec{A}_y = A_x\hat{\imath} + A_y\hat{\jmath}$$

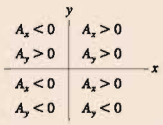
In the figure at the right, for example:

$A_x = A\cos\theta$ $\quad A = \sqrt{A_x^2 + A_y^2}$

$A_y = A\sin\theta$ $\quad \theta = \tan^{-1}(A_y/A_x)$

▶ Minus signs need to be included if the vector points down or left.





The components A_x and A_y are the magnitudes of the component vectors $\vec{A}_x$ and $\vec{A}_y$ *and* a plus or minus sign to show whether the component vector points toward the positive end or the negative end of the axis.

Working Graphically

Working Algebraically

Vector calculations are done component by component:

$$\vec{C} = 2\vec{A} + \vec{B} \quad \text{means} \quad \begin{cases} C_x = 2A_x + B_x \\ C_y = 2A_y + B_y \end{cases}$$

The magnitude of $\vec{C}$ is then $C = \sqrt{C_x^2 + C_y^2}$ and its direction is found using $\tan^{-1}$.

Terms and Notation

scalar quantity	graphical addition	quadrants	component
vector quantity	zero vector, $\vec{0}$	component vector	unit vector, $\hat{\imath}$ or $\hat{\jmath}$
magnitude	Cartesian coordinates	decomposition	algebraic addition
resultant vector			

For homework assigned on MasteringPhysics, go to www.masteringphysics.com

Problem difficulty is labeled as | (straightforward) to ||| (challenging).

CONCEPTUAL QUESTIONS

1. Can the magnitude of the displacement vector be more than the distance traveled? Less than the distance traveled? Explain.
2. If $\vec{C} = \vec{A} + \vec{B}$, can $C = A + B$? Can $C > A + B$? For each, show how or explain why not.
3. If $\vec{C} = \vec{A} + \vec{B}$, can $C = 0$? Can $C < 0$? For each, show how or explain why not.
4. Is it possible to add a scalar to a vector? If so, demonstrate. If not, explain why not.
5. How would you define the *zero vector* $\vec{0}$?
6. Can a vector have a component equal to zero and still have nonzero magnitude? Explain.
7. Can a vector have zero magnitude if one of its components is nonzero? Explain.
8. Suppose two vectors have unequal magnitudes. Can their sum be zero? Explain.

EXERCISES AND PROBLEMS

Exercises

Section 3.1 Vectors

Section 3.2 Properties of Vectors

1. | Trace the vectors in FIGURE EX3.1 onto your paper. Then find (a) $\vec{A} + \vec{B}$ and (b) $\vec{A} - \vec{B}$.

FIGURE EX3.1

FIGURE EX3.2

2. | Trace the vectors in FIGURE EX3.2 onto your paper. Then find (a) $\vec{A} + \vec{B}$ and (b) $\vec{A} - \vec{B}$.

Section 3.3 Coordinate Systems and Vector Components

3. | a. What are the x- and y-components of vector $\vec{E}$ in terms of the angle θ and the magnitude E shown in FIGURE EX3.3?
   b. For the same vector, what are the x- and y-components in terms of the angle ϕ and the magnitude E?

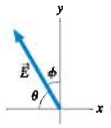


FIGURE EX3.3

4. | A position vector in the first quadrant has an x-component of 6 m and a magnitude of 10 m. What is the value of its y-component?
5. || A velocity vector 40° below the positive x-axis has a y-component of −10 m/s. What is the value of its x-component?
6. | Draw each of the following vectors, then find its x- and y-components.
   a. $\vec{r}$ = (100 m, 45° below positive x-axis)
   b. $\vec{v}$ = (300 m/s, 20° above positive x-axis)
   c. $\vec{a}$ = (5.0 m/s^2, negative y-direction)
7. | Draw each of the following vectors, then find its x- and y-components.
   a. $\vec{v}$ = (5.0 cm/s, negative x-direction)
   b. $\vec{a}$ = (10 m/s^2, 40° left of negative y-axis)
   c. $\vec{F}$ = (50 N, 36.9° right of positive y-axis)
8. | Let $\vec{C}$ = (3.15 m, 15° above the negative x-axis) and $\vec{D}$ = (25.6 m, 30° to the right of the negative y-axis). Find the magnitude, the x-component, and the y-component of each vector.
9. | The quantity called the *electric field* is a vector. The electric field inside a scientific instrument is $\vec{E} = (125\hat{i} - 250\hat{j})$ V/m, where V/m stands for volts per meter. What are the magnitude and direction of the electric field?

Section 3.4 Vector Algebra

10. | Draw each of the following vectors, label an angle that specifies the vector's direction, then find its magnitude and direction.
    a. $\vec{B} = -4\hat{i} + 4\hat{j}$
    b. $\vec{r} = (-2.0\hat{i} - 1.0\hat{j})$ cm
    c. $\vec{v} = (-10\hat{i} - 100\hat{j})$ m/s
    d. $\vec{a} = (20\hat{i} + 10\hat{j})$ m/s^2
11. | Draw each of the following vectors, label an angle that specifies the vector's direction, then find the vector's magnitude and direction.
    a. $\vec{A} = 4\hat{i} - 6\hat{j}$
    b. $\vec{r} = (50\hat{i} + 80\hat{j})$ m
    c. $\vec{v} = (-20\hat{i} + 40\hat{j})$ m/s
    d. $\vec{a} = (2.0\hat{i} - 6.0\hat{j})$ m/s^2
12. | Let $\vec{A} = 2\hat{i} + 3\hat{j}$ and $\vec{B} = 4\hat{i} - 2\hat{j}$.
    a. Draw a coordinate system and on it show vectors $\vec{A}$ and $\vec{B}$.
    b. Use graphical vector subtraction to find $\vec{C} = \vec{A} - \vec{B}$.
13. | Let $\vec{A} = 5\hat{i} + 2\hat{j}$, $\vec{B} = -3\hat{i} - 5\hat{j}$, and $\vec{C} = \vec{A} + \vec{B}$.
    a. Write vector $\vec{C}$ in component form.
    b. Draw a coordinate system and on it show vectors $\vec{A}$, $\vec{B}$, and $\vec{C}$.
    c. What are the magnitude and direction of vector $\vec{C}$?
14. | Let $\vec{A} = 5\hat{i} + 2\hat{j}$, $\vec{B} = -3\hat{i} - 5\hat{j}$, and $\vec{D} = \vec{A} - \vec{B}$.
    a. Write vector $\vec{D}$ in component form.
    b. Draw a coordinate system and on it show vectors $\vec{A}$, $\vec{B}$, and $\vec{D}$.
    c. What are the magnitude and direction of vector $\vec{D}$?

15. | Let $\vec{A} = 5\hat{i} + 2\hat{j}$, $\vec{B} = -3\hat{i} - 5\hat{j}$, and $\vec{E} = 2\vec{A} + 3\vec{B}$.
    a. Write vector $\vec{E}$ in component form.
    b. Draw a coordinate system and on it show vectors $\vec{A}$, $\vec{B}$, and $\vec{E}$.
    c. What are the magnitude and direction of vector $\vec{E}$?
16. | Let $\vec{A} = 5\hat{i} + 2\hat{j}$, $\vec{B} = -3\hat{i} - 5\hat{j}$, and $\vec{F} = \vec{A} - 4\vec{B}$.
    a. Write vector $\vec{F}$ in component form.
    b. Draw a coordinate system and on it show vectors $\vec{A}$, $\vec{B}$, and $\vec{F}$.
    c. What are the magnitude and direction of vector $\vec{F}$?
17. | Are the following statements true or false? Explain your answer.
    a. The magnitude of a vector can be different in different coordinate systems.
    b. The direction of a vector can be different in different coordinate systems.
    c. The components of a vector can be different in different coordinate systems.
18. | Let $\vec{B}$ = (5.0 m, 60° counterclockwise from vertical). Find the x- and y-components of $\vec{B}$ in each of the two coordinate systems shown in FIGURE EX3.18.

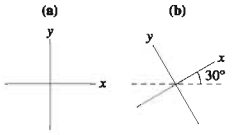


FIGURE EX3.18

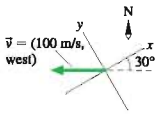


FIGURE EX3.19

19. | What are the x- and y-components of the velocity vector shown in FIGURE EX3.19?

Problems

20. | Let $\vec{A}$ = (3.0 m, 20° south of east), $\vec{B}$ = (2.0 m, north), and $\vec{C}$ = (5.0 m, 70° south of west).
    a. Draw and label $\vec{A}$, $\vec{B}$, and $\vec{C}$ with their tails at the origin. Use a coordinate system with the x-axis to the east.
    b. Write $\vec{A}$, $\vec{B}$, and $\vec{C}$ in component form, using unit vectors.
    c. Find the magnitude and the direction of $\vec{D} = \vec{A} + \vec{B} + \vec{C}$.
21. | Trace the vectors in FIGURE P3.21 onto your paper. Use the graphical method of vector addition and subtraction to find the following.

FIGURE P3.21

    a. $\vec{D} + \vec{E} + \vec{F}$ b. $\vec{D} + 2\vec{E}$ c. $\vec{D} - 2\vec{E} + \vec{F}$
22. | Let $\vec{E} = 2\hat{i} + 3\hat{j}$ and $\vec{F} = 2\hat{i} - 2\hat{j}$. Find the magnitude of
    a. $\vec{E}$ and $\vec{F}$ b. $\vec{E} + \vec{F}$ c. $-\vec{E} - 2\vec{F}$
23. | The position of a particle as a function of time is given by $\vec{r} = (5.0\hat{i} + 4.0\hat{j})t^2$ m, where t is in seconds.
    a. What is the particle's distance from the origin at t = 0, 2, and 5 s?
    b. Find an expression for the particle's velocity $\vec{v}$ as a function of time.
    c. What is the particle's speed at t = 0, 2, and 5 s?
24. || FIGURE P3.24 shows vectors $\vec{A}$ and $\vec{B}$. Let $\vec{C} = \vec{A} + \vec{B}$.
    a. Reproduce the figure on your page as accurately as possible, using a ruler and protractor. Draw vector $\vec{C}$ on your figure, using the graphical addition of $\vec{A}$ and $\vec{B}$. Then determine the magnitude and direction of $\vec{C}$ by *measuring* it with a ruler and protractor.
    b. Based on your figure of part a, use geometry and trigonometry to *calculate* the magnitude and direction of $\vec{C}$.
    c. Decompose vectors $\vec{A}$ and $\vec{B}$ into components, then use these to calculate algebraically the magnitude and direction of $\vec{C}$.

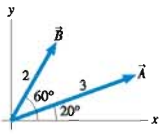


FIGURE P3.24

25. || For the three vectors shown in FIGURE P3.25, $\vec{A} + \vec{B} + \vec{C} = -2\hat{i}$. What is vector $\vec{B}$?
    a. Write $\vec{B}$ in component form.
    b. Write $\vec{B}$ as a magnitude and a direction.

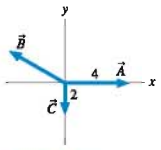


FIGURE P3.25

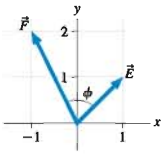


FIGURE P3.26

26. || a. What is the angle ϕ between vectors $\vec{E}$ and $\vec{F}$ in FIGURE P3.26?
    b. Use geometry and trigonometry to determine the magnitude and direction of $\vec{G} = \vec{E} + \vec{F}$.
    c. Use components to determine the magnitude and direction of $\vec{G} = \vec{E} + \vec{F}$.
27. || FIGURE P3.27 shows vectors $\vec{A}$ and $\vec{B}$. Find vector $\vec{C}$ such that $\vec{A} + \vec{B} + \vec{C} = \vec{0}$. Write your answer in component form.

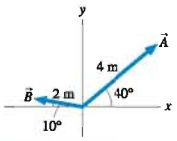


FIGURE P3.27

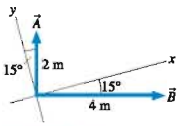


FIGURE P3.28

28. ||| FIGURE P3.28 shows vectors $\vec{A}$ and $\vec{B}$. Find $\vec{D} = 2\vec{A} + \vec{B}$. Write your answer in component form.
29. || Find a vector that points in the same direction as the vector $(\hat{i} + \hat{j})$ and whose magnitude is 1.
30. | Carlos runs with velocity $\vec{v}$ = (5.0 m/s, 25° north of east) for 10 minutes. How far to the north of his starting position does Carlos end up?
31. || While vacationing in the mountains you do some hiking. In the morning, your displacement is $\vec{s}_{\text{morning}}$ = (2000 m, east) + (3000 m, north) + (200 m, vertical). After lunch, your displacement is $\vec{s}_{\text{afternoon}}$ = (1500 m, west) + (2000 m, north) − (300 m, vertical).
    a. At the end of the hike, how much higher or lower are you compared to your starting point?
    b. What is the magnitude of your net displacement for the day?
32. || The minute hand on a watch is 2.0 cm in length. What is the displacement vector of the tip of the minute hand
    a. From 8:00 to 8:20 A.M.?
    b. From 8:00 to 9:00 A.M.?
33. || Bob walks 200 m south, then jogs 400 m southwest, then walks 200 m in a direction 30° east of north.
    a. Draw an accurate graphical representation of Bob's motion. Use a ruler and a protractor!

b. Use either trigonometry or components to find the displacement that will return Bob to his starting point by the most direct route. Give your answer as a distance and a direction.
c. Does your answer to part b agree with what you can measure on your diagram of part a?

34. || Jim's dog Sparky runs 50 m northeast to a tree, then 70 m west to a second tree, and finally 20 m south to a third tree.
a. Draw a picture and establish a coordinate system.
b. Calculate Sparky's net displacement in component form.
c. Calculate Sparky's net displacement as a magnitude and an angle.

35. || A field mouse trying to escape a hawk runs east for 5.0 m, darts southeast for 3.0 m, then drops 1.0 m straight down a hole into its burrow. What is the magnitude of the net displacement of the mouse?

36. | A cannon tilted upward at 30° fires a cannonball with a speed of 100 m/s. What is the component of the cannonball's velocity parallel to the ground?

37. | Jack and Jill ran up the hill at 3.0 m/s. The horizontal component of Jill's velocity vector was 2.5 m/s.
a. What was the angle of the hill?
b. What was the vertical component of Jill's velocity?

38. | A pine cone falls straight down from a pine tree growing on a 20° slope. The pine cone hits the ground with a speed of 10 m/s. What is the component of the pine cone's impact velocity (a) parallel to the ground and (b) perpendicular to the ground?

39. | Mary needs to row her boat across a 100-m-wide river that is flowing to the east at a speed of 1.0 m/s. Mary can row the boat with a speed of 2.0 m/s relative to the water.
a. If Mary rows straight north, how far downstream will she land?
b. Draw a picture showing Mary's displacement due to rowing, her displacement due to the river's motion, and her net displacement.

40. || The treasure map in FIGURE P3.40 gives the following directions to the buried treasure: "Start at the old oak tree, walk due north for 500 paces, then due east for 100 paces. Dig." But when you arrive, you find an angry dragon just north of the tree. To avoid the dragon, you set off along the yellow brick road at an angle 60° east of north. After walking 300 paces you see an opening through the woods. Which direction should you go, and how far, to reach the treasure?

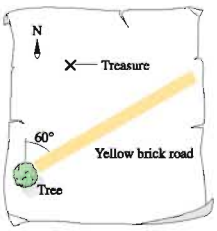


FIGURE P3.40

41. || A jet plane is flying horizontally with a speed of 500 m/s over a hill that slopes upward with a 3% grade (i.e., the "rise" is 3% of the "run"). What is the component of the plane's velocity perpendicular to the ground?

42. || A flock of ducks is trying to migrate south for the winter, but they keep being blown off course by a wind blowing from the west at 6.0 m/s. A wise elder duck finally realizes that the solution is to fly at an angle to the wind. If the ducks can fly at 8.0 m/s relative to the air, what direction should they head in order to move directly south?

43. || The car in FIGURE P3.43 speeds up as it turns a quarter-circle curve from north to east. When exactly halfway around the curve, the car's acceleration is $\vec{a} = (2.0\ \text{m/s}^2, 15°\ \text{south of east})$. At this point, what is the component of $\vec{a}$ (a) tangent to the circle and (b) perpendicular to the circle?

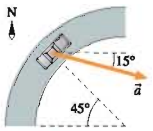


FIGURE P3.43

44. || FIGURE P3.44 shows three ropes tied together in a knot. One of your friends pulls on a rope with 3.0 units of force and another pulls on a second rope with 5.0 units of force. How hard and in what direction must you pull on the third rope to keep the knot from moving?

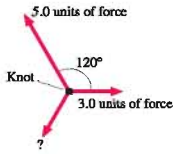


FIGURE P3.44

45. || Three forces are exerted on an object placed on a tilted floor in FIGURE P3.45. The forces are measured in newtons (N). Assuming that forces are vectors,
a. What is the component of the *net force* $\vec{F}_{\text{net}} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3$ parallel to the floor?
b. What is the component of $\vec{F}_{\text{net}}$ perpendicular to the floor?
c. What are the magnitude and direction of $\vec{F}_{\text{net}}$?

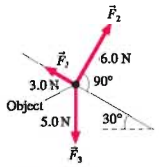


FIGURE P3.45

46. || FIGURE P3.46 shows four electric charges located at the corners of a rectangle. Like charges, you will recall, repel each other while opposite charges attract. Charge B exerts a repulsive force (directly *away from* B) on charge A of 3.0 N. Charge C exerts an attractive force (directly *toward* C) on charge A of 6.0 N. Finally, charge D exerts an attractive force of 2.0 N on charge A. Assuming that forces are vectors, what are the magnitude and direction of the net force $\vec{F}_{\text{net}}$ exerted on charge A?

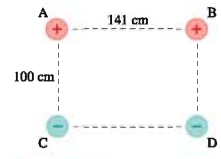


FIGURE P3.46

STOP TO THINK ANSWERS

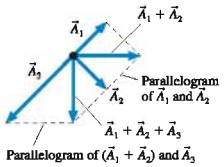
Stop to Think 3.1: c. The graphical construction of $\vec{A}_1 + \vec{A}_2 + \vec{A}_3$ is shown at right.

Stop to Think 3.2: a. The graphical construction of $2\vec{A} - \vec{B}$ is shown at right.

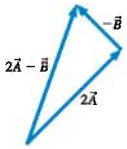
Stop to Think 3.3: $C_x = -4\ \text{cm}, C_y = 2\ \text{cm}$.

Stop to Think 3.4: c. Vector $\vec{C}$ points to the left and down, so both C_x and C_y are negative. C_x is in the numerator because it is the side opposite ϕ.

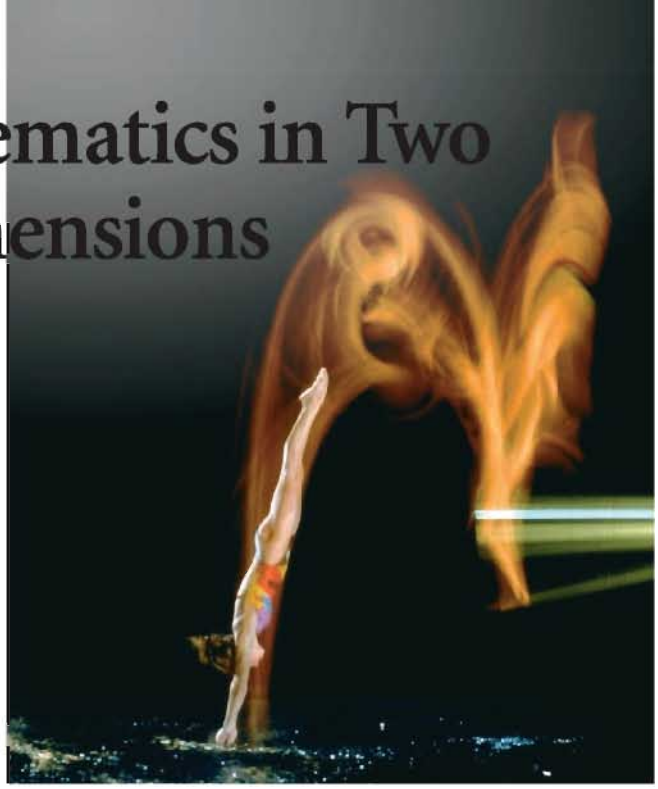
STOP TO THINK 3.1



STOP TO THINK 3.2



4 Kinematics in Two Dimensions



This diver is a spinning projectile following a parabolic trajectory.

▶ Looking Ahead

The goal of Chapter 4 is to learn to solve problems about motion in a plane. In this chapter you will learn to:

- Use kinematics in two dimensions.
- Understand projectile motion.
- Explore the issues of relative motion.
- Understand the mathematics of circular kinematics.

◀ Looking Back

This chapter uses vectors to extend kinematics to two dimensions. Please review:

- Section 1.5 Finding acceleration vectors on a motion diagram.
- Sections 2.5 and 2.6 Constant-acceleration kinematics and free fall.
- Sections 3.3 and 3.4 Decomposing vectors into components.

The one-dimensional motion of Chapter 2 has many interesting applications, but motion in the real world is often more complex. A car turning a corner, a basketball sailing toward the hoop, a planet orbiting the sun, and the diver in the photograph are examples of two-dimensional motion or, equivalently, motion in a plane.

This chapter will continue to focus on kinematics, the mathematical description of motion. We'll begin with motion in which two perpendicular components of acceleration are independent of each other. The most important such motion is that of a projectile. Then we'll turn to circular motion, analyzing particles in circular trajectories and rigid objects rotating on an axle. We'll then be ready, in the next chapter, to take up the *cause* of motion.

4.1 Acceleration

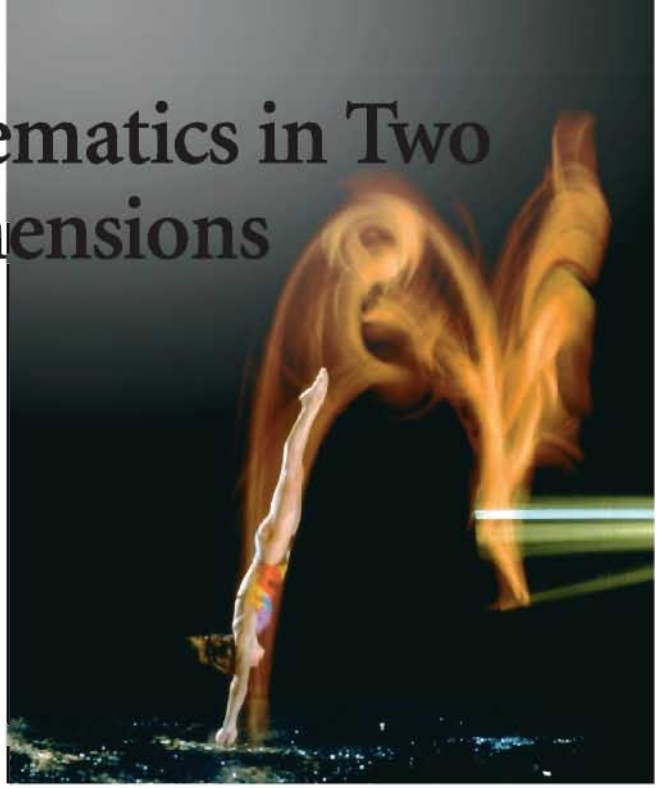
In Chapter 1 we defined the *average acceleration* $\vec{a}_{\text{avg}}$ of a moving object to be the vector

$$\vec{a}_{\text{avg}} = \frac{\Delta \vec{v}}{\Delta t} \tag{4.1}$$

From its definition, we see that **$\vec{a}$ points in the same direction as $\Delta\vec{v}$**, the change of velocity. As an object moves, its velocity vector can change in two possible ways:

1. The magnitude of $\vec{v}$ can change, indicating a change in speed, or
2. The direction of $\vec{v}$ can change, indicating that the object has changed direction.

4 Kinematics in Two Dimensions



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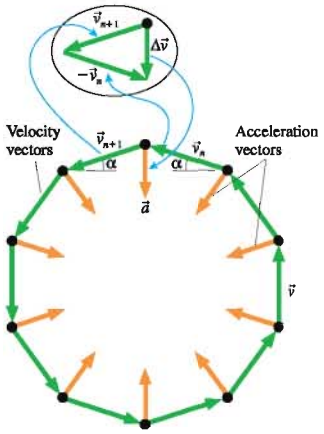
2. The direction of $\vec{v}$ can change, indicating that the object has changed direction.

FIGURE 4.2 Finding Maria's acceleration.



triangle and vector $\Delta\vec{v}$ is exactly vertical. When $\vec{a}$ is drawn on the motion diagram, in the same direction as $\Delta\vec{v}$, we see that Maria's acceleration vector points directly to the center of the circle.

No matter which dot you select on the motion diagram in Figure 4.2, the velocity vectors leading toward and away from that dot change in such a way as to cause the acceleration to point to the center of the circle. You should convince yourself of this by finding $\vec{a}$ at several other points around the circle. An acceleration that always points directly toward the center of a circle is called a *centripetal acceleration*. The word "centripetal" comes from a Greek root meaning "center seeking." We will have a lot to say about centripetal acceleration later in the chapter.

EXAMPLE 4.1 Through the valley

A ball rolls down a long hill, through the valley, and back up the other side. Draw a complete motion diagram of the ball, showing velocity and acceleration vectors.

MODEL Model the ball as a particle.

VISUALIZE FIGURE 4.3 is the motion diagram. Where the particle moves along a *straight line,* it speeds up if $\vec{a}$ and $\vec{v}$ point in the same direction and slows down if $\vec{a}$ and $\vec{v}$ point in opposite directions. This idea was the basis for the one-dimensional kinematics we developed in Chapter 2. For linear motion, acceleration is a change of speed. When the direction of $\vec{v}$ changes, as it does when the ball goes through the valley, we need to use vector subtraction to find the direction of $\Delta\vec{v}$ and thus of $\vec{a}$. The procedure is shown at two points in the motion diagram. Notice that the point at the bottom of the valley is much like the top point of Maria's motion diagram in Figure 4.2.

FIGURE 4.3 The motion diagram of the ball of Example 4.1.

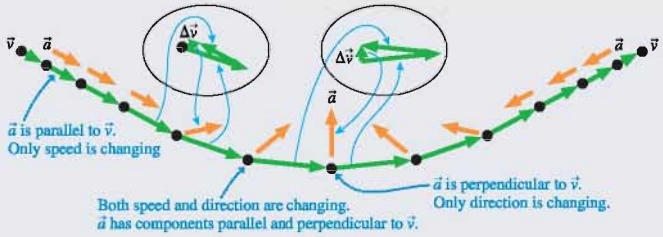
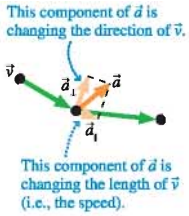


FIGURE 4.4 Decomposing the acceleration vector.

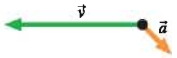


Chapter 3 showed how to decompose a vector into two components perpendicular and parallel to a line. In FIGURE 4.4, the acceleration vector $\vec{a}$ at one point in the motion diagram of Figure 4.3 has been decomposed into a piece $\vec{a}_\parallel$ parallel to $\vec{v}$ and a piece $\vec{a}_\perp$ perpendicular to $\vec{v}$. **$\vec{a}_\parallel$ is the piece of the acceleration vector that changes the speed.** In this case, the ball is speeding up because $\vec{a}_\parallel$ is parallel to the motion. **The component $\vec{a}_\perp$ is the piece of the acceleration that causes the velocity to change direction.** Notice that $\vec{a}$ *always* has a perpendicular component at points where the ball is changing directions.

On the straight sections of the hill, where only the speed is changing, the perpendicular component vanishes and $\vec{a}$ is parallel to $\vec{v}$. At the very bottom of the hill, where only the direction is changing, not the speed, the parallel component vanishes and $\vec{a}$ is perpendicular to $\vec{v}$. The important point to remember is that either changing speed *or* changing direction requires an acceleration.

STOP TO THINK 4.1 This acceleration will cause the particle to

a. Speed up and curve upward.
b. Speed up and curve downward.
c. Slow down and curve upward.
d. Slow down and curve downward.
e. Move to the right and down.
f. Reverse direction.



4.2 Two-Dimensional Kinematics

Motion diagrams are an important tool for visualizing motion, but we also need to develop a mathematical description of motion in two dimensions. We're going to begin with motion in which the horizontal and vertical components of acceleration are independent of each other. It will be easier to use x- and y-components of vectors, rather than components parallel and perpendicular to the motion. We'll point out, as we go along, the connection between these two points of view. For convenience, we'll say that the motion is in the xy-plane regardless of whether the plane of motion is horizontal or vertical.

FIGURE 4.5 shows a particle moving along a curved path—its *trajectory*—in the xy-plane. We can locate the particle in terms of its position vector

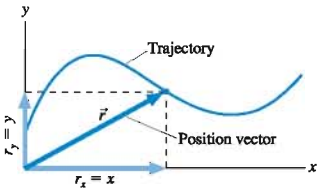
$$\vec{r} = r_x\hat{\imath} + r_y\hat{\jmath}$$

where, as you'll recall, $\hat{\imath}$ and $\hat{\jmath}$ are unit vectors along the x- and y-axes and r_x and r_y are, respectively, the x- and y-components of $\vec{r}$. But r_x is simply x, the x-coordinate of the point. Similarly, r_y is the y-coordinate y. Hence the position vector is

$$\vec{r} = x\hat{\imath} + y\hat{\jmath} \tag{4.2}$$

NOTE ▶ In Chapter 2 we made extensive use of position-versus-time graphs, either x versus t or y versus t. Figure 4.5, like many of the graphs we'll use in this chapter, is a graph of y versus x. In other words, it's an actual *picture* of the trajectory, not an abstract representation of the motion. ◀

FIGURE 4.5 A particle moving along a trajectory in the xy-plane.



As the particle in FIGURE 4.6 moves from position $\vec{r}_1$ at time t_1 to position $\vec{r}_2$ at time t_2, its *displacement*, the vector from point 1 to point 2, is

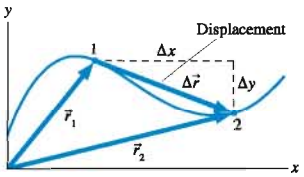
$$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1$$

We can write the displacement vector in component form as

$$\Delta\vec{r} = \Delta x\hat{\imath} + \Delta y\hat{\jmath} \tag{4.3}$$

where $\Delta x = x_2 - x_1$ and $\Delta y = y_2 - y_1$ are the horizontal and vertical changes of position.

FIGURE 4.6 The particle undergoes displacement $\Delta\vec{r}$.



Chapter 1 defined the *average velocity* of a particle moving through a displacement $\Delta\vec{r}$ in a time interval Δt as

$$\vec{v}_{\text{avg}} = \frac{\Delta\vec{r}}{\Delta t} = \frac{\Delta x}{\Delta t}\hat{\imath} + \frac{\Delta y}{\Delta t}\hat{\jmath} \tag{4.4}$$

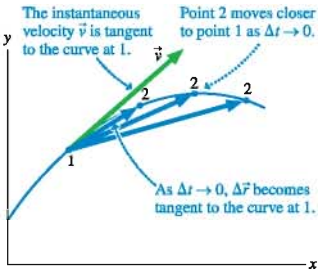
You learned in Chapter 2 that the *instantaneous velocity* is the limit of $\vec{v}_{\text{avg}}$ as $\Delta t \to 0$. Taking the limit of Equation 4.4 gives the instantaneous velocity in two dimensions:

$$\vec{v} = \lim_{\Delta t \to 0}\frac{\Delta\vec{r}}{\Delta t} = \frac{d\vec{r}}{dt} = \frac{dx}{dt}\hat{\imath} + \frac{dy}{dt}\hat{\jmath} \tag{4.5}$$

But we can also write the velocity vector in terms of its x- and y-components as

$$\vec{v} = v_x\hat{\imath} + v_y\hat{\jmath} \tag{4.6}$$

FIGURE 4.7 The instantaneous velocity vector $\vec{v}$ is tangent to the trajectory.



Comparing Equations 4.5 and 4.6, you can see that the velocity vector $\vec{v}$ has x- and y-components

$$v_x = \frac{dx}{dt} \quad \text{and} \quad v_y = \frac{dy}{dt} \tag{4.7}$$

That is, the x-component v_x of the velocity vector is the rate dx/dt at which the particle's x-coordinate is changing. The y-component is similar.

The average velocity $\vec{v}_{avg}$ points in the direction of $\Delta\vec{r}$, a fact we used in Chapter 1 to draw the velocity vectors on motion diagrams. FIGURE 4.7 shows that $\Delta\vec{r}$ becomes tangent to the trajectory as $\Delta t \to 0$. Consequently, **the instantaneous velocity vector $\vec{v}$ is tangent to the trajectory.**

FIGURE 4.8 illustrates another important feature of the velocity vector. If the vector's angle θ is measured from the positive x-axis, the velocity vector components are

$$\begin{aligned} v_x &= v\cos\theta \\ v_y &= v\sin\theta \end{aligned} \tag{4.8}$$

where

$$v = \sqrt{v_x^2 + v_y^2} \tag{4.9}$$

is the particle's *speed* at that point. Speed is always a positive number (or zero), whereas the components are *signed* quantities (i.e., they can be positive or negative) to convey information about the direction of the velocity vector. Conversely, we can use the two velocity components to determine the direction of motion:

$$\tan\theta = \frac{v_y}{v_x} \tag{4.10}$$

NOTE ▶ In Chapter 2, you learned that the *value* of the velocity component v_s at time t is given by the *slope* of the position-versus-time graph at time t. Now we see that the *direction* of the velocity vector $\vec{v}$ is given by the *tangent* to the y-versus-x graph of the trajectory. FIGURE 4.9 reminds you that these two graphs use different interpretations of the tangent lines. The tangent to the trajectory does not tell us anything about how fast the particle is moving, only its direction. ◀

FIGURE 4.8 Relating the components of $\vec{v}$ to the speed and direction.

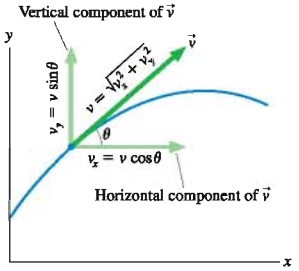
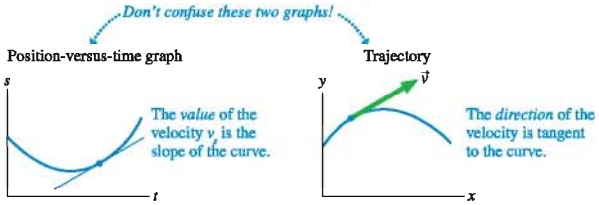


FIGURE 4.9 Two different uses of tangent lines.



EXAMPLE 4.2 Describing the motion with graphs

A particle's motion is described by the two equations

$$x = 2t^2 \text{ m/s}$$
$$y = (5t + 5) \text{ m/s}$$

where the time t is in s.

a. Draw a graph of the particle's trajectory.
b. Draw a graph of the particle's speed as a function of time.

MODEL These are *parametric equations* that give the particle's coordinates x and y separately in terms of the parameter t.

SOLVE a. The trajectory is a curve in the xy-plane. The easiest way to proceed is to calculate x and y at several instants of time.

t (s)	x (m)	y (m)	v (m/s)
0	0	5	5.0
1	2	10	6.4
2	8	15	9.4
3	18	20	13.0
4	32	25	16.8

These points are plotted in FIGURE 4.10a, then a smooth curve is drawn through them to show the trajectory.

b. The particle's speed is given by Equation 4.9. We first need to use Equation 4.7 to find the components of the velocity vector:

$$v_x = \frac{dx}{dt} = 4t \text{ m/s} \quad \text{and} \quad v_y = \frac{dy}{dt} = 5 \text{ m/s}$$

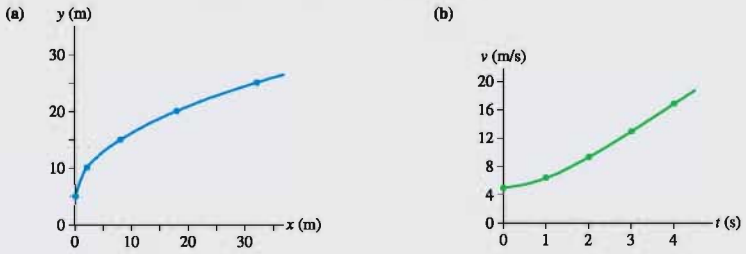
Using these gives the particle's speed at time t:

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{16t^2 + 25} \text{ m/s}$$

The speed was computed in the table and is graphed in FIGURE 4.10b.

ASSESS The y-versus-x graph of Figure 4.10a is a trajectory, not a position-versus-time graph. Thus the slope is *not* the particle's speed. The particle is speeding up, as you can see in the second graph, even though the slope of the trajectory is decreasing.

FIGURE 4.10 Two motion graphs for the particle of Example 4.2.



Acceleration

Let's return to the particle moving along a trajectory in the xy-plane. FIGURE 4.11a shows the instantaneous velocity $\vec{v}_1$ at point 1 and, a short time later, velocity $\vec{v}_2$ at point 2. These two vectors are tangent to the trajectory. We can use the vector-subtraction technique, shown in the inset, to find $\vec{a}_{avg}$ on this segment of the trajectory.

If we now take the limit $\Delta t \to 0$, the *instantaneous acceleration* is

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt} \tag{4.11}$$

As $\Delta t \to 0$, points 1 and 2 in Figure 4.11a merge, and the instantaneous acceleration $\vec{a}$ is found at the same point on the trajectory (and the same instant of time) as the instantaneous velocity $\vec{v}$. This is shown in FIGURE 4.11b.

FIGURE 4.11 The average and instantaneous acceleration vectors on a curved trajectory.

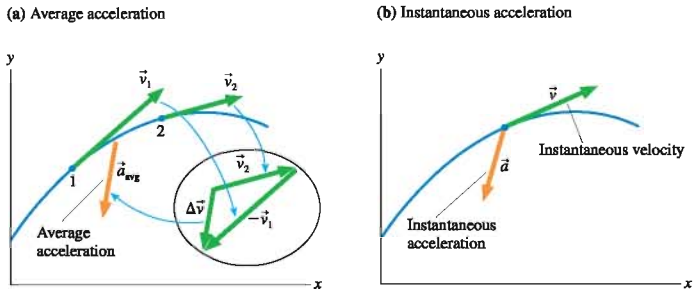
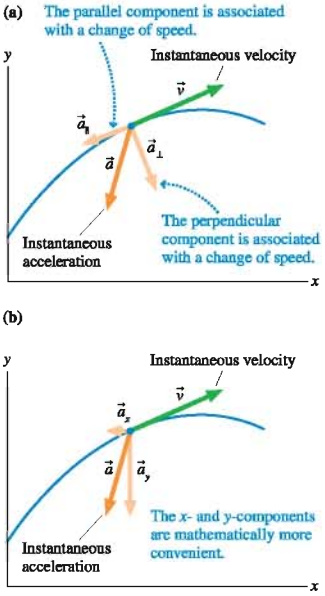


FIGURE 4.12 Decomposition of the instantaneous acceleration $\vec{a}$.



By definition, the acceleration vector $\vec{a}$ is the rate at which the velocity $\vec{v}$ is changing at that instant. To show this, FIGURE 4.12a decomposes $\vec{a}$ into components $\vec{a}_\parallel$ and $\vec{a}_\perp$ that are parallel and perpendicular to the trajectory. $\vec{a}_\parallel$ is associated with a change of speed, and $\vec{a}_\perp$ is associated with a change of direction. Both kinds of changes are accelerations. Notice that $\vec{a}_\perp$ always points toward the "inside" of the curve because that is the direction in which $\vec{v}$ is changing.

The parallel and perpendicular components of $\vec{a}$ convey important ideas about acceleration, but it's usually more practical to write $\vec{a}$ in terms of the x- and y-components shown in FIGURE 4.12b. Because $\vec{v} = v_x\hat{\imath} + v_y\hat{\jmath}$, we find

$$\vec{a} = a_x\hat{\imath} + a_y\hat{\jmath} = \frac{d\vec{v}}{dt} = \frac{dv_x}{dt}\hat{\imath} + \frac{dv_y}{dt}\hat{\jmath} \tag{4.12}$$

from which we see that

$$a_x = \frac{dv_x}{dt} \quad \text{and} \quad a_y = \frac{dv_y}{dt} \tag{4.13}$$

That is, the x-component of $\vec{a}$ is the rate dv_x/dt at which the x-component of velocity is changing.

Constant Acceleration

If the acceleration $\vec{a} = a_x\hat{\imath} + a_y\hat{\jmath}$ is constant, then the two components a_x and a_y are both constant (or zero). In this case, everything you learned about constant-acceleration kinematics in Chapter 2 carries over to the x- and y-components of two-dimensional motion.

Consider a particle that moves with constant acceleration from an initial position $\vec{r}_i = x_i\hat{\imath} + y_i\hat{\jmath}$, starting with initial velocity $\vec{v}_i = v_{ix}\hat{\imath} + v_{iy}\hat{\jmath}$. Its position and velocity at a final point f are

$$\begin{aligned} x_f &= x_i + v_{ix}\,\Delta t + \tfrac{1}{2}a_x(\Delta t)^2 & y_f &= y_i + v_{iy}\,\Delta t + \tfrac{1}{2}a_y(\Delta t)^2 \\ v_{fx} &= v_{ix} + a_x\,\Delta t & v_{fy} &= v_{iy} + a_y\,\Delta t \end{aligned} \tag{4.14}$$

There are *many* quantities to keep track of in two-dimensional kinematics, making the pictorial representation all the more important as a problem-solving tool.

NOTE ▶ For constant acceleration, the x-component of the motion and the y-component of the motion are independent of each other. However, they remain connected through the fact that Δt must be the same for both. ◀

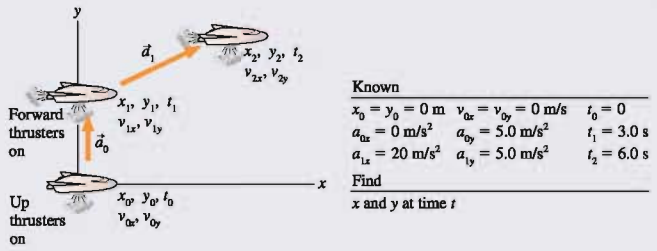
EXAMPLE 4.3 Plotting the trajectory of the shuttlecraft

The up thrusters on the shuttlecraft of the starship *Enterprise* give it an upward acceleration of 5.0 m/s^2. Its forward thrusters provide a forward acceleration of 20 m/s^2. As it leaves the *Enterprise*, the shuttlecraft turns on only the up thrusters. After clearing the flight deck, 3.0 s later, it adds the forward thrusters. Plot a trajectory of the shuttlecraft for its first 6 s.

MODEL Represent the shuttlecraft as a particle. There are two segments of constant-acceleration motion.

VISUALIZE FIGURE 4.13 shows a pictorial representation. The coordinate system has been chosen so that the shuttlecraft starts at the origin and initially moves along the y-axis. The craft moves vertically for 3.0 s, then begins to acquire a forward motion. There are three points in the motion: the beginning, the end, and the point at the which forward thrusters are turned on. These points are labeled (x_0, y_0), (x_1, y_1), and (x_2, y_2). The velocities are (v_{0x}, v_{0y}), (v_{1x}, v_{1y}), and (v_{2x}, v_{2y}). This will be our standard labeling scheme for trajectories, where it is essential to keep the x-components and y-components separate.

FIGURE 4.13 Pictorial representation of the motion of the shuttlecraft.



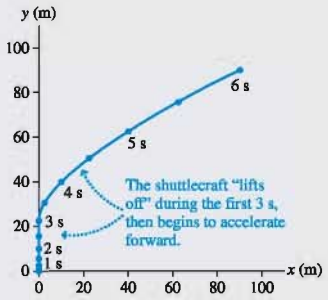
Known

$x_0 = y_0 = 0$ m	$v_{0x} = v_{0y} = 0$ m/s	$t_0 = 0$
$a_{0x} = 0$ m/s^2	$a_{0y} = 5.0$ m/s^2	$t_1 = 3.0$ s
$a_{1x} = 20$ m/s^2	$a_{1y} = 5.0$ m/s^2	$t_2 = 6.0$ s

Find

x and y at time t

FIGURE 4.14 The shuttlecraft trajectory.



SOLVE During the first phase of the acceleration, when $a_{0x} = 0$ m/s^2 and $a_{0y} = 5.0$ m/s^2, the motion is described by

$$y = y_0 + v_{0y}(t - t_0) + \tfrac{1}{2}a_{0y}(t - t_0)^2 = 2.5t^2 \text{ m}$$

$$v_y = v_{0y} + a_{0y}(t - t_0) = 5.0t \text{ m/s}$$

where the time t is in s. These equations allow us to calculate the position and velocity at any time t. At $t_1 = 3.0$ s, when the first phase of the motion ends, we find that

$$x_1 = 0 \text{ m} \qquad v_{1x} = 0 \text{ m/s}$$

$$y_1 = 22.5 \text{ m} \qquad v_{1y} = 15 \text{ m/s}$$

During the next 3 s, when $a_{1x} = 20$ m/s^2 and $a_{1y} = 5.0$ m/s^2, the x- and y-coordinates are

$$x = x_1 + v_{1x}(t - t_1) + \tfrac{1}{2}a_{1x}(t - t_1)^2$$

$$= 10(t - 3.0)^2 \text{ m}$$

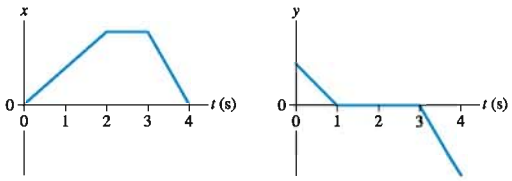
$$y = y_1 + v_{1y}(t - t_1) + \tfrac{1}{2}a_{1y}(t - t_1)^2$$

$$= \left(22.5 + 15(t - 3.0) + 2.5(t - 3.0)^2\right) \text{ m}$$

where, again, t is in s. To show the trajectory, we've calculated x and y every 0.5 s, plotted the points in FIGURE 4.14, and drawn a smooth curve through the points.

STOP TO THINK 4.2 During which time interval or intervals is the particle described by these position graphs at rest? More than one may be correct.

a. 0–1 s
b. 1–2 s
c. 2–3 s
d. 3–4 s



4.3 Projectile Motion

Baseballs and tennis balls flying through the air, Olympic divers, and daredevils shot from cannons all exhibit what we call *projectile motion*. A **projectile** is an object that moves in two dimensions under the influence of only gravity. Projectile motion is an extension of the free-fall motion we studied in Chapter 2. We will continue to neglect the influence of air resistance, leading to results that are a good approximation of reality for relatively heavy objects moving relatively slowly over relatively short distances. As we'll see, projectiles in two dimensions follow a *parabolic trajectory* like the one seen in FIGURE 4.15.

FIGURE 4.15 The parabolic trajectory of a bouncing ball.

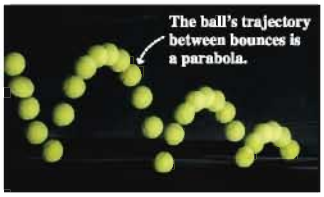
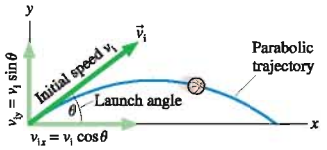


FIGURE 4.16 A projectile launched with initial velocity $\vec{v}_i$.



3.1–3.7 Activ ONLINE Physics

The start of a projectile's motion, be it thrown by hand or shot from a gun, is called the *launch*, and the angle θ of the initial velocity $\vec{v}_i$ above the horizontal (i.e., above the x-axis) is called the **launch angle.** FIGURE 4.16 illustrates the relationship between the initial velocity vector $\vec{v}_i$ and the initial values of the components v_{ix} and v_{iy}. You can see that

$$\begin{aligned} v_{ix} &= v_i \cos\theta \\ v_{iy} &= v_i \sin\theta \end{aligned} \qquad (4.15)$$

where v_i is the initial speed.

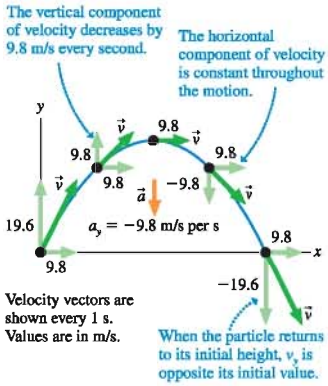
NOTE ▶ The components v_{ix} and v_{iy} are not necessarily positive. In particular, a projectile launched at an angle *below* the horizontal (such as a ball thrown downward from the roof of a building) has *negative* values for θ and v_{iy}. However, the speed v_i is always positive. ◀

Gravity acts downward, and we know that objects released from rest fall straight down, not sideways. Hence it's reasonable to assume—and we will justify this assumption in Chapter 8—that a projectile has no horizontal acceleration. Its vertical acceleration is simply that of free fall; thus

$$\begin{aligned} a_x &= 0 \\ a_y &= -g \quad \text{(projectile motion)} \end{aligned} \qquad (4.16)$$

In other words, **the vertical component of acceleration a_y is just the familiar $-g$ of free fall, while the horizontal component a_x is zero. Projectiles are in free fall.**

FIGURE 4.17 The velocity and acceleration vectors of a projectile moving along a parabolic trajectory.



To see how these conditions influence the motion, FIGURE 4.17 shows a projectile launched from $(x_i, y_i) = (0\text{ m}, 0\text{ m})$ with an initial velocity $\vec{v}_i = (9.8\hat{\imath} + 19.6\hat{\jmath})$ m/s. The velocity and acceleration vectors are then shown every 1.0 s. The value of v_x never changes because there's no horizontal acceleration, but v_y decreases by 9.8 m/s every second. This is what it *means* to accelerate at $a_y = -9.8\text{ m/s}^2 = (-9.8\text{ m/s})$ per second.

You can see from Figure 4.17 that **projectile motion is made up of two independent motions:** uniform motion at constant velocity in the horizontal direction and free-fall motion in the vertical direction. The kinematic equations that describe these two motions are

$$\begin{aligned} x_f &= x_i + v_{ix}\,\Delta t & y_f &= y_i + v_{iy}\,\Delta t - \tfrac{1}{2}g(\Delta t)^2 \\ v_{fx} &= v_{ix} = \text{constant} & v_{fy} &= v_{iy} - g\,\Delta t \end{aligned} \qquad (4.17)$$

These are parametric equations for the parabolic trajectory of a projectile.

EXAMPLE 4.4 Don't try this at home!

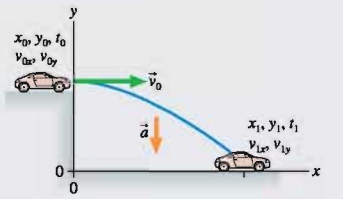
A stunt man drives a car off a 10.0-m-high cliff at a speed of 20.0 m/s. How far does the car land from the base of the cliff?

MODEL Represent the car as a particle in free fall. Assume that the car is moving horizontally as it leaves the cliff.

VISUALIZE The pictorial representation, shown in FIGURE 4.18, is *very* important because the number of quantities to keep track of in projectile motion problems is quite large. We have chosen to put the origin at the base of the cliff. The assumption that the car is moving horizontally as it leaves the cliff leads to $v_{0x} = v_0$ and $v_{0y} = 0$ m/s. A motion diagram is not essential in projectile motion problems because we already know that the projectile follows a parabolic trajectory with $\vec{a} = -g\hat{\jmath}$.

SOLVE Each point on the trajectory has x- and y-components of position, velocity, and acceleration but only *one* value of time.

FIGURE 4.18 Pictorial representation for the car of Example 4.4.



Known			Find
$x_0 = 0$ m	$v_{0y} = 0$ m/s	$t_0 = 0$ s	x_1
$y_0 = 10.0$ m	$v_{0x} = v_0 = 20.0$ m/s		
$a_x = 0\text{ m/s}^2$	$a_y = -g$	$y_1 = 0$ m	

The time needed to move horizontally to x_1 is the *same* time needed to fall vertically through distance y_0. **Although the horizontal and vertical motions are independent, they are connected through the time t.** This is a critical observation for solving projectile motion problems. The kinematics equations are

$$x_1 = x_0 + v_{0x}(t_1 - t_0) = v_0 t_1$$

$$y_1 = 0 = y_0 + v_{0y}(t_1 - t_0) - \tfrac{1}{2}g(t_1 - t_0)^2 = y_0 - \tfrac{1}{2}gt_1^2$$

We can use the vertical equation to determine the time t_1 needed to fall distance y_0:

$$t_1 = \sqrt{\frac{2y_0}{g}} = \sqrt{\frac{2(10.0\text{ m})}{9.80\text{ m/s}^2}} = 1.43\text{ s}$$

We then insert this expression for t into the horizontal equation to find the distance traveled:

$$x_1 = v_0 t_1 = (20.0\text{ m/s})(1.43\text{ s}) = 28.6\text{ m}$$

ASSESS The cliff height is ≈33 ft and the initial speed is $v_0 \approx$ 40 mph. Traveling $x_1 = 29$ m ≈ 95 ft before hitting the ground seems reasonable.

The x- and y-equations of Example 4.4 are parametric equations. It's not hard to eliminate t and write an expression for y as a function of x. From the x_1 equation, $t_1 = x_1/v_0$. Substituting this into the y_1 equation, we find

$$y = y_0 - \frac{g}{2v_0^2}x^2 \qquad (4.18)$$

The graph of $y = ax^2$ is a parabola, so Equation 4.18 represents an inverted parabola that starts from height y_0. This proves, as we asserted above, that a projectile follows a parabolic trajectory.

Reasoning About Projectile Motion

Think about the following question:

> A heavy ball is thrown exactly horizontally at height h above a horizontal field. At the exact instant that the ball is thrown, a second ball is simply dropped from height h. Which ball hits the ground first?

It may seem hard to believe, but—if air resistance is neglected—the balls hit the ground *simultaneously*. They do so because the horizontal and vertical components of projectile motion are independent of each other. The initial horizontal velocity of the first ball has *no* influence over its vertical motion. Neither ball has any initial motion in the vertical direction, so both fall distance h in the same amount of time. You can see this in FIGURE 4.19. The *vertical* motions of the two balls are identical, and they hit the floor simultaneously.

FIGURE 4.20a shows a useful way to think about the trajectory of a projectile. Without gravity, a projectile would follow a straight line. Because of gravity, the particle at time t has "fallen" a distance $\frac{1}{2}gt^2$ below this line. The separation grows as $\frac{1}{2}gt^2$, giving the trajectory its parabolic shape.

FIGURE 4.19 A projectile launched horizontally falls in the same time as a projectile that is released from rest.

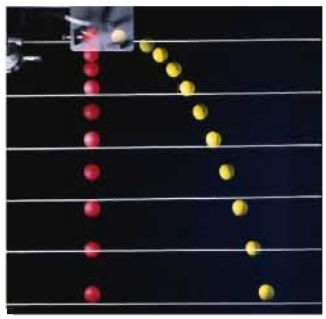
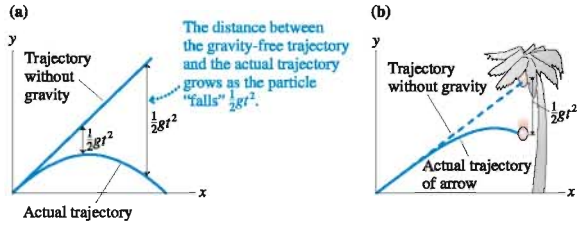


FIGURE 4.20 A projectile follows a parabolic trajectory because it "falls" a distance $\frac{1}{2}gt^2$ below a straight-line trajectory.



Use this idea to think about the following "classic" problem in physics:

A hungry bow-and-arrow hunter in the jungle wants to shoot down a coconut that is hanging from the branch of a tree. He points his arrow directly at the coconut, but as luck would have it, the coconut falls from the branch at the *exact* instant the hunter releases the string. Does the arrow hit the coconut?

You might think that the arrow will miss the falling coconut, but it doesn't. Although the arrow travels very fast, it follows a slightly curved parabolic trajectory, not a straight line. Had the coconut stayed on the tree, the arrow would have curved under its target as gravity causes it to fall a distance $\frac{1}{2}gt^2$ below the straight line. But $\frac{1}{2}gt^2$ is also the distance the coconut falls while the arrow is in flight. Thus, as FIGURE 4.20b shows, the arrow and the coconut fall the same distance and meet at the same point!

Solving Projectile Motion Problems

This information about projectiles is the basis of a problem-solving strategy.

PROBLEM-SOLVING STRATEGY 4.1 Projectile motion problems

MODEL Make simplifying assumptions, such as treating the object as a particle. Is it reasonable to ignore air resistance?

VISUALIZE Use a pictorial representation. Establish a coordinate system with the x-axis horizontal and the y-axis vertical. Show important points in the motion on a sketch. Define symbols and identify what the problem is trying to find.

SOLVE The acceleration is known: $a_x = 0$ and $a_y = -g$. Thus the problem is one of two-dimensional kinematics. The kinematic equations are

$$x_f = x_i + v_{ix}\,\Delta t \qquad y_f = y_i + v_{iy}\,\Delta t - \tfrac{1}{2}g(\Delta t)^2$$

$$v_{fx} = v_{ix} = \text{constant} \qquad v_{fy} = v_{iy} - g\,\Delta t$$

Δt is the same for the horizontal and vertical components of the motion. Find Δt from one component, then use that value for the other component.

ASSESS Check that your result has the correct units, is reasonable, and answers the question.

EXAMPLE 4.5 The distance of a fly ball

A baseball is hit at angle θ and is caught at the height from which it was hit. If the ball is hit at a 30.0° angle, with what speed must it leave the bat to travel 100 m?

MODEL Represent the ball as a particle. A baseball is fairly heavy and dense, so ignore air resistance.

VISUALIZE FIGURE 4.21 shows the pictorial representation. The height above the ground at which the ball was hit and caught is not relevant because it stays the same, so we have placed the origin at the point where the ball is hit. The ball travels distance x_1.

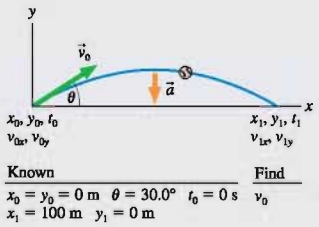
SOLVE The initial x- and y-components of the ball's velocity are

$$v_{0x} = v_0\cos\theta$$

$$v_{0y} = v_0\sin\theta$$

where v_0 is the initial speed that we need to find.

FIGURE 4.21 Pictorial representation for the baseball of Example 4.5.



Known	Find
$x_0 = y_0 = 0$ m $\quad \theta = 30.0°$ $\quad t_0 = 0$ s $x_1 = 100$ m $\quad y_1 = 0$ m	v_0

The kinematic equations of projectile motion are

$$\begin{aligned} x_1 &= x_0 + v_{0x}(t_1 - t_0) \\ &= (v_0\cos\theta)t_1 \\ y_1 = 0 &= y_0 + v_{0y}(t_1 - t_0) - \tfrac{1}{2}g(t_1 - t_0)^2 \\ &= (v_0\sin\theta)t_1 - \tfrac{1}{2}gt_1^2 \end{aligned}$$

We can use the vertical equation to find the time of flight:

$$0 = (v_0\sin\theta)t_1 - \tfrac{1}{2}gt_1^2 = (v_0\sin\theta - \tfrac{1}{2}gt_1)t_1$$

and thus

$$t_1 = 0 \quad \text{or} \quad \frac{2v_0\sin\theta}{g}$$

Both values are legitimate solutions. The first corresponds to the instant when $y = 0$ at the beginning of the trajectory and the second to when $y = 0$ at the end. Clearly, though, we want the second solution. Substituting this expression for t_1 into the equation for x_1 gives

$$x_1 = (v_0\cos\theta)\frac{2v_0\sin\theta}{g} = \frac{2v_0^2\sin\theta\cos\theta}{g}$$

We can simplify this result by using the trigonometric identity $2\sin\theta\cos\theta = \sin(2\theta)$. The distance traveled by the ball when hit at angle θ is

$$x_1 = \frac{v_0^2\sin(2\theta)}{g}$$

Setting $x_1 = 100$ m and solving for the speed v_0 give

$$v_0 = \sqrt{\frac{gx_1}{\sin(2\theta)}} = \sqrt{\frac{(9.80\text{ m/s}^2)(100\text{ m})}{\sin 60.0^\circ}} = 33.6\text{ m/s}$$

ASSESS A speed of 33.6 m/s ≈ 70 mph seems quite reasonable for a batted ball.

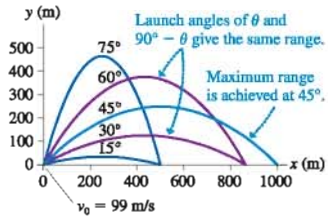
As Example 4.5 found, a projectile that lands at the same elevation from which it was fired travels distance

$$\text{distance} = \frac{v_0^2\sin(2\theta)}{g} \tag{4.19}$$

The maximum distance occurs for $\theta = 45°$, where $\sin(2\theta) = 1$. But there's more that we can learn from this equation. Because $\sin(180° - x) = \sin x$, it follows that $\sin(2(90° - \theta)) = \sin(2\theta)$. Consequently, a projectile launched either at angle θ *or* at angle $(90° - \theta)$ will travel the same distance. FIGURE 4.22 shows the trajectories of projectiles launched with the same initial speed in 15° increments of angle.

NOTE ▶ Equation 4.19 is *not* a general result. It applies *only* in situations where the projectile lands at the same elevation from which it was fired. ◀

FIGURE 4.22 Trajectories of a projectile launched at different angles with a speed of 99 m/s.



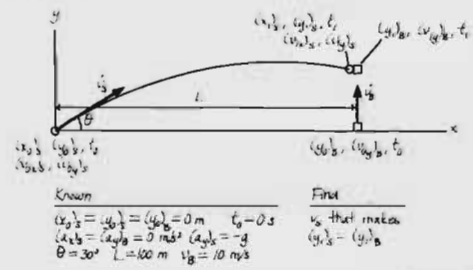
EXAMPLE 4.6 Hit the box

Students at an engineering contest use a compressed-air cannon to shoot a softball at a box being hoisted straight up at 10 m/s by a crane. The cannon, tilted upward at a 30° angle, is 100 m from the box and fires by remote control the instant the box leaves the ground. Students can control the launch speed of the softball by setting the air pressure. What launch speed should the students use to hit the box?

MODEL Represent both the softball and the box as particles. Ignore air resistance.

VISUALIZE FIGURE 4.23 shows the pictorial representation. There's a *large* amount of information to keep track of in a problem like this, making the pictorial representation essential for success.

FIGURE 4.23 The pictorial representation of the softball and the box.



Continued

SOLVE The softball collides with the box if they have the same vertical position at exactly the same instant they have equal horizontal positions. Because the box moves vertically, the two will have the same horizontal position when the ball has traveled distance L to the right. This happens at time

$$t_1 = \frac{L}{(v_{0x})_S} = \frac{L}{v_S \cos\theta}$$

Equating the vertical positions at this instant, $(y_1)_S = (y_1)_B$, gives

$$(v_{0y})_S t_1 - \tfrac{1}{2} g t_1^2 = (v_{0y})_B t_1$$

One factor of t_1 cancels from each term. We can then use $(v_{0y})_S = v_S \sin\theta$, $(v_{0y})_B = v_B$, and the above expression for t_1 to write

$$(v_{0y})_S - \tfrac{1}{2} g t_1 - (v_{0y})_B = v_S \sin\theta - \tfrac{1}{2} g \frac{L}{v_S \cos\theta} - v_B = 0$$

Multiplying through by $2v_S \cos\theta$ gives a quadratic equation for v_S:

$$2\sin\theta\cos\theta v_S^2 - 2v_B \cos\theta v_S - gL = 0$$

It's important to learn to work problems with symbols rather than numbers. It you had plugged in numbers immediately, you wouldn't have recognized that t_1 cancels in the vertical equation and you would have done a lot of unnecessary calculations. Even so, writing the quadratic-equation solution symbolically is going to produce an ungainly mess. This is a reasonable time to insert numeric values—as long as all values are in SI units! With known values for θ, L, and v_B, the quadratic equation becomes

$$0.866 v_S^2 - 17.3 v_S - 980 = 0$$

The solutions to this equation are

$$v_S = \frac{17.3 \pm \sqrt{(17.3)^2 + 4(0.866)(980)}}{2(0.866)} = 45 \text{ m/s and } -25 \text{ m/s}$$

Speed must be positive, so the negative answer is not physically meaningful. The ball needs to be launched at a speed of 45 m/s.

ASSESS A speed of 45 m/s ≈ 90 mph. That seems quite reasonable for a ball shot at a target 100 m away.

STOP TO THINK 4.3 A 50 g marble rolls off a table and lands 2 m from the base of the table. A 100 g marble rolls off the same table with the same speed. It lands at distance

a. Less than 1 m. b. 1 m. c. Between 1 m and 2 m.
d. 2 m. e. Between 2 m and 4 m. f. 4 m.

4.4 Relative Motion

You've now dealt many times with problems that say something like "A car travels at 30 m/s" or "A plane travels at 300 m/s." But just what do these statements really mean?

In FIGURE 4.24, Amy, Bill, and Carlos are watching a runner. According to Amy, the runner's velocity is $v_x = 5$ m/s. But to Bill, who's riding alongside, the runner is lifting his legs up and down but going neither forward nor backward relative to Bill. As far as Bill is concerned, the runner's velocity is $v_x = 0$ m/s. Carlos sees the runner receding in his rearview mirror, in the *negative* x-direction, getting 10 m farther away from him every second. According to Carlos, the runner's velocity is $v_x = -10$ m/s. Which is the runner's *true* velocity?

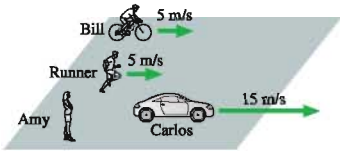


FIGURE 4.24 Amy, Bill, and Carlos each measure the velocity of the runner. The velocities shown are in Amy's reference frame.

Velocity is not a concept that can be true or false. The runner's velocity *relative to Amy* is 5 m/s. That is, his velocity is 5 m/s in a coordinate system attached to Amy and in which Amy is at rest. The runner's velocity relative to Bill is 0 m/s, and the velocity relative to Carlos is −10 m/s. These are all valid descriptions of the runner's motion.

Relative Position

Suppose that Amy and Bill each have a coordinate system attached to their bodies. As Bill bicycles past Amy, he carries his coordinate system with him. Each is at rest in his or her coordinate system. Further, let's imagine that Amy and Bill each have helpers with meter sticks and stopwatches in their coordinate systems. Amy and Bill, with their helpers, are able to measure the position at which a physical event takes

place and the time at which it occurs. A coordinate system in which an experimenter (possibly with the assistance of helpers) makes position and time measurements of physical events is called a **reference frame.** Amy and Bill each have their own reference frame.

Let's define two reference frames, shown in FIGURE 4.25, that we'll call frame S and frame S′. (The symbol ′ is called a *prime,* and S′ is pronounced "S prime.") The coordinate axes in frame S are x and y, while those in S′ are x' and y'. Frame S′ is moving with velocity $\vec{V}$ relative to frame S. That is, if an experimenter at rest in S measures the motion of the origin of S′ as it goes past, she finds that the origin of S′ has velocity $\vec{V}$. Of course, an experimenter at rest in S′ would say that frame S has velocity $-\vec{V}$. We'll use an uppercase V for the velocity of reference frames, reserving lowercase v for the velocity of objects that move in the reference frames.

NOTE ▶ There's no implication that either reference frame is "at rest." All we know is that the two frames are moving *relative* to each other with velocity $\vec{V}$. ◀

We will stipulate four conditions for reference frames:

1. The frames are oriented the same, with the x- and x'-axes parallel to each other.
2. The origins of frame S and frame S′ coincide at $t = 0$.
3. All motion is in the xy-plane, so we don't need to consider the z-axis.
4. The relative velocity $\vec{V}$ is *constant.*

The first three are a matter of how we define the coordinate systems. Item 4, by contrast, is a choice with consequences. It says that we will consider only reference frames that move with constant speed in a straight line. These are called **inertial reference frames,** and we'll find in Chapter 5 that these are the reference frames in which the laws of motion are valid.

Suppose a light bulb flashes at time t. Experimenters in both reference frames see the flash and measure its position. Observers in S place the flash at position $\vec{r}$, as measured with respect to the coordinate system of frame S. Similarly, experimenters in S′ determine that the flash occurred at position $\vec{r}'$, relative to the origin of S′. (We'll use primes to indicate positions and velocities measured in frame S′.)

What is the relationship between the position vectors $\vec{r}$ and $\vec{r}'$? It's not hard to see, from FIGURE 4.26, that

$$\vec{r} = \vec{r}' + \vec{R}$$

where $\vec{R}$ is the position vector of the origin of frame S′ as measured in frame S.

Frame S′ is traveling with velocity $\vec{V}$ relative to frame S, and their origins coincided at $t = 0$. At time t, when the light flashes, the origin of S′ has moved to position $\vec{R} = t\vec{V}$. (We've written $t\vec{V}$ rather than $\vec{V}t$ because it is customary to write the scalar first.) Thus

$$\vec{r} = \vec{r}' + t\vec{V} \quad \text{or} \quad \vec{r}' = \vec{r} - t\vec{V} \tag{4.20}$$

Equation 4.20 is called the **Galilean transformation of position.** It will be easiest for most purposes to write this in terms of components:

$$\begin{aligned} x &= x' + V_x t \\ y &= y' + V_y t \end{aligned} \quad \text{or} \quad \begin{aligned} x' &= x - V_x t \\ y' &= y - V_y t \end{aligned} \tag{4.21}$$

If we know *where* and *when* an event occurred in one reference frame, we can *transform* that position into any other reference frame that moves relative to the first with constant velocity $\vec{V}$.

FIGURE 4.25 Reference frame S′ moves with velocity $\vec{V}$ relative to reference frame S.

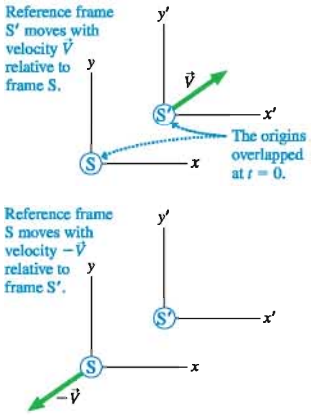
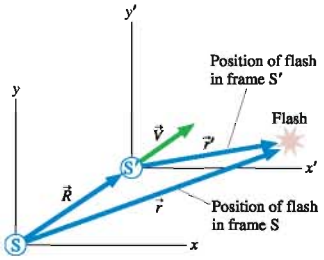


FIGURE 4.26 Measurements made in frames S and S′.



EXAMPLE 4.7 Watching a ball toss

Miguel throws a ball upward at a 63.0° angle with a speed of 22.0 m/s. Nancy rides past Miguel on her bicycle at 10.0 m/s at the instant he releases the ball.

a. Find and graph the ball's trajectory as seen by Miguel.
b. Find and graph the ball's trajectory as seen by Nancy.

SOLVE a. For Miguel, the ball is a projectile that follows a parabolic trajectory. This problem is almost exactly the same as Example 4.5. The components of the initial velocity are

$$v_{0x} = v_0\cos\theta = (22.0\text{ m/s})\cos 63.0° = 10.0\text{ m/s}$$

$$v_{0y} = v_0\sin\theta = (22.0\text{ m/s})\sin 63.0° = 19.6\text{ m/s}$$

The x- and y-equations of motion for the ball's position at time t are

$$x = x_0 + v_{0x}(t - t_0) = 10.0t\text{ m}$$

$$y = y_0 + v_{0y}(t - t_0) - \tfrac{1}{2}g(t - t_0)^2 = (19.6t - 4.90t^2)\text{ m}$$

where t is in s. It's not hard to show that the ball reaches height $y_{max} = 19.6$ m at $t = 2.0$ s and hits the ground at $x_{max} = 40$ m at $t = 4.0$ s. The trajectory is shown in FIGURE 4.27a.

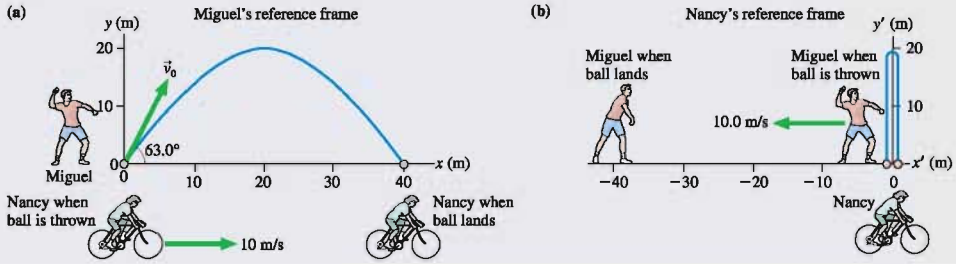
b. We can determine the trajectory Nancy sees by using Equations 4.21 to transform the ball's position from Miguel's reference frame into Nancy's reference frame. Let Miguel be in frame S and Nancy in frame S′. Nancy moves with velocity $\vec{V} = 10.0\hat{\imath}$ m/s relative to S. In terms of components, $V_x = 10.0$ m/s and $V_y = 0$ m/s. When Miguel, at time t, measures the ball at position (x, y) in frame S, Nancy finds the ball at

$$x' = x - V_x t = 10.0t - 10.0t = 0$$

$$y' = y - V_y t = y$$

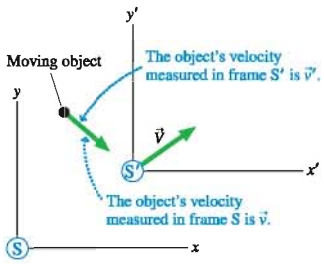
Because Nancy's horizontal motion is the same as the ball's ($V_x = v_x = 10.0$ m/s in Miguel's frame), she doesn't see the ball moving either right or left. Nancy's experience is like that of Bill riding beside the runner in Figure 4.24. The ball moves *vertically* up and down in frame S′. Further, the vertical position y' in S′ is the same as the vertical position y in S. According to Nancy, the ball goes straight up, reaches a height of 19.6 m, and falls straight back down. It hits the ground right beside her bicycle at $t = 4.0$ s. This is seen in FIGURE 4.27b.

FIGURE 4.27 A ball's trajectory as seen by Miguel and Nancy.



In Chapter 2 we studied *free fall,* vertical motion straight up and down. In Section 4.3 we studied the parabolic trajectories of *projectile motion.* Now, from Example 4.7 we see that **free-fall motion and projectile motion are really the same motion, simply seen from two different reference frames.** The motion is vertical in the *one* reference frame whose horizontal motion is the same as the ball's. The trajectory is a parabola in any other reference frame.

FIGURE 4.28 A velocity of a moving object is measured by experimenters in two different reference frames.



Relative Velocity

Let's think a bit more about Example 4.7. According to an observer in Miguel's reference frame, Miguel throws the ball with velocity $\vec{v}_0 = (10.0\hat{\imath} + 19.6\hat{\jmath})$ m/s. The ball's initial speed is $v_0 = 22.0$ m/s. But in frame S′, where Nancy sees the ball go straight up and down, Miguel throws the ball with velocity $\vec{v}_0' = 19.6\hat{\jmath}$ m/s. An object's velocity measured in frame S is *not* the same as its velocity measured in frame S′.

FIGURE 4.28 shows a *moving object* that is observed from reference frames S and S′. Experimenters in frame S locate the object at position $\vec{r}$ and measure its velocity to be $\vec{v}$. Simultaneously, experimenters in S′ measure position $\vec{r}'$ and velocity $\vec{v}'$. The position vectors, which are related by $\vec{r} = \vec{r}' + \vec{R}$, change as the object moves. In addi-

tion, $\vec{R}$ changes as the reference frames move relative to each other. The *rate* of change is

$$\frac{d\vec{r}}{dt} = \frac{d\vec{r}'}{dt} + \frac{d\vec{R}}{dt} \qquad (4.22)$$

The derivative $d\vec{r}/dt$, by definition, is the object's velocity $\vec{v}$ measured in frame S. Similarly, $d\vec{r}'/dt$ is the object's velocity $\vec{v}'$ measured in frame S′, and $d\vec{R}/dt$ is the velocity $\vec{V}$ of frame S′ relative to frame S. Consequently, Equation 4.22 tells us that

$$\vec{v} = \vec{v}' + \vec{V} \quad \text{or} \quad \vec{v}' = \vec{v} - \vec{V} \qquad (4.23)$$

Equation 4.23 is the **Galilean transformation of velocity.** If we know an object's velocity measured in one reference frame, we can transform it into the velocity that would be measured by an experimenter in a different reference frame. As FIGURE 4.29 shows, doing so is an exercise in vector addition.

We will often find it convenient, as we did with position, to write Equation 4.23 in terms of components:

$$\begin{aligned} v_x &= v_x' + V_x & \text{or} \quad v_x' &= v_x - V_x \\ v_y &= v_y' + V_y & v_y' &= v_y - V_y \end{aligned} \qquad (4.24)$$

This relationship between velocities measured by experimenters in different frames of reference was recognized by Galileo in his pioneering studies of motion, hence its name.

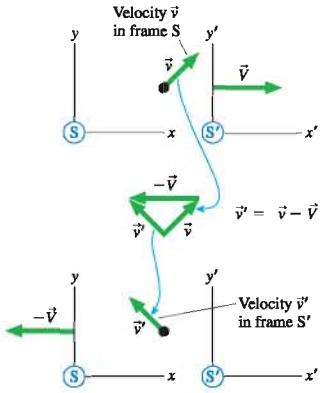
Let's apply Equation 4.24 to Miguel and Nancy. We've already noted that the ball's initial velocity in Miguel's frame, frame S, is $\vec{v}_0 = (10.0\hat{\imath} + 19.6\hat{\jmath})$ m/s. Nancy was moving relative to Miguel at velocity $\vec{V} = 10.0\hat{\imath}$ m/s. We can use Equation 4.24 to transform the velocity to Nancy's frame, frame S′, finding

$$v_x' = v_x - V_x = 10.0 \text{ m/s} - 10.0 \text{ m/s} = 0 \text{ m/s}$$
$$v_y' = v_y - V_y = 19.6 \text{ m/s} - 0 \text{ m/s} = 19.6 \text{ m/s}$$

Thus $\vec{v}_0' = 19.6\hat{\jmath}$ m/s. This agrees with our conclusion from Example 4.7.

It's important to understand the distinction between the three velocities $\vec{v}$, $\vec{v}'$, and $\vec{V}$. $\vec{v}$ and $\vec{v}'$ are the velocities of an *object* that is observed from two different reference frames. Experimenters in S use their meter sticks and stopwatches to measure the object's velocity $\vec{v}$ in their reference frame. At the same time, experimenters in S′ measure the velocity of the same object to be $\vec{v}'$. $\vec{V}$ is the relative velocity between two *reference frames;* the velocity of S′ as measured by an experimenter in S. $\vec{V}$ has nothing to do with the object. It may happen that either $\vec{v}$ or $\vec{v}'$ is zero, meaning that the object is at rest in one reference frame, but we still must distinguish between the object and the reference frame.

FIGURE 4.29 Velocities $\vec{v}$ and $\vec{v}'$, as measured in frames S and S′, are related by vector addition.



EXAMPLE 4.8 A speeding bullet

The police are chasing a bank robber. While driving at 50 m/s, they fire a bullet to shoot out a tire of his car. The police gun shoots bullets at 300 m/s. What is the bullet's speed as measured by a TV camera crew parked beside the road?

MODEL Assume that all motion is along the *x*-axis. Let the earth be frame S and a frame attached to the police car be S′. Frame S′ moves relative to frame S with $V_x = 50$ m/s.

SOLVE The bullet is the moving object that will be observed from both frames. The gun is in frame S′, so the bullet travels in this frame with $v_x' = 300$ m/s. We can use Equation 4.24 to transform the bullet's velocity into the earth reference frame:

$$v_x = v_x' + V_x = 300 \text{ m/s} + 50 \text{ m/s} = 350 \text{ m/s}$$

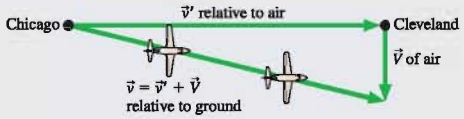
The Galilean velocity transformations are pretty much common sense for one-dimensional motion. Their real usefulness appears when an object travels in a *medium* that moves with respect to the earth. For example, a boat moves relative to the water. What is the boat's net motion if the water is a flowing river? Airplanes fly relative to the air, but the air at high altitudes often flows at high speed. Navigation of boats and planes requires knowing both the motion of the vessel in the medium and the motion of the medium relative to the earth.

EXAMPLE 4.9 Flying to Cleveland I

Cleveland is 300 miles east of Chicago. A plane leaves Chicago flying due east at 500 mph. The pilot forgot to check the weather and doesn't know that the wind is blowing to the south at 50 mph. What is the plane's ground speed? Where is the plane 0.60 hour later, when the pilot expects to land in Cleveland?

MODEL Let the earth be reference frame S. Chicago and Cleveland are at rest in the earth's frame. Let the air be frame S′. If the x-axis points east and the y-axis north, then the air is moving with respect to the earth at $\vec{V} = -50\hat{j}$ mph. The plane flies in the air, so its velocity in frame S′ is $\vec{v}' = 500\hat{i}$ mph.

FIGURE 4.30 The wind causes a plane flying due east in the air to move to the southeast relative to the earth.



SOLVE The velocity transformation equation $\vec{v} = \vec{v}' + \vec{V}$ is a vector-addition equation. **FIGURE 4.30** shows graphically what happens. Although the nose of the plane points east, the wind carries the plane in a direction somewhat south of east. The plane's velocity relative to the ground is

$$\vec{v} = \vec{v}' + \vec{V} = (500\hat{i} - 50\hat{j}) \text{ mph}$$

The plane's ground speed, its speed in frame S, is

$$v = \sqrt{v_x^2 + v_y^2} = 502 \text{ mph}$$

After flying for 0.60 hour at this velocity, the plane's location (relative to Chicago) is

$$x = v_x t = (500 \text{ mph})(0.60 \text{ hr}) = 300 \text{ mi}$$

$$y = v_y t = (-50 \text{ mph})(0.60 \text{ hr}) = -30 \text{ mi}$$

The plane is 30 mi due south of Cleveland! Although the pilot thought he was flying to the east, his actual heading has been $\tan^{-1}(V/v) = \tan^{-1}(0.10) = 5.72°$ south of east.

EXAMPLE 4.10 Flying to Cleveland II

A wiser pilot flying from Chicago to Cleveland on the same day plots a course that will take her directly to Cleveland. In which direction does she fly the plane? How long does it take to reach Cleveland?

MODEL Let the earth be reference frame S. Let the air be frame S′. If the x-axis points east and the y-axis north, then the air is moving with respect to the earth at $\vec{V} = -50\hat{j}$ mph.

SOLVE The objective of navigation is to move between two points on the earth's surface, in frame S. The wiser pilot, who knows that the wind will affect her plane, draws the vector picture of **FIGURE 4.31**. The plane's velocity in frame S is

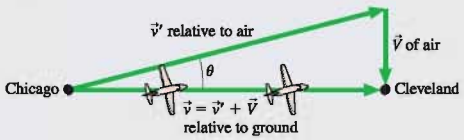
$$v_x = v_x' + V_x = (500 \text{ mph}) \cos\theta$$

$$v_y = v_y' + V_y = (500 \text{ mph}) \sin\theta - 50 \text{ mph}$$

In plotting her course, the pilot knows that she wants $v_y = 0$ in order to fly due east to Cleveland in the earth's frame. To achieve this, she'll actually have to point the nose of the plane somewhat north of east. The proper heading, found from $v_y = 0$, is

$$\theta = \sin^{-1}\left(\frac{50 \text{ mph}}{500 \text{ mph}}\right) = 5.74°$$

FIGURE 4.31 To travel due east in a south wind, a pilot has to point the plane somewhat to the northeast.



The plane's velocity in frame S is then $\vec{v} = (500 \text{ mph}) \times \cos 5.74\hat{i} = 497\hat{i}$ mph. You can see from Figure 4.31 that the plane's speed v in the earth's frame is slower than its speed v' in the air's reference frame. The time needed to fly to Cleveland at this speed is

$$t = \frac{300 \text{ mi}}{497 \text{ mph}} = 0.604 \text{ hr}$$

It takes 0.004 hr = 14 s longer to reach Cleveland than it would on a day without wind.

ASSESS A boat crossing a river or an ocean current faces the same difficulties. These are exactly the kinds of calculations performed by pilots of boats and planes as part of navigation.

STOP TO THINK 4.4 A plane traveling horizontally to the right at 100 m/s flies past a helicopter that is going straight up at 20 m/s. From the helicopter's perspective, the plane's direction and speed are

a. Right and up, less than 100 m/s.
b. Right and up, 100 m/s.
c. Right and up, more than100 m/s.
d. Right and down, less than 100 m/s.
e. Right and down, 100 m/s.
f. Right and down, more than 100 m/s.

4.5 Uniform Circular Motion

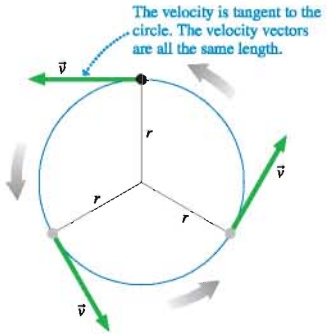
FIGURE 4.32 shows a particle moving around a circle of radius r. The particle might be a satellite in an orbit, a ball on the end of a string, or even just a dot painted on the side of a rotating wheel. Circular motion is another example of motion in a plane, but it is quite different from projectile motion.

To begin the study of circular motion, consider a particle that moves at *constant speed* around a circle of radius r. This is called **uniform circular motion.** Regardless of what the particle represents, its velocity vector $\vec{v}$ is always tangent to the circle. The particle's speed v is constant, so vector $\vec{v}$ is always the same length.

The time interval it takes the particle to go around the circle once, completing one revolution (abbreviated rev), is called the **period** of the motion. Period is represented by the symbol T. It's easy to relate the particle's period T to its speed v. For a particle moving with constant speed, speed is simply distance/time. In one period, the particle moves once around a circle of radius r and travels the circumference $2\pi r$. Thus

$$v = \frac{1 \text{ circumference}}{1 \text{ period}} = \frac{2\pi r}{T} \qquad (4.25)$$

FIGURE 4.32 A particle in uniform circular motion.



EXAMPLE 4.11 A rotating crankshaft

A 4.0-cm-diameter crankshaft turns at 2400 rpm (revolutions per minute). What is the speed of a point on the surface of the crankshaft?

SOLVE We need to determine the time it takes the crankshaft to make 1 rev. First, we convert 2400 rpm to revolutions per second:

$$\frac{2400 \text{ rev}}{1 \text{ min}} \times \frac{1 \text{ min}}{60 \text{ s}} = 40 \text{ rev/s}$$

If the crankshaft turns 40 times in 1 s, the time for 1 rev is

$$T = \frac{1}{40} \text{ s} = 0.025 \text{ s}$$

Thus the speed of a point on the surface, where $r = 2.0 \text{ cm} = 0.020 \text{ m}$, is

$$v = \frac{2\pi r}{T} = \frac{2\pi(0.020 \text{ m})}{0.025 \text{ s}} = 5.0 \text{ m/s}$$

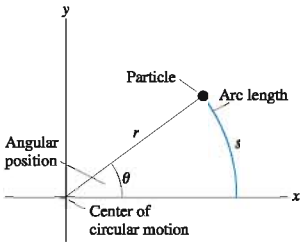
Circular motion is one of the most important types of motion.

Angular Position

Rather than using xy-coordinates, it will be more convenient to describe the position of a particle in circular motion by its distance r from the center of the circle (labeled O) and its angle θ from the positive x-axis. This is shown in FIGURE 4.33 on the next page. The angle θ is the **angular position** of the particle.

We can distinguish a position above the x-axis from a position that is an equal angle below the x-axis by *defining* θ to be positive when measured *counterclockwise* (ccw) from the positive x-axis. An angle measured clockwise (cw) from the positive x-axis has a negative value. "Clockwise" and "counterclockwise" in circular motion are analogous, respectively, to "left of the origin" and "right of the origin" in linear motion, which we associated with negative and positive values of x. A particle 30° below the positive x-axis is equally well described by either $\theta = -30°$ or $\theta = +330°$. We could also describe this particle by $\theta = \frac{11}{12}$ rev, where *revolutions* are another way to measure the angle.

FIGURE 4.33 A particle's position is described by distance r and angle θ.



Although degrees and revolutions are widely used measures of angle, mathematicians and scientists usually find it more useful to measure the angle θ in Figure 4.33 by using the **arc length** s that the particle travels along the edge of a circle of radius r. We define the angular unit of **radians** such that

$$\theta(\text{radians}) \equiv \frac{s}{r} \tag{4.26}$$

The radian, which is abbreviated rad, is the SI unit of an angle. An angle of 1 rad has an arc length s exactly equal to the radius r.

The arc length completely around a circle is the circle's circumference $2\pi r$. Thus the angle of a full circle is

$$\theta_{\text{full circle}} = \frac{2\pi r}{r} = 2\pi \text{ rad}$$

This relationship is the basis for the well-known conversion factors

$$1 \text{ rev} = 360° = 2\pi \text{ rad}$$

As a simple example of converting between radians and degrees, let's convert an angle of 1 rad to degrees:

$$1 \text{ rad} = 1 \text{ rad} \times \frac{360°}{2\pi \text{ rad}} = 57.3°$$

Thus a rough approximation is 1 rad $\approx 60°$. We will often specify angles in degrees, but keep in mind that the SI unit is the radian.

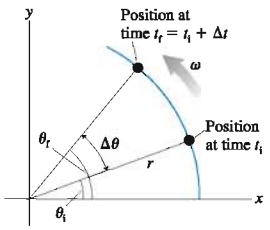
An important consequence of Equation 4.26 is that the arc length spanning angle θ is

$$s = r\theta \qquad (\text{with } \theta \text{ in rad}) \tag{4.27}$$

This is a result that we will use often, but it is valid *only* if θ is measured in radians and not in degrees. This very simple relationship between angle and arc length is one of the primary motivations for using radians.

NOTE ▶ Units of angle are often troublesome. Unlike the kilogram or the second, for which we have standards, the radian is a *defined* unit. Further, its definition as a ratio of two lengths makes it a *pure number* without dimensions. Thus the unit of angle, be it radians or degrees or revolutions, is really just a *name* to remind us that we're dealing with an angle. Consequently, the radian unit sometimes appears or disappears without warning. This seems rather mysterious until you get used to it. This textbook will call your attention to such behavior the first few times it occurs. With a little practice, you'll soon learn when the rad unit is needed and when it's not. ◀

FIGURE 4.34 A particle moves with angular velocity ω.



Angular Velocity

FIGURE 4.34 shows a particle moving in a circle from an initial angular position θ_i at time t_i to a final angular position θ_f at a later time t_f. The change $\Delta\theta = \theta_f - \theta_i$ is called the **angular displacement.** We can measure the particle's circular motion in terms of the

rate of change of θ, just as we measured the particle's linear motion in terms of the rate of change of its position s.

In analogy with linear motion, let's define the *average angular velocity* to be

$$\text{average angular velocity} \equiv \frac{\Delta\theta}{\Delta t} \tag{4.28}$$

As the time interval Δt becomes very small, $\Delta t \to 0$, we arrive at the definition of the instantaneous **angular velocity**

$$\omega \equiv \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt} \qquad \text{(angular velocity)} \tag{4.29}$$

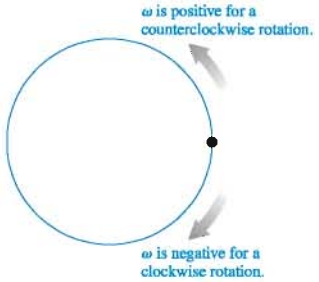
The symbol ω is a lowercase Greek omega, *not* an ordinary w. The SI unit of angular velocity is rad/s, but °/s, rev/s, and rev/min are also common units. Revolutions per minute is abbreviated rpm.

Angular velocity is the *rate* at which a particle's angular position is changing as it moves around a circle. A particle that starts from $\theta = 0$ rad with an angular velocity of 0.5 rad/s will be at angle $\theta = 0.5$ rad after 1 s, at $\theta = 1.0$ rad after 2 s, at $\theta = 1.5$ rad after 3 s, and so on. Its angular position is increasing at the *rate* of 0.5 radian per second. In analogy with uniform linear motion, which you studied in Chapter 2, uniform circular motion is motion in which the angle increases at a *constant* rate: **A particle moves with uniform circular motion if and only if its angular velocity ω is constant and unchanging.**

Angular velocity, like the velocity v_s of one-dimensional motion, can be positive or negative. The signs shown in FIGURE 4.35 are based on the fact that θ was defined to be positive for a counterclockwise rotation. Because the definition $\omega = d\theta/dt$ for circular motion parallels the definition $v_s = ds/dt$ for linear motion, the graphical relationships we found between v_s and s in Chapter 2 apply equally well to ω and θ:

- ω = slope of the θ-versus-t graph at time t
- $\theta_f = \theta_i$ + area under the ω-versus-t graph between t_i and t_f

FIGURE 4.35 Positive and negative angular velocities.



EXAMPLE 4.12 A graphical representation of circular motion

FIGURE 4.36 shows the angular position of a particle moving around a circle of radius r. Describe the particle's motion and draw an ω-versus-t graph.

FIGURE 4.36 Angular position graph for the particle of Example 4.12.

SOLVE Although circular motion seems to "start over" every revolution (every 2π rad), the angular position θ continues to increase. $\theta = 6\pi$ rad corresponds to three revolutions. This particle makes 3 ccw rev (because θ is getting more positive) in 3 s, immediately reverses direction and makes 1 cw rev in 2 s, then stops at $t = 5$ s and holds the position $\theta = 4\pi$ rad. The angular velocity is found by measuring the slope of the graph:

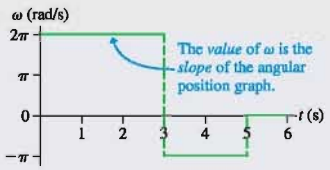
$t = 0$–3 s slope = $\Delta\theta/\Delta t = 6\pi$ rad/3 s = 2π rad/s

$t = 3$–5 s slope = $\Delta\theta/\Delta t = -2\pi$ rad/2 s = $-\pi$ rad/s

$t > 5$ s slope = $\Delta\theta/\Delta t = 0$ rad/s

These results are shown as an ω-versus-t graph in FIGURE 4.37. For the first 3 s, the motion is uniform circular motion with $\omega = 2\pi$ rad/s. The particle then changes to a different uniform circular motion with $\omega = -\pi$ rad/s for 2 s, then stops.

FIGURE 4.37 ω-versus-t graph for the particle of Example 4.12.



NOTE ▶ In physics, we nearly always want to give results as numerical values. Example 4.11 had a π in the equation, but we used its numerical value to compute $v = 5.0$ m/s. However, angles in radians are an exception to this rule. It's okay to leave a π in the value of θ or ω, and we have done so in Example 4.12. ◀

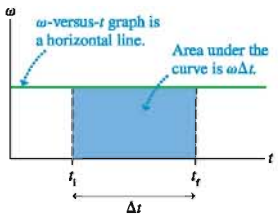


FIGURE 4.38 The ω-versus-t graph for uniform circular motion is a horizontal line.

The angular velocity is constant during uniform circular motion, so the ω-versus-t graph is a horizontal line. It's easy to see from FIGURE 4.38 that the area under the curve from t_i to t_f is simply $\omega\Delta t$. Consequently,

$$\theta_f = \theta_i + \omega\Delta t \qquad \text{(uniform circular motion)} \qquad (4.30)$$

Equation 4.30 is equivalent, with different variables, to the result $s_f = s_i + v_s\Delta t$ for uniform linear motion. You will see many more instances where circular motion is analogous to linear motion with angular variables replacing linear variables. Thus much of what you learned about linear kinematics carries over to circular motion.

Not surprisingly, the angular velocity ω is closely related to the *period* T of the motion. As a particle goes around a circle one time, its angular displacement is $\Delta\theta = 2\pi$ rad during the interval $\Delta t = T$. Thus, using the definition of angular velocity, we find

$$|\omega| = \frac{2\pi \text{ rad}}{T} \qquad \text{or} \qquad T = \frac{2\pi \text{ rad}}{|\omega|} \qquad (4.31)$$

The period alone gives only the absolute value of $|\omega|$. You need to know the direction of motion to determine the sign of ω.

EXAMPLE 4.13 At the roulette wheel

A small steel roulette ball rolls ccw around the inside of a 30-cm-diameter roulette wheel. The ball completes 2.0 rev in 1.20 s.

a. What is the ball's angular velocity?
b. What is the ball's position at $t = 2.0$ s? Assume $\theta_i = 0$.

MODEL Model the ball as a particle in uniform circular motion.

SOLVE a. The period of the ball's motion, the time for 1 rev, is $T = 0.60$ s. Angular velocity is positive for ccw motion, so

$$\omega = \frac{2\pi \text{ rad}}{T} = \frac{2\pi \text{ rad}}{0.60 \text{ s}} = 10.47 \text{ rad/s}$$

b. The ball starts at $\theta_i = 0$ rad. After $\Delta t = 2.0$ s, its position is given by Equation 4.30:

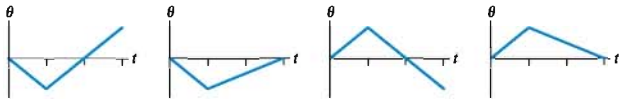
$$\theta_f = 0 \text{ rad} + (10.47 \text{ rad/s})(2.0 \text{ s}) = 20.94 \text{ rad}$$

where we've kept an extra significant figure to avoid round-off error. Although this is a mathematically acceptable answer, an observer would say that the ball is always located somewhere between 0° and 360°. Thus it is common practice to subtract an integer number of 2π rad, representing the completed revolutions. Because $20.94/2\pi = 3.333$, we can write

$$\theta_f = 20.94 \text{ rad} = 3.333 \times 2\pi \text{ rad}$$
$$= 3 \times 2\pi \text{ rad} + 0.333 \times 2\pi \text{ rad}$$
$$= 3 \times 2\pi \text{ rad} + 2.09 \text{ rad}$$

In other words, at $t = 2.0$ s the ball has completed 3 rev and is 2.09 rad = 120° into its fourth revolution. An observer would say that the ball's position is $\theta_f = 120°$.

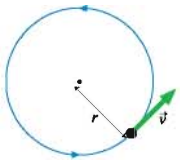
STOP TO THINK 4.5 A particle moves cw around a circle at constant speed for 2.0 s. It then reverses direction and moves ccw at half the original speed until it has traveled through the same angle. Which is the particle's angle-versus-time graph?



4.6 Velocity and Acceleration in Uniform Circular Motion

For a particle in circular motion, such as the one in FIGURE 4.39, the velocity vector $\vec{v}$ is always tangent to the circle. In other words, the velocity vector has only a *tangential component,* which we will designate v_t.

FIGURE 4.39 The velocity vector $\vec{v}$ has only a tangential component v_t.



The tangential velocity component v_t is the rate ds/dt at which the particle moves *around* the circle, where s is the arc length measured from the positive x-axis. From Equation 4.27, the arc length is $s = r\theta$. Taking the derivative, we find

$$v_t = \frac{ds}{dt} = r\frac{d\theta}{dt}$$

But $d\theta/dt$ is the angular velocity ω. Thus the tangential velocity and the angular velocity are related by

$$v_t = \omega r \qquad \text{(with } \omega \text{ in rad/s)} \tag{4.32}$$

NOTE ▶ ω is restricted to rad/s because the relationship $s = r\theta$ is the definition of radians. While it may be convenient in some problems to measure ω in rev/s or rpm, you must convert to SI units of rad/s before using Equation 4.32. ◀

The tangential velocity v_t is positive for ccw motion, negative for cw motion. Because v_t is the only nonzero component of $\vec{v}$, the particle's speed is $v = |v_t| = |\omega|r$. We'll sometimes write this as $v = \omega r$ if there's no ambiguity about the sign of ω. Because a particle in uniform circular motion has constant speed, you can see that it must also rotate with constant angular velocity.

As a simple example, a particle moving cw at 2.0 m/s in a circle of radius 40 cm has angular velocity

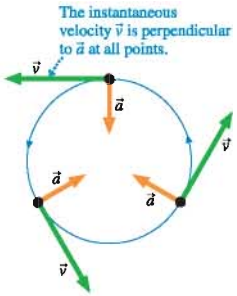
$$\omega = \frac{v_t}{r} = \frac{-2.0 \text{ m/s}}{0.40 \text{ m}} = -5.0 \text{ rad/s}$$

where v_t and ω are negative because the motion is clockwise. Notice the units. Velocity divided by distance has units of s^{-1}. But because the division, in this case, gives us an angular quantity, we've inserted the *dimensionless* unit rad to give ω the appropriate units of rad/s.

Acceleration

Figures 4.1 and 4.2 at the beginning of this chapter looked at the uniform circular motion of a Ferris wheel. You are strongly encouraged to review those figures. There we found that a particle in uniform circular motion, although moving with constant speed, has an acceleration because the *direction* of the velocity vector $\vec{v}$ is always changing. The motion-diagram analysis showed that the **acceleration $\vec{a}$ points toward the center of the circle.** The instantaneous velocity is tangent to the circle, so $\vec{v}$ and $\vec{a}$ are perpendicular to each other at all points on the circle, as FIGURE 4.40 shows.

FIGURE 4.40 For uniform circular motion, the acceleration $\vec{a}$ always points to the center.



The acceleration of uniform circular motion is called **centripetal acceleration,** a term from a Greek root meaning "center seeking." Centripetal acceleration is not a new type of acceleration; all we are doing is *naming* an acceleration that corresponds to a particular type of motion. The magnitude of the centripetal acceleration is constant because each successive $\Delta\vec{v}$ in the motion diagram has the same length.

The motion diagram tells us the direction of $\vec{a}$, but it doesn't give us a value for a. To complete our description of uniform circular motion, we need to find a quantitative

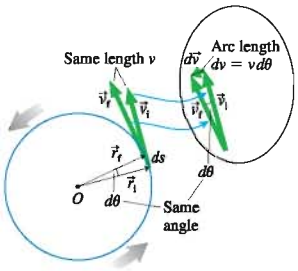


FIGURE 4.41 Finding the acceleration of circular motion.

relationship between a and the particle's speed v. FIGURE 4.41 shows the position $\vec{r}_i$ and velocity $\vec{v}_i$ at one instant of motion and the position $\vec{r}_f$ and velocity $\vec{v}_f$ an infinitesimal amount of time dt later. During this small interval of time, the particle turned through the infinitesimal angle $d\theta$ and traveled distance $ds = r\,d\theta$.

By definition, the acceleration is $\vec{a} = d\vec{v}/dt$. We can see from the inset to Figure 4.41 that $d\vec{v}$ points toward the center of the circle—that is, $\vec{a}$ is a centripetal acceleration. To find the magnitude of $\vec{a}$, we can see from the isosceles triangle of velocity vectors that, if $d\theta$ is in radians,

$$dv = |d\vec{v}| = v\,d\theta \tag{4.33}$$

For uniform circular motion at constant speed, $v = ds/dt = r\,d\theta/dt$ and thus the time to turn through angle $d\theta$ is

$$dt = \frac{r\,d\theta}{v} \tag{4.34}$$

Combining Equations 4.33 and 4.44, we see that the acceleration has magnitude

$$a = |\vec{a}| = \frac{|d\vec{v}|}{dt} = \frac{v\,d\theta}{r\,d\theta/v} = \frac{v^2}{r}$$

4.1 Activ Physics ONLINE

In vector notation, utilizing our knowledge that $\vec{a}$ points toward the center, we can write

$$\vec{a} = \left(\frac{v^2}{r}, \text{ toward center of circle}\right) \qquad \text{(centripetal acceleration)} \tag{4.35}$$

Using Equation 4.32, $v = \omega r$, we can also express the magnitude of the centripetal acceleration in terms of the angular velocity ω as

$$a = \omega^2 r \tag{4.36}$$

NOTE ▶ Centripetal acceleration is not a constant acceleration. The magnitude of the centripetal acceleration is constant during uniform circular motion, but the direction of $\vec{a}$ is constantly changing. Thus the constant-acceleration kinematics equations of Chapter 2 do *not* apply to circular motion. ◀

EXAMPLE 4.14 The acceleration of a Ferris wheel

A typical carnival Ferris wheel has a radius of 9.0 m and rotates 4.0 times per minute. What magnitude acceleration do the riders experience?

MODEL Model the rider as a particle in uniform circular motion.

SOLVE The period is $T = \frac{1}{4}$ min $= 15$ s. From Equation 4.25, a rider's speed is

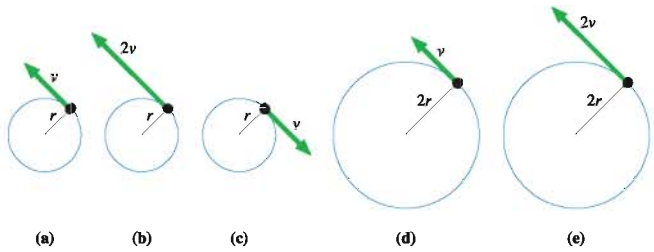
$$v = \frac{2\pi r}{T} = \frac{2\pi(9.0\text{ m})}{15\text{ s}} = 3.77\text{ m/s}$$

Consequently, the centripetal acceleration is

$$a = \frac{v^2}{r} = \frac{(3.77\text{ m/s})^2}{9.0\text{ m}} = 1.6\text{ m/s}^2$$

ASSESS This was not intended to be a profound problem, merely to illustrate how centripetal acceleration is computed. The acceleration is enough to be noticed and make the ride interesting, but not enough to be scary.

STOP TO THINK 4.6 Rank in order, from largest to smallest, the centripetal accelerations a_a to a_e of particles a to e.



4.7 Nonuniform Circular Motion and Angular Acceleration

A roller coaster car doing a loop-the-loop slows down as it goes up one side, speeds up as it comes back down the other. The ball in a roulette wheel gradually slows until it stops. Circular motion with a changing speed is called **nonuniform circular motion.**

Figure 4.12 showed that the acceleration vector for motion in a plane can be decomposed into components $\vec{a}_\parallel$ (parallel to the trajectory) associated with a change of speed and $\vec{a}_\perp$ (perpendicular to the trajectory) associated with a change of direction. Centripetal acceleration is $\vec{a}_\perp$; it is always perpendicular to $\vec{v}$, and it is responsible for the particle constantly changing directions as it moves around the circle. For a particle to speed up or slow down as it moves around a circle, it needs—in addition to the centripetal acceleration—an acceleration parallel to the trajectory or, equivalently, parallel to $\vec{v}$. We'll call this the **tangential acceleration** a_t because, like the velocity v_t, it is always tangent to the circle.

FIGURE 4.42 shows a particle in nonuniform circular motion. *Any* circular motion, whether uniform or nonuniform, has a centripetal acceleration because the particle is changing direction. The centripetal acceleration, which points radially in toward the center of the circle, will now be called the **radial acceleration** a_r. Because of the tangential acceleration, **the acceleration vector $\vec{a}$ of a particle in nonuniform circular motion does *not* point toward the center of the circle.** It points "ahead" of center for a particle that is speeding up, as in Figure 4.42, but it would point "behind" center for a particle slowing down. You can see from Figure 4.42 that the magnitude of the acceleration is

$$a = \sqrt{a_r^2 + a_t^2} \tag{4.37}$$

As the particle speeds up or slows down, the tangential acceleration is simply the rate at which the tangential velocity changes:

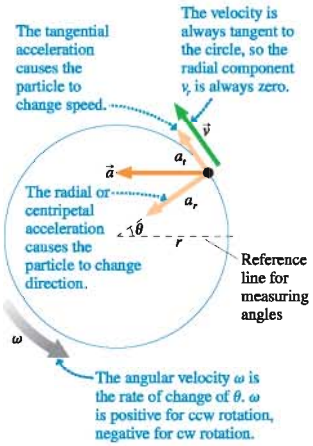
$$a_t = \frac{dv_t}{dt} \tag{4.38}$$

If a_t is constant, then the arc length s traveled by the particle around the circle and the tangential velocity v_t are found from constant-acceleration kinematics:

$$\begin{aligned} s_f &= s_i + v_{it}\,\Delta t + \tfrac{1}{2}a_t(\Delta t)^2 \\ v_{ft} &= v_{it} + a_t\,\Delta t \end{aligned} \tag{4.39}$$

The roller coaster is in nonuniform circular motion as it goes around the loop.

FIGURE 4.42 Nonuniform circular motion.



EXAMPLE 4.15 Circular rocket motion

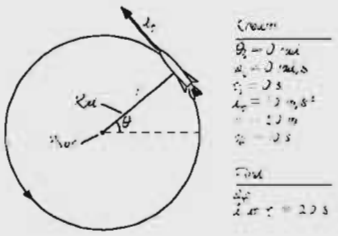
A model rocket is attached to the end of a 2.0-m-long rigid rod. The other end of the rod rotates on a frictionless pivot, causing the rocket to move in a horizontal circle. The rocket accelerates at 1.0 m/s² for 10 s, starting from rest, then runs out of fuel.

a. What is the magnitude of $\vec{a}$ at $t = 2.0$ s?
b. What is the rocket's angular velocity, in rpm, when it runs out of fuel?

MODEL Model the rocket as a particle in nonuniform circular motion. Assume that the rocket starts from rest.

VISUALIZE FIGURE 4.43 is a pictorial representation of the situation. The acceleration caused by the rocket motor is the tangential acceleration, $a_t = 1.0$ m/s².

FIGURE 4.43 The nonuniform circular motion of a model rocket.



SOLVE a. The rocket motor creates the tangential acceleration $a_t = 1.0$ m/s². As the rocket speeds up, it acquires a radial, or centripetal, acceleration $a_r = v_t^2/r$. At $t = 2.0$ s,

$$v_{2s} = v_i + a_t \Delta t = 0 + (1.0 \text{ m/s}^2)(2.0 \text{ s}) = 2.0 \text{ m/s}$$

$$a_r = \frac{v_{2s}^2}{r} = \frac{(2.0 \text{ m/s})^2}{2.0 \text{ m}} = 2.0 \text{ m/s}^2$$

Thus the magnitude of the acceleration at this instant is

$$a = \sqrt{a_r^2 + a_t^2} = \sqrt{(2.0 \text{ m/s}^2)^2 + (1.0 \text{ m/s}^2)^2} = 2.2 \text{ m/s}^2$$

b. The tangential velocity after 10 s is

$$v_{10s} = v_i + a_t \Delta t = 0 + (1.0 \text{ m/s}^2)(10.0 \text{ s}) = 10.0 \text{ m/s}$$

and thus the angular velocity is

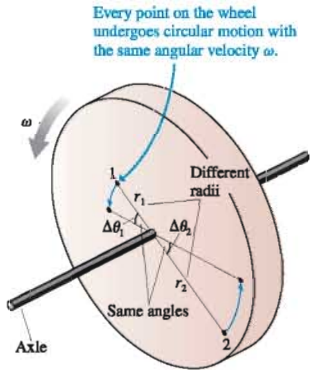
$$\omega_f = \frac{v_{10s}}{r} = \frac{10.0 \text{ m/s}}{2.0 \text{ m}} = 5.0 \text{ rad/s}$$

This is another situation in which we explicitly inserted the rad unit. Converting to rpm:

$$\omega_f = \frac{5.0 \text{ rad}}{1 \text{ s}} \times \frac{1 \text{ rev}}{2\pi \text{ rad}} \times \frac{60 \text{ s}}{1 \text{ min}} = 48 \text{ rpm}$$

Angular Acceleration

FIGURE 4.44 The two points on the wheel rotate with the same angular velocity.



The kinematics of circular motion applies not only to particles but also to rotating solid objects. FIGURE 4.44 shows a wheel rotating on an axle. Notice that two points on the wheel, marked with dots, turn through the *same angle* as the wheel rotates, even though their radii may be different. That is, $\Delta\theta_1 = \Delta\theta_2$ during some time interval Δt. As a consequence, the two points have equal angular velocities: $\omega_1 = \omega_2$. Thus we can refer to the angular velocity ω *of the wheel.*

Two points in a rotating object may have the same angular velocity, but they have *different* tangential velocities v_t if they have different distances from the point of rotation. Consequently, angular velocity is more useful than tangential velocity for describing a rotating object.

Suppose an object's rotation speeds up or slows down; that is, points on the object have a tangential acceleration a_t. We've seen that $a_t = dv_t/dt$ and that $v_t = r\omega$. Combining these two equations, we find

$$a_t = \frac{d(r\omega)}{dt} = r\frac{d\omega}{dt} \tag{4.40}$$

In taking the derivative, we used the fact that r is a constant for circular motion.

We originally defined acceleration as $a = dv/dt$, the rate of change of the velocity. The derivative in Equation 4.40 is the rate of change of the *angular* velocity. By analogy, let's define the **angular acceleration** α (Greek alpha) to be

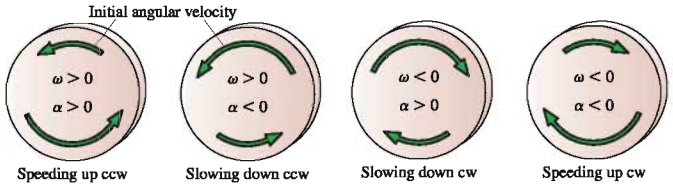
$$\alpha \equiv \frac{d\omega}{dt} \qquad \text{(angular acceleration)} \tag{4.41}$$

The units of angular acceleration are rad/s^2. Angular acceleration is the *rate* at which the angular velocity ω changes, just as linear acceleration is the rate at which the linear velocity v changes. FIGURE 4.45 illustrates the idea.

NOTE ▶ Be careful with the sign of α. You learned in Chapter 2 that positive and negative values of the acceleration can't be interpreted as simply "speeding up" and "slowing down." ◀

Because linear acceleration $\vec{a}$ is a vector, positive a_x means that v_x is increasing to the right or decreasing to the left. Negative a_x means that v_x is increasing to the left or decreasing to the right. For rotational motion, α is positive if ω is increasing ccw or decreasing cw, negative if ω is increasing cw or decreasing ccw. These cases are illustrated in FIGURE 4.46.

FIGURE 4.46 The signs of angular velocity and acceleration.



Comparing Equations 4.40 and 4.41, we see that the tangential and angular accelerations are related by

$$a_t = r\alpha \tag{4.42}$$

Two points on a rotating object have the *same* angular acceleration α, but in general they have *different* tangential accelerations because they are moving in circles of different radii. Notice the analogy between Equation 4.42 and the similar equation $v_t = r\omega$ for tangential and angular velocity.

Because α is the time derivative of ω, we can use exactly the same graphical relationships that we found for linear motion:

- α = slope of the ω-versus-t graph at time t
- $\omega_f = \omega_i$ + area under the α-versus-t graph between t_i and t_f

These relationships involving slopes and areas are illustrated in FIGURE 4.47.

FIGURE 4.45 A wheel with angular acceleration $\alpha = 2$ rad/s^2.

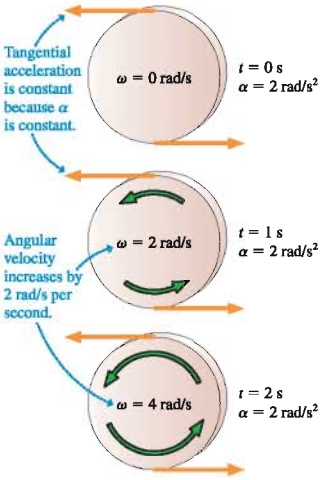
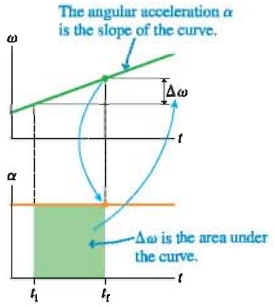


FIGURE 4.47 The graphical relationships between angular velocity and acceleration.



EXAMPLE 4.16 A rotating wheel

FIGURE 4.48a is a graph of angular velocity versus time for a rotating wheel. Describe the motion and draw a graph of angular acceleration versus time.

SOLVE This is a wheel that starts from rest, gradually speeds up *counterclockwise* until reaching top speed at t_1, maintains a constant angular velocity until t_2, then gradually slows down until stopping at t_3. The motion is always ccw because ω is always positive. The angular acceleration graph of FIGURE 4.48b is based on the fact that α is the slope of the ω-versus-t graph.

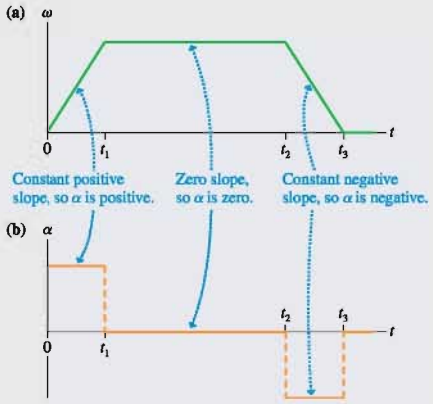


FIGURE 4.48 ω-versus-t graph and the corresponding α-versus-t graph for a rotating wheel.

The circular kinematic equations, Equation 4.39, can be written in terms of angular quantities if we divide both sides by the radius r:

$$\frac{s_f}{r} = \frac{s_i}{r} + \frac{v_{it}}{r}\Delta t + \frac{1}{2}\frac{a_t}{r}(\Delta t)^2$$
$$\frac{v_{ft}}{r} = \frac{v_{it}}{r} + \frac{a_t}{r}\Delta t$$

You'll recognize that s/r is the angular position θ, v_t/r is the angular velocity ω, and a_t/r is the angular acceleration α. Thus the angular position and velocity after undergoing angular acceleration α are

$$\theta_f = \theta_i + \omega_i\Delta t + \tfrac{1}{2}\alpha(\Delta t)^2$$
$$\omega_f = \omega_i + \alpha t \qquad \text{(nonuniform circular motion)} \qquad (4.43)$$

7.7 Activ Physics ONLINE

In addition, the centripetal acceleration equation $a_r = v^2/r = \omega^2 r$ is still valid.

Table 4.1 shows the kinematic equations for constant angular acceleration. These equations apply to a particle in circular motion or to the rotation of a rigid object. Notice that the rotational kinematic equations are exactly analogous to the linear kinematic equations.

TABLE 4.1 Rotational and linear kinematics for constant acceleration

Rotational kinematics	Linear kinematics
$\omega_f = \omega_i + \alpha\Delta t$	$v_{fs} = v_{is} + a_s\Delta t$
$\theta_f = \theta_i + \omega_i\Delta t + \frac{1}{2}\alpha(\Delta t)^2$	$s_f = s_i + v_{is}\Delta t + \frac{1}{2}a_s(\Delta t)^2$
$\omega_f^2 = \omega_i^2 + 2\alpha\Delta\theta$	$v_{fs}^2 = v_{is}^2 + 2a_s\Delta s$

EXAMPLE 4.17 Back to the roulette wheel

A small steel roulette ball rolls around the inside of a 30-cm-diameter roulette wheel. It is spun at 150 rpm, but it slows to 60 rpm over an interval of 5.0 s. How many revolutions does the ball make during these 5.0 s?

MODEL The ball is a particle in nonuniform circular motion. Assume constant angular acceleration as it slows.

SOLVE During these 5.0 s the ball rotates through angle

$$\Delta\theta = \theta_f - \theta_i = \omega_i\Delta t + \tfrac{1}{2}\alpha(\Delta t)^2$$

where $\Delta t = 5.0$ s. We can find the angular acceleration from the initial and final angular velocities, but first they must be converted to SI units:

$$\omega_i = 150\frac{\text{rev}}{\text{min}} \times \frac{1\text{ min}}{60\text{ s}} \times \frac{2\pi\text{ rad}}{1\text{ rev}} = 15.71\text{ rad/s}$$
$$\omega_f = 60\frac{\text{rev}}{\text{min}} = 0.40\omega_i = 6.28\text{ rad/s}$$

The angular acceleration α is

$$\alpha = \frac{\Delta\omega}{\Delta t} = \frac{6.28\text{ rad/s} - 15.71\text{ rad/s}}{5.0\text{ s}} = -1.89\text{ rad/s}^2$$

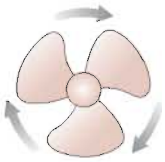
Thus the ball rotates through angle

$$\Delta\theta = (15.71\text{ rad/s})(5.0\text{ s}) + \tfrac{1}{2}(-1.89\text{ rad/s}^2)(5.0\text{ s})^2 = 54.9\text{ rad}$$

Because $54.9/2\pi = 8.75$, the ball completes $8\frac{3}{4}$ revolutions as it slows to 60 rpm.

ASSESS This problem is solved just like the linear kinematics problems you learned to solve in Chapter 2.

STOP TO THINK 4.7 The fan blade is slowing down. What are the signs of ω and α?



a. ω is positive and α is positive.
b. ω is positive and α is negative.
c. ω is negative and α is positive.
d. ω is negative and α is negative.

SUMMARY

The goal of Chapter 4 has been to learn to solve problems about motion in a plane.

General Principles

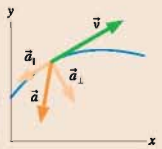
The **instantaneous velocity**

$$\vec{v} = d\vec{r}/dt$$

is a vector tangent to the trajectory.

The **instantaneous acceleration** is

$$\vec{a} = d\vec{v}/dt$$



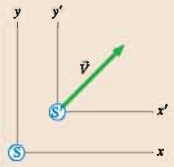
$\vec{a}_\parallel$, the component of $\vec{a}$ parallel to $\vec{v}$, is responsible for change of *speed*. $\vec{a}_\perp$, the component of $\vec{a}$ perpendicular to $\vec{v}$, is responsible for change of *direction*.

Relative motion

Inertial reference frames move relative to each other with constant velocity $\vec{V}$. Measurements of position and velocity measured in frame S are related to measurements in frame S′ by the Galilean transformations:

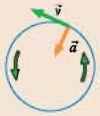
$$x' = x - V_x t \qquad v'_x = v_x - V_x$$

$$y' = y - V_y t \qquad v'_y = v_y - V_y$$



Important Concepts

Uniform Circular Motion



Angular velocity $\omega = d\theta/dt$.

v_t and ω are constant:

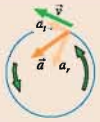
$$v_t = \omega r$$

The centripetal acceleration points toward the center of the circle:

$$a = \frac{v^2}{r} = \omega^2 r$$

It changes the particle's direction but not its speed.

Nonuniform Circular Motion



Angular acceleration $\alpha = d\omega/dt$.

The radial acceleration

$$a_r = \frac{v^2}{r} = \omega^2 r$$

changes the particle's direction. The tangential component

$$a_t = \alpha r$$

changes the particle's speed.

Applications

Kinematics in two dimensions

If $\vec{a}$ is constant, then the x- and y-components of motion are independent of each other.

$$x_f = x_i + v_{ix}\Delta t + \tfrac{1}{2}a_x(\Delta t)^2$$

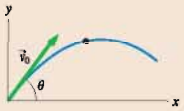
$$y_f = y_i + v_{iy}\Delta t + \tfrac{1}{2}a_y(\Delta t)^2$$

$$v_{fx} = v_{ix} + a_x\Delta t$$

$$v_{fy} = v_{iy} + a_y\Delta t$$

Projectile motion occurs if the object moves under the influence of only gravity. The motion is a parabola.

- Uniform motion in the horizontal direction with $v_{0x} = v_0\cos\theta$.
- Free-fall motion in the vertical direction with $a_y = -g$ and $v_{0y} = v_0\sin\theta$.
- The x and y kinematic equations have the *same* value for Δt.



Circular motion kinematics

Period $T = \dfrac{2\pi r}{v} = \dfrac{2\pi}{\omega}$

Angular position $\theta = \dfrac{s}{r}$

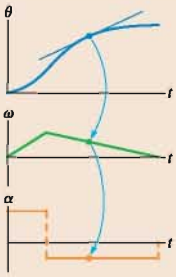
$$\omega_f = \omega_i + \alpha\Delta t$$

$$\theta_f = \theta_i + \omega_i\Delta t + \tfrac{1}{2}\alpha(\Delta t)^2$$

$$\omega_f^2 = \omega_i^2 + 2\alpha\Delta\theta$$

Angle, angular velocity, and angular acceleration are related graphically.

- The angular velocity is the slope of the angular position graph.
- The angular acceleration is the slope of the angular velocity graph.



Terms and Notation

projectile	Galilean transformation of velocity	radians	radial acceleration, a_r
launch angle, θ	uniform circular motion	angular displacement, $\Delta\theta$	angular acceleration, α
reference frame	period, T	angular velocity, ω	
inertial reference frame	angular position, θ	centripetal acceleration, a	
Galilean transformation of position	arc length, s	nonuniform circular motion	
		tangential acceleration, a_t	

MP For homework assigned on MasteringPhysics, go to www.masteringphysics.com

Problem difficulty is labeled as I (straightforward) to III (challenging).

Problems labeled can be done on a Dynamics Worksheet.

Problems labeled integrate significant material from earlier chapters.

CONCEPTUAL QUESTIONS

1. a. Is the particle in FIGURE Q4.1 speeding up, slowing down, or traveling at constant speed?
   b. Is this particle curving to the right, curving to the left, or traveling straight?

FIGURE Q4.1

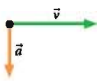


FIGURE Q4.2

2. Is the particle in FIGURE Q4.2 following a straight trajectory, a parabolic trajectory, or a circular trajectory? Or is not possible to tell? Explain.
3. Tarzan swings through the jungle by hanging from a vine.
   a. Immediately after stepping off a branch to swing over to another tree, is Tarzan's acceleration $\vec{a}$ zero or not zero? If not zero, which way does it point? Explain.
   b. Answer the same question at the lowest point in Tarzan's swing.
4. A projectile is launched over horizontal ground at an angle between 0° and 90°.
   a. Is there any point on the trajectory where $\vec{v}$ and $\vec{a}$ are parallel to each other? If so, where?
   b. Is there any point where $\vec{v}$ and $\vec{a}$ are perpendicular to each other? If so, where?
5. For a projectile, which of the following quantities are constant during the flight: x, y, r, v_x, v_y, v, a_x, a_y? Which of these quantities are zero throughout the flight?
6. A cart that is rolling at constant velocity on a level table fires a ball straight up.
   a. When the ball comes back down, will it land in front of the launching tube, behind the launching tube, or directly in the tube? Explain.
   b. Will your answer change if the cart is accelerating in the forward direction? If so, how?
7. A rock is thrown from a bridge at an angle 30° below horizontal.
   a. Immediately after the rock is released, is the magnitude of its acceleration greater than, less than, or equal to g? Explain.
   b. At the instant of impact, is the rock's speed greater than, less than, or equal to the speed with which it was thrown? Explain.
8. Rank in order, from shortest to longest (some may be simultaneous), the amount of time it takes each of the projectiles in FIGURE Q4.8 to hit the ground. Ignore air resistance.

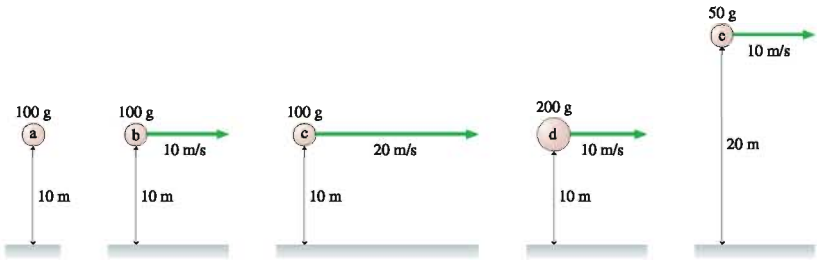


FIGURE Q4.8

9. Anita is running to the right at 5 m/s in FIGURE Q4.9. Balls 1 and 2 are thrown toward her by friends standing on the ground. According to Anita, both balls are approaching her at 10 m/s. Which ball was thrown at a faster speed? Or were they thrown with the same speed? Explain.

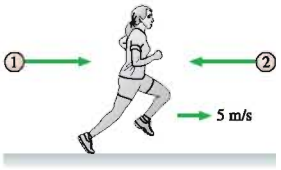


FIGURE Q4.9

10. An electromagnet on the ceiling of an airplane holds a steel ball. When a button is pushed, the magnet releases the ball. The experiment is first done while the plane is parked on the ground, and the point where the ball hits the floor is marked with an X. Then the experiment is repeated while the plane is flying level at a steady 500 mph. Does the ball land slightly in front of the X (toward the nose of the plane), on the X, or slightly behind the X (toward the tail of the plane)? Explain.
11. Zack is driving past his house in FIGURE Q4.11. He wants to toss his physics book out the window and have it land in his driveway. If he lets go of the book exactly as he passes the end of the driveway, should he direct his throw outward and toward the front of the car (throw 1), straight outward (throw 2), or outward and toward the back of the car (throw 3)? Explain.

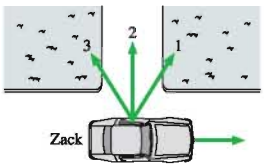


FIGURE Q4.11

12. In FIGURE Q4.12, Yvette and Zack are driving down the freeway side by side with their windows down. Zack wants to toss his physics book out the window and have it land in Yvette's front seat. Ignoring air resistance, should he direct his throw outward and toward the front of the car (throw 1), straight outward (throw 2), or outward and toward the back of the car (throw 3)? Explain.

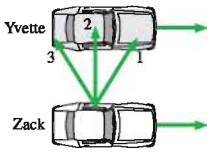


FIGURE Q4.12

13. In uniform circular motion, which of the following quantities are constant: speed, instantaneous velocity, tangential velocity, radial acceleration, tangential acceleration? Which of these quantities are zero throughout the motion?
14. FIGURE Q4.14 shows three points on a steadily rotating wheel.
    a. Rank in order, from largest to smallest, the angular velocities ω_1, ω_2, and ω_3 of these points. Explain.
    b. Rank in order, from largest to smallest, the speeds v_1, v_2, and v_3 of these points. Explain.

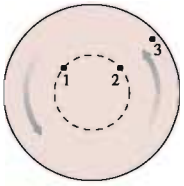


FIGURE Q4.14

15. FIGURE Q14.15 shows four rotating wheels. For each, determine the signs (+ or −) of ω and α.

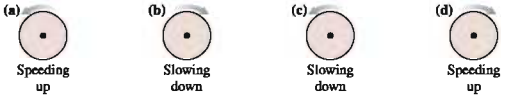


FIGURE Q4.15

16. FIGURE Q4.16 shows a pendulum at one end point of its arc.
    a. At this point, is ω positive, negative, or zero? Explain.
    b. At this point, is α positive, negative, or zero? Explain.

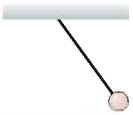


FIGURE Q4.16

EXERCISES AND PROBLEMS

Exercises

Section 4.1 Acceleration

Problems 1 through 3 show a partial motion diagram. For each:

a. Complete the motion diagram by adding acceleration vectors.
b. Write a physics *problem* for which this is the correct motion diagram. Be imaginative! Don't forget to include enough information to make the problem complete and to state clearly what is to be found.

1. I

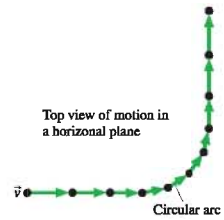


FIGURE EX4.1

2. I

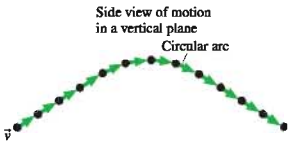


FIGURE EX4.2

3. I

FIGURE EX4.3

4. II Consider a pendulum swinging back and forth on a string. Use a motion diagram analysis and a written explanation to answer the following questions.
   a. At the lowest point in the motion, is the velocity zero or nonzero? Is the acceleration zero or nonzero? If these vectors aren't zero, which way do they point?
   b. At the end of its arc, when the pendulum is at the highest point on the right or left side, is the velocity zero or nonzero? Is the acceleration zero or nonzero? If these vectors aren't zero, which way do they point?

Section 4.2 Two-Dimensional Kinematics

5. II A sailboat is traveling east at 5.0 m/s. A sudden gust of wind gives the boat an acceleration $\vec{a} = (0.80\ \text{m/s}^2, 40°\ \text{north of east})$. What are the boat's speed and direction 6.0 s later when the gust subsides?
6. II A particle's trajectory is described by $x = \left(\frac{1}{2}t^3 - 2t^2\right)$ m and $y = \left(\frac{1}{2}t^2 - 2t\right)$ m, where t is in s.
   a. What are the particle's position and speed at $t = 0$ s and $t = 4$ s?
   b. What is the particle's direction of motion, measured as an angle from the x-axis, at $t = 0$ s and $t = 4$ s?
7. II A flying saucer maneuvering with constant acceleration is observed with the positions and velocities shown in FIGURE EX4.7. What is the saucer's acceleration $\vec{a}$?

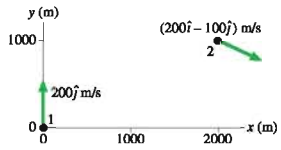


FIGURE EX4.7

8. II A rocket-powered hockey puck moves on a horizontal frictionless table. FIGURE EX4.8 at the top of the next column shows graphs of v_x and v_y, the x- and y-components of the puck's velocity. The puck starts at the origin.
   a. In which direction is the puck moving at $t = 2$ s? Give your answer as an angle from the x-axis.
   b. How far from the origin is the puck at $t = 5$ s?

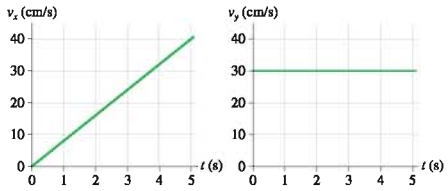


FIGURE EX4.8

9. II A rocket-powered hockey puck moves on a horizontal frictionless table. FIGURE EX4.9 shows graphs of v_x and v_y, the x- and y-components of the puck's velocity. The puck starts at the origin.
   a. What is the magnitude of the puck's acceleration?
   b. How far from the origin is the puck at $t = 0$ s, 5 s, and 10 s?

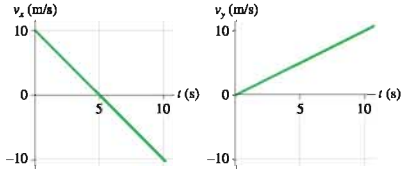


FIGURE EX4.9

Section 4.3 Projectile Motion

10. I A physics student on Planet Exidor throws a ball, and it follows the parabolic trajectory shown in FIGURE EX4.10. The ball's position is shown at 1 s intervals until $t = 3$ s. At $t = 1$ s, the ball's velocity is $\vec{v} = (2.0\hat{i} + 2.0\hat{j})$ m/s.

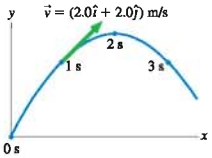


FIGURE EX4.10

   a. Determine the ball's velocity at $t = 0$ s, 2 s, and 3 s.
   b. What is the value of g on Planet Exidor?
   c. What was the ball's launch angle?
11. I A ball thrown horizontally at 25 m/s travels a horizontal distance of 50 m before hitting the ground. From what height was the ball thrown?
12. II A rifle is aimed horizontally at a target 50 m away. The bullet hits the target 2.0 cm below the aim point.
   a. What was the bullet's flight time?
   b. What was the bullet's speed as it left the barrel?
13. I A supply plane needs to drop a package of food to scientists working on a glacier in Greenland. The plane flies 100 m above the glacier at a speed of 150 m/s. How far short of the target should it drop the package?
14. I A sailor climbs to the top of the mast, 15 m above the deck, to look for land while his ship moves steadily forward through calm waters at 4.0 m/s. Unfortunately, he drops his spyglass to the deck below.
   a. Where does it land with respect to the base of the mast below him?
   b. Where does it land with respect to a fisherman sitting at rest in his dinghy as the ship goes past? Assume that the fisherman is even with the mast at the instant the spyglass is dropped.

Section 4.4 Relative Motion

15. | Ted is sitting in his lawn chair when Stella flies directly overhead, going southeast at 100 m/s. Five seconds later, a firecracker explodes 200 m east of Ted. What are the coordinates of the explosion in Stella's reference frame? Let Stella be at the origin, with her x-axis pointing to the east.
16. || A boat takes 3.0 hours to travel 30 km down a river, then 5.0 hours to return. How fast is the river flowing?
17. || When the moving sidewalk at the airport is broken, as it often seems to be, it takes you 50 s to walk from your gate to baggage claim. When it is working and you stand on the moving sidewalk the entire way, without walking, it takes 75 s to travel the same distance. How long will it take you to travel from the gate to baggage claim if you walk while riding on the moving sidewalk?
18. | Mary needs to row her boat across a 100-m-wide river that is flowing to the east at a speed of 3.0 m/s. Mary can row with a speed of 2.0 m/s.
    a. If Mary rows straight north, where will she land?
    b. Draw a picture showing her displacement due to rowing, her displacement due to the river's motion, and her net displacement.
19. | Susan, driving north at 60 mph, and Shawn, driving east at 45 mph, are approaching an intersection. What is Shawn's speed relative to Susan's reference frame?

Section 4.5 Uniform Circular Motion

20. | FIGURE EX4.20 shows the angular-position-versus-time graph for a particle moving in a circle.
    a. Write a description of the particle's motion.
    b. Draw the angular-velocity-versus-time graph.

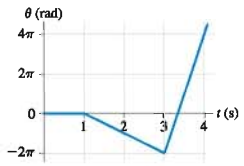


FIGURE EX4.20

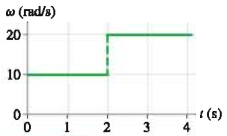


FIGURE EX4.21

21. || FIGURE EX4.21 shows the angular-velocity-versus-time graph for a particle moving in a circle. How many revolutions does the object make during the first 4 s?
22. || FIGURE EX4.22 shows the angular-velocity-versus-time graph for a particle moving in a circle, starting from $\theta_0 = 0$ rad at $t = 0$ s. Draw the angular-position-versus-time graph. Include an appropriate scale on both axes.

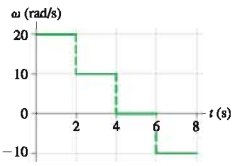


FIGURE EX4.22

23. | An old-fashioned single-play vinyl record rotates on a turntable at 45 rpm. What are (a) the angular velocity in rad/s and (b) the period of the motion?
24. || The earth's radius is about 4000 miles. Kampala, the capital of Uganda, and Singapore are both nearly on the equator. The distance between them is 5000 miles.
    a. Through what angle do you turn, relative to the earth, if you fly from Kampala to Singapore? Give your answer in both radians and degrees.
    b. The flight from Kampala to Singapore takes 9 hours. What is the plane's angular velocity relative to the earth?

Section 4.6 Velocity and Acceleration in Uniform Circular Motion

25. || A 300-m-tall tower is built on the equator. How much faster does a point at the top of the tower move than a point at the bottom? The earth's radius is 6400 km.
26. | How fast must a plane fly along the earth's equator so that the sun stands still relative to the passengers? In which direction must the plane fly, east to west or west to east? Give your answer in both km/hr and mph. The earth's radius is 6400 km.
27. | To withstand "g-forces" of up to 10 g's, caused by suddenly pulling out of a steep dive, fighter jet pilots train on a "human centrifuge." 10 g's is an acceleration of 98 m/s^2. If the length of the centrifuge arm is 12 m, at what speed is the rider moving when she experiences 10 g's?
28. | The radius of the earth's very nearly circular orbit around the sun is 1.5×10^{11} m. Find the magnitude of the earth's (a) velocity, (b) angular velocity, and (c) centripetal acceleration as it travels around the sun. Assume a year of 365 days.
29. || Your roommate is working on his bicycle and has the bike upside down. He spins the 60-cm-diameter wheel, and you notice that a pebble stuck in the tread goes by three times every second. What are the pebble's speed and acceleration?

Section 4.7 Nonuniform Circular Motion and Angular Acceleration

30. | FIGURE EX4.30 shows the angular velocity graph of the crankshaft in a car. Draw a graph of the angular acceleration versus time. Include appropriate numerical scales on both axes.

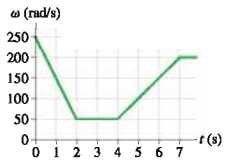


FIGURE EX4.30

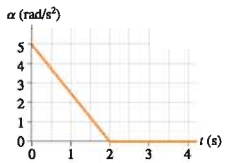


FIGURE EX4.31

31. | FIGURE EX4.31 shows the angular acceleration graph of a turntable that starts from rest. Draw a graph of the angular velocity versus time. Include appropriate numerical scales on both axes.
32. | FIGURE EX4.32 shows the angular-velocity-versus-time graph for a particle moving in a circle. How many revolutions does the object make during the first 4 s?

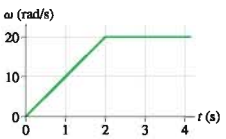


FIGURE EX4.32

33. || a. FIGURE EX4.33a shows angular velocity versus time. Draw the corresponding graph of angular acceleration versus time.
b. FIGURE EX4.33b shows angular acceleration versus time. Draw the corresponding graph of angular velocity versus time. Assume $\omega_0 = 0$.

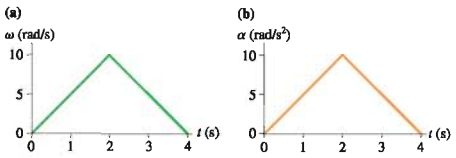


FIGURE EX4.33

34. || A car speeds up as it turns from traveling due south to heading due east. When exactly halfway around the curve, the car's acceleration is 3.0 m/s², 20° north of east. What are the radial and tangential components of the acceleration at that point?
35. || A 5.0-m-diameter merry-go-round is initially turning with a 4.0 s period. It slows down and stops in 20 s.
a. Before slowing, what is the speed of a child on the rim?
b. How many revolutions does the merry-go-round make as it stops?
36. || A 3.0-cm-diameter crankshaft that is rotating at 2500 rpm comes to a halt in 1.5 s.
a. What is the tangential acceleration of a point on the surface?
b. How many revolutions does the crankshaft make as it stops?
37. | An electric fan goes from rest to 1800 rpm in 4.0 s. What is its angular acceleration?
38. | A bicycle wheel is rotating at 50 rpm when the cyclist begins to pedal harder, giving the wheel a constant angular acceleration of 0.50 rad/s².
a. What is the wheel's angular velocity, in rpm, 10 s later?
b. How many revolutions does the wheel make during this time?

Problems

39. || A particle starts from rest at $\vec{r}_0 = 9.0\hat{j}$ m and moves in the xy-plane with the velocity shown in FIGURE P4.39. The particle passes through a wire hoop located at $\vec{r}_1 = 20\hat{i}$ m, then continues onward.
a. At what time does the particle pass through the hoop?
b. What is the value of v_{4y}, the y-component of the particle's velocity at $t = 4$ s?
c. Calculate and plot the particle's trajectory.

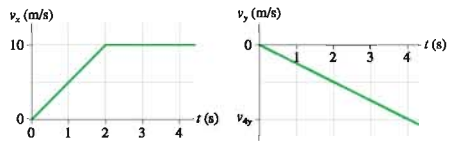


FIGURE P4.39

40. || A projectile's horizontal range on level ground is $R = v_0^2 \sin 2\theta / g$. At what launch angle or angles will the projectile land at half of its maximum possible range.
41. || a. A projectile is launched with speed v_0 and angle θ. Derive an expression for the projectile's maximum height h.
b. A baseball is hit with a speed of 33.6 m/s. Calculate its height and the distance traveled if it is hit at angles of 30.0°, 45.0°, and 60.0°.
42. || A projectile is fired with an initial speed of 30 m/s at an angle of 60° above the horizontal. The object hits the ground 7.5 s later.
a. How much higher or lower is the launch point relative to the point where the projectile hits the ground?
b. To what maximum height above the launch point does the projectile rise?
c. What are the magnitude and direction of the projectile's velocity at the instant it hits the ground?
43. || In the Olympic shotput event, an athlete throws the shot with an initial speed of 12.0 m/s at a 40.0° angle from the horizontal. The shot leaves her hand at a height of 1.80 m above the ground.
a. How far does the shot travel?
b. Repeat the calculation of part (a) for angles 42.5°, 45.0°, and 47.5°. Put all your results, including 40.0°, in a table. At what angle of release does she throw the farthest?
44. || On the Apollo 14 mission to the moon, astronaut Alan Shepard hit a golf ball with a 6 iron. The free-fall acceleration on the moon is 1/6 of its value on earth. Suppose he hit the ball with a speed of 25 m/s at an angle 30° above the horizontal.
a. How much farther did the ball travel on the moon than it would have on earth?
b. For how much more time was the ball in flight?
45. || A ball is thrown toward a cliff of height h with a speed of 30 m/s and an angle of 60° above horizontal. It lands on the edge of the cliff 4.0 s later.
a. How high is the cliff?
b. What was the maximum height of the ball?
c. What is the ball's impact speed?
46. || A tennis player hits a ball 2.0 m above the ground. The ball leaves his racquet with a speed of 20.0 m/s at an angle 5.0° above the horizontal. The horizontal distance to the net is 7.0 m, and the net is 1.0 m high. Does the ball clear the net? If so, by how much? If not, by how much does it miss?
47. || A baseball player friend of yours wants to determine his pitching speed. You have him stand on a ledge and throw the ball horizontally from an elevation 4.0 m above the ground. The ball lands 25 m away.
a. What is his pitching speed?
b. As you think about it, you're not sure he threw the ball exactly horizontally. As you watch him throw, the pitches seem to vary from 5° below horizontal to 5° above horizontal. What is the *range* of speeds with which the ball might have left his hand?
48. || You are playing right field for the baseball team. Your team is up by one run in the bottom of the last inning of the game when a ground ball slips through the infield and comes straight toward you. As you pick up the ball 65 m from home plate, you see a runner rounding third base and heading for home with the tying run. You throw the ball at an angle of 30° above the horizontal with just the right speed so that the ball is caught by the catcher, standing on home plate, at the same height as you threw it. As you release the ball, the runner is 20.0 m from home plate and running full speed at 8.0 m/s. Will the ball arrive in time for your team's catcher to make the tag and win the game?

49. || You're 6.0 m from one wall of a house. You want to toss a ball to your friend who is 6.0 m from the opposite wall. The throw and catch each occur 1.0 m above the ground.
    a. What minimum speed will allow the ball to clear the roof?
    b. At what angle should you toss the ball?

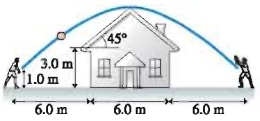


FIGURE P4.49

50. || Sand moves without slipping at 6.0 m/s down a conveyer that is tilted at 15°. The sand enters a pipe 3.0 m below the end of the conveyer belt, as shown in FIGURE P4.50. What is the horizontal distance d between the conveyer belt and the pipe?

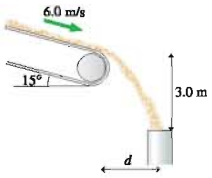


FIGURE P4.50

51. || King Arthur's knights fire a cannon from the top of the castle wall. The cannonball is fired at a speed of 50 m/s and an angle of 30°. A cannonball that was accidentally dropped hits the moat below in 1.5 s.
    a. How far from the castle wall does the cannonball hit the ground?
    b. What is the ball's maximum height above the ground?
52. || A stunt man drives a car at a speed of 20 m/s off a 30-m-high cliff. The road leading to the cliff is inclined upward at an angle of 20°.
    a. How far from the base of the cliff does the car land?
    b. What is the car's impact speed?
53. || A cat is chasing a mouse. The mouse runs in a straight line at a speed of 1.5 m/s. If the cat leaps off the floor at a 30° angle and a speed of 4.0 m/s, at what distance behind the mouse should the cat leap in order to land on the poor mouse?
54. || An assembly line has a staple gun that rolls to the left at 1.0 m/s while parts to be stapled roll past it to the right at 3.0 m/s. The staple gun fires 10 staples per second. How far apart are the staples in the finished part?
55. || Ships A and B leave port together. For the next two hours, ship A travels at 20 mph in a direction 30° west of north while the ship B travels 20° east of north at 25 mph.
    a. What is the distance between the two ships two hours after they depart?
    b. What is the speed of ship A as seen by ship B?
56. || A kayaker needs to paddle north across a 100-m-wide harbor. The tide is going out, creating a tidal current that flows to the east at 2.0 m/s. The kayaker can paddle with a speed of 3.0 m/s.
    a. In which direction should he paddle in order to travel straight across the harbor?
    b. How long will it take him to cross?
57. || Mike throws a ball upward and toward the east at a 63° angle with a speed of 22 m/s. Nancy drives east past Mike at 30 m/s at the instant he releases the ball.
    a. What is the ball's initial angle in Nancy's reference frame?
    b. Find and graph the ball's trajectory as seen by Nancy.
58. || A sailboat is sailing due east at 8.0 mph. The wind appears to blow from the southwest at 12.0 mph.
    a. What are the true wind speed and direction?
    b. What are the true wind speed and direction if the wind appears to blow from the northeast at 12.0 mph?
59. || While driving north at 25 m/s during a rainstorm you notice that the rain makes an angle of 38° with the vertical. While driving back home moments later at the same speed but in the opposite direction, you see that the rain is falling straight down. From these observations, determine the speed and angle of the raindrops relative to the ground.
60. || A plane has an airspeed of 200 mph. The pilot wishes to reach a destination 600 mi due east, but a wind is blowing at 50 mph in the direction 30° north of east.
    a. In what direction must the pilot head the plane in order to reach her destination?
    b. How long will the trip take?
61. || A typical laboratory centrifuge rotates at 4000 rpm. Test tubes have to be placed into a centrifuge very carefully because of the very large accelerations.
    a. What is the acceleration at the end of a test tube that is 10 cm from the axis of rotation?
    b. For comparison, what is the magnitude of the acceleration a test tube would experience if dropped from a height of 1.0 m and stopped in a 1.0-ms-long encounter with a hard floor?
62. || Astronauts use a centrifuge to simulate the acceleration of a rocket launch. The centrifuge takes 30 s to speed up from rest to its top speed of 1 rotation every 1.3 s. The astronaut is strapped into a seat 6.0 m from the axis.
    a. What is the astronaut's tangential acceleration during the first 30 s?
    b. How many g's of acceleration does the astronaut experience when the device is rotating at top speed? Each 9.8 m/s^2 of acceleration is 1 g.
63. ||| A car starts from rest on a curve with a radius of 120 m and accelerates at 1.0 m/s^2. Through what angle will the car have traveled when the magnitude of its total acceleration is 2.0 m/s^2?
64. || As the earth rotates, what is the speed of (a) a physics student in Miami, Florida, at latitude 26°, and (b) a physics student in Fairbanks, Alaska, at latitude 65°? Ignore the revolution of the earth around the sun. The radius of the earth is 6400 km.
65. || Communications satellites are placed in a circular orbit where they stay directly over a fixed point on the equator as the earth rotates. These are called *geosynchronous orbits*. The radius of the earth is 6.37×10^6 m, and the altitude of a geosynchronous orbit is 3.58×10^7 m ($\approx$22,000 miles). What are (a) the speed and (b) the magnitude of the acceleration of a satellite in a geosynchronous orbit?
66. || A magnetic computer disk 8.0 cm in diameter is initially at rest. A small dot is painted on the edge of the disk. The disk accelerates at 600 rad/s^2 for $\frac{1}{2}$ s, then coasts at a steady angular velocity for another $\frac{1}{2}$ s. What is the speed of the dot at $t = 1.0$ s? Through how many revolutions has the disk turned?
67. || A high-speed drill rotating ccw at 2400 rpm comes to a halt in 2.5 s.
    a. What is the drill's angular acceleration?
    b. How many revolutions does it make as it stops?
68. || An electric-generator turbine spins at 3600 rpm. Friction is so small that it takes the turbine 10 min to coast to a stop. How many revolutions does it make while stopping?

69. ‖ A wheel initially rotating at 60 rpm experiences the angular acceleration shown in FIGURE P4.69. What is the wheel's angular velocity, in rpm, at $t = 3.0$ s?

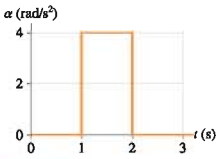


FIGURE P4.69

70. ‖ If you step on your car's brakes hard, the wheels stop turning (i.e., the wheels "lock") after 1.0 revolution. At the same constant acceleration, how many revolutions do the wheels make before stopping if your initial speed is twice as high?
71. ‖ A well-lubricated bicycle wheel spins a long time before stopping. Suppose a wheel initially rotating at 100 rpm takes 60 s to stop. If the angular acceleration is constant, how many revolutions does the wheel make while stopping?
72. ‖ A rock stuck in the tread of a 60.0-cm-diameter bicycle wheel has a tangential speed of 3.00 m/s. When the brakes are applied, the rock's tangential deceleration is 1.00 m/s^2.
    a. What are the magnitudes of the rock's angular velocity and angular acceleration at $t = 1.50$ s?
    b. At what time is the magnitude of the rock's acceleration equal to g?
73. ‖ A long string is wrapped around a 6.0-cm-diameter cylinder, initially at rest, that is free to rotate on an axle. The string is then pulled with a constant acceleration of 1.5 m/s^2 until 1.0 m of string has been unwound. If the string unwinds without slipping, what is the cylinder's angular speed, in rpm, at this time?

In Problems 74 through 76 you are given the equations that are used to solve a problem. For each of these, you are to
    a. Write a realistic problem for which these are the correct equations. Be sure that the answer your problem requests is consistent with the equations given.
    b. Finish the solution of the problem, including a pictorial representation.

74. $100\text{ m} = 0\text{ m} + (50\cos\theta\text{ m/s})t_1$
    $0\text{ m} = 0\text{ m} + (50\sin\theta\text{ m/s})t_1 - \frac{1}{2}(9.80\text{ m/s}^2)t_1^2$
75. $v_x = -(6.0\cos 45°)\text{ m/s} + 3.0\text{ m/s}$
    $v_y = (6.0\sin 45°)\text{ m/s} + 0\text{ m/s}$,
    $100\text{ m} = v_y t_1$, $x_1 = v_x t_1$
76. $2.5\text{ rad} = 0\text{ rad} + \omega_i(10\text{ s}) + \big((1.5\text{ m/s}^2)/2(50\text{ m})\big)(10\text{ s})^2$
    $\omega_f = \omega_i + \big((1.5\text{ m/s}^2)/(50\text{ m})\big)(10\text{ s})$
77. Write a realistic problem for which the x-versus-t and y-versus-t graphs shown in FIGURE P4.77 represent the motion of an object. Be sure the answer your problem requests is consistent with the graphs. Then finish the solution of the problem.

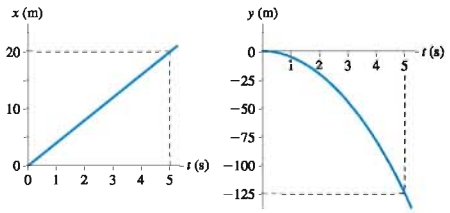


FIGURE P4.77

Challenge Problems

78. You are asked to consult for the city's research hospital, where a group of doctors is investigating the bombardment of cancer tumors with high-energy ions. The ions are fired directly toward the center of the tumor at speeds of 5.0×10^6 m/s. To cover the entire tumor area, the ions are deflected sideways by passing them between two charged metal plates that accelerate the ions perpendicular to the direction of their initial motion. The acceleration region is 5.0 cm long, and the ends of the acceleration plates are 1.5 m from the patient. What acceleration is required to deflect an ion 2.0 cm to one side?

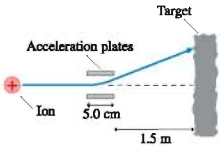


FIGURE CP4.78

79. In one contest at the county fair, a spring-loaded plunger launches a ball at a speed of 3.0 m/s from one corner of a smooth, flat board that is tilted up at a 20° angle. To win, you must make the ball hit a small target at the adjacent corner, 2.50 m away. At what angle θ should you tilt the ball launcher?

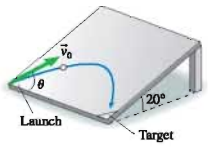


FIGURE CP4.79

80. You are watching an archery tournament when you start wondering how fast an arrow is shot from the bow. Remembering your physics, you ask one of the archers to shoot an arrow parallel to the ground. You find the arrow stuck in the ground 60 m away, making a 3° angle with the ground. How fast was the arrow shot?
81. An archer standing on a 15° slope shoots an arrow 20° above the horizontal, as shown in FIGURE CP4.81. How far down the slope does the arrow hit if it is shot with a speed of 50 m/s from 1.75 m above the ground?

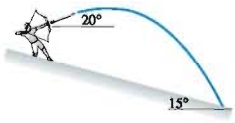


FIGURE CP4.81

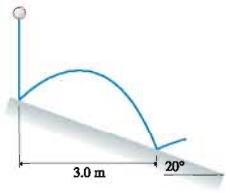


FIGURE CP4.82

82. A rubber ball is dropped onto a ramp that is tilted at 20°, as shown in FIGURE CP4.82. A bouncing ball obeys the "law of reflection," which says that the ball leaves the surface at the same angle it approached the surface. The ball's next bounce is 3.0 m to the right of its first bounce. What is the ball's rebound speed on its first bounce?
83. A skateboarder starts up a 1.0-m-high, 30° ramp at a speed of 7.0 m/s. The skateboard wheels roll without friction. How far from the end of the ramp does the skateboarder touch down?
84. A motorcycle daredevil wants to set a record for jumping over burning school buses. He has hired you to help with the design. He intends to ride off a horizontal platform at 40 m/s, cross the burning buses in a pit below him, then land on a ramp sloping down at 20°. It's very important that he not bounce when he hits

the landing ramp because that could cause him to lose control and crash. You immediately recognize that he won't bounce if his velocity is parallel to the ramp as he touches down. This can be accomplished if the ramp is tangent to his trajectory *and* if he lands right on the front edge of the ramp. There's no room for error! Your task is to determine where to place the landing ramp. That is, how far from the edge of the launching platform should the front edge of the landing ramp be horizontally and how far below it? There's a clause in your contract that requires you to test your design before the hero goes on national television to set the record.

85. A cannon on a train car fires a projectile to the right with speed v_0, relative to the train, from a barrel elevated at angle θ. The cannon fires just as the train, which had been cruising to the right along a level track with speed v_{train}, begins to accelerate with acceleration a. Find an expression for the angle at which the projectile should be fired so that it lands as far as possible from the cannon. You can ignore the small height of the cannon above the track.
86. A child in danger of drowning in a river is being carried downstream by a current that flows uniformly with a speed of 2.0 m/s. The child is 200 m from the shore and 1500 m upstream of the boat dock from which the rescue team sets out. If their boat speed is 8.0 m/s with respect to the water, at what angle from the shore should the pilot leave the shore to go directly to the child?
87. Uri is on a flight from Boston to Los Angeles. His plane is traveling 20° south of west at 500 mph. Val is on a flight from Miami to Seattle. Her plane is traveling 30° north of west at 500 mph. Somewhere over Kansas, Uri's plane passes 1000 ft directly over Val's plane. Uri is sitting on the right side and can see Val's plane below him after they pass. Uri notices that the fuselage of Val's plane doesn't point in the direction that her plane is moving. What is the angle between the fuselage and the direction of motion?
88. An amusement park game, shown in FIGURE CP4.88, launches a marble toward a small cup. The marble is placed directly on top of a spring-loaded wheel and held with a clamp. When released, the wheel spins around clockwise at constant angular acceleration, opening the clamp and releasing the marble after making $\frac{11}{12}$ revolution. What angular acceleration is needed for the ball to land in the cup?

FIGURE CP4.88

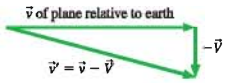
STOP TO THINK ANSWERS

Stop to Think 4.1: d. The parallel component of $\vec{a}$ is opposite $\vec{v}$ and will cause the particle to slow down. The perpendicular component of $\vec{a}$ will cause the particle to change direction downward.

Stop to Think 4.2: c. $v = 0$ requires both $v_x = 0$ and $v_y = 0$. Neither x nor y can be changing.

Stop to Think 4.3: d. A projectile's acceleration $\vec{a} = -g\hat{j}$ does not depend on its mass. The second marble has the same initial velocity and the same acceleration, so it follows the same trajectory and lands at the same position.

Stop to Think 4.4: f. The helicopter frame S′ moves with $\vec{V} = 20\hat{j}$ m/s relative to the earth frame S. The plane moves with $\vec{V} = 100\hat{i}$ m/s in the earth's frame. The vector addition in the figure shows that $\vec{v}'$ is longer than $\vec{v}$.

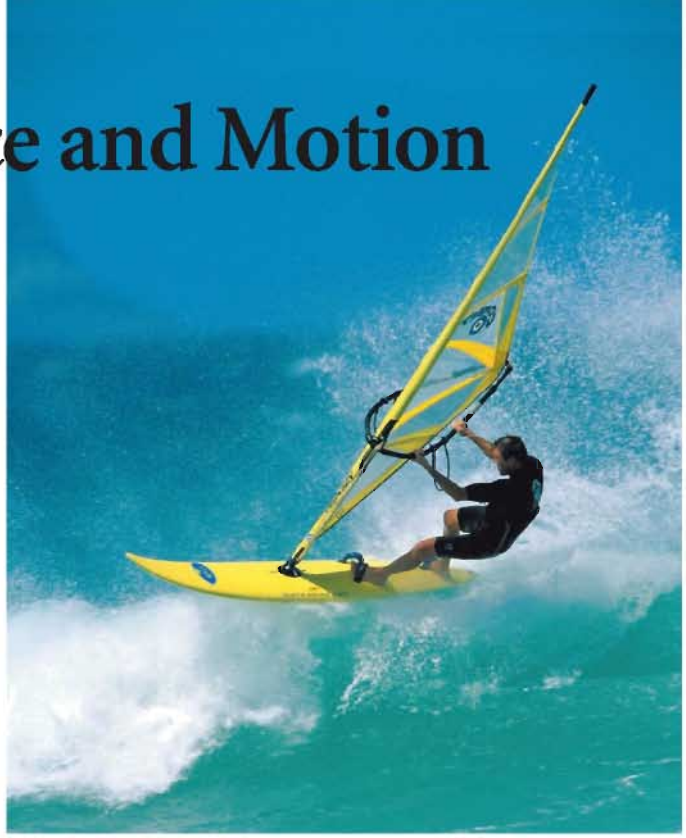


Stop to Think 4.5: b. An initial cw rotation causes the particle's angular position to become increasingly negative. The speed drops to half after reversing direction, so the slope becomes positive and is half as steep as the initial slope. Turning through the same angle returns the particle to $\theta = 0°$.

Stop to Think 4.6: $a_b > a_e > a_n = a_c > a_d$. Centripetal acceleration is v^2/r. Doubling r decreases a_r by a factor of 2. Doubling v increases a_r by a factor of 4. Reversing direction doesn't change a_r.

Stop to Think 4.7: c. ω is negative because the rotation is cw. Because ω is negative but becoming *less* negative, the change $\Delta\omega$ is *positive*. So α is positive.

5 Force and Motion



Wind surfing is a memorable example of the connection between force and motion.

A strong gust of wind can send this wind surfer blasting across the water. We could use kinematics to describe the surfer's motion with pictures, graphs, and equations. By defining position, velocity, and acceleration and dressing them in mathematical clothing, kinematics provides a language to describe *how* something moves. But kinematics would tell us nothing about *why* the wind surfer moves. For the more fundamental task of understanding the *cause* of motion, we turn our attention to **dynamics.** Dynamics joins with kinematics to form **mechanics,** the general science of motion. We study dynamics qualitatively in this chapter, then develop it quantitatively in the next three chapters.

The theory of mechanics originated in the mid-1600s when Sir Isaac Newton formulated his laws of motion. These fundamental principles of mechanics explain how motion occurs as a consequence of forces. Newton's laws are more than 300 years old, but they still form the basis for our contemporary understanding of motion.

A challenge in learning physics is that a textbook is not an experiment. The book can assert that an experiment will have a certain outcome, but you may not be convinced unless you see or do the experiment yourself. Newton's laws are frequently contrary to our intuition, and a lack of familiarity with the evidence for Newton's laws is a source of difficulty for many people. You will have an opportunity through lecture demonstrations and in the laboratory to see for yourself the evidence supporting Newton's laws. Physics is not an arbitrary collection of definitions and formulas, but a consistent theory as to how the universe really works. It is only with experience and evidence that we learn to separate physical fact from fantasy.

▶ Looking Ahead

The goal of Chapter 5 is to establish a connection between force and motion. In this chapter you will learn to:

- Recognize what a force is and is not.
- Identify the specific forces acting on an object.
- Draw free-body diagrams.
- Understand the connection between force and motion.

◀ Looking Back

To master the material introduced in this chapter, you must understand how acceleration is determined and how vectors are used. Please review:

- Section 1.5 Acceleration.
- Section 3.2 Properties of vectors.

5.1 Force

If you kick a ball, it rolls across the floor. If you pull on a door handle, the door opens. You know, from many years of experience, that some sort of *force* is required to move these objects. Our goal is to understand *why* motion occurs, and the observation that force and motion are related is a good place to start.

The two major issues that this chapter will examine are:

- What is a force?
- What is the connection between force and motion?

We begin with the first of these questions in the table below.

What is a force?

A force is a push or a pull.

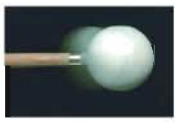
Our commonsense idea of a **force** is that it is a *push* or a *pull*. We will refine this idea as we go along, but it is an adequate starting point. Notice our careful choice of words: We refer to "*a* force," rather than simply "force." We want to think of a force as a very specific *action*, so that we can talk about a single force or perhaps about two or three individual forces that we can clearly distinguish. Hence the concrete idea of "a force" acting on an object.

A force acts on an object.

Implicit in our concept of force is that a **force acts on an object.** In other words, pushes and pulls are applied *to* something—an object. From the object's perspective, it has a force *exerted* on it. Forces do not exist in isolation from the object that experiences them.

A force requires an agent.

Every force has an **agent,** something that acts or exerts power. That is, a force has a specific, identifiable *cause*. As you throw a ball, it is your hand, while in contact with the ball, that is the agent or the cause of the force exerted on the ball. *If* a force is being exerted on an object, you must be able to identify a specific cause (i.e., the agent) of that force. Conversely, a force is not exerted on an object *unless* you can identify a specific cause or agent. Although this idea may seem to be stating the obvious, you will find it to be a powerful tool for avoiding some common misconceptions about what is and is not a force.



A force is a vector.

If you push an object, you can push either gently or very hard. Similarly, you can push either left or right, up or down. To quantify a push, we need to specify both a magnitude *and* a direction. It should thus come as no surprise that a force is a vector quantity. The general symbol for a force is the vector symbol $\vec{F}$. The size or strength of a force is its magnitude F.

A force can be either a contact force . . .

There are two basic classes of forces, depending on whether the agent touches the object or not. **Contact forces** are forces that act on an object by touching it at a point of contact. The bat must touch the ball to hit it. A string must be tied to an object to pull it. The majority of forces that we will examine are contact forces.

. . . or a long-range force.

Long-range forces are forces that act on an object without physical contact. Magnetism is an example of a long-range force. You have undoubtedly held a magnet over a paper clip and seen the paper clip leap up to the magnet. A coffee cup released from your hand is pulled to the earth by the long-range force of gravity.

Let's summarize these ideas as our definition of force:

- A force is a push or a pull on an object.
- A force is a vector. It has both a magnitude and a direction.
- A force requires an agent. Something does the pushing or pulling.
- A force is either a contact force or a long-range force. Gravity is the only long-range force we will deal with until much later in the book.

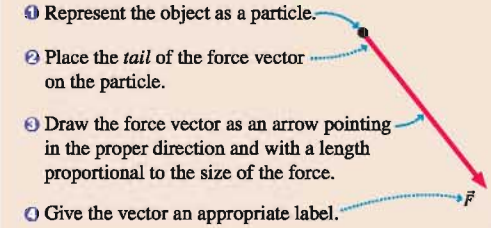
NOTE ▶ In the particle model, objects cannot exert forces on themselves. A force on an object will always have an agent or cause external to the object. Now, there are certainly objects that have internal forces (think of all the forces inside the engine of your car!), but the particle model is not valid if you need to consider those internal forces. If you are going to treat your car as a particle and look only at the overall motion of the car as a whole, that motion will be a consequence of external forces acting on the car. ◀

Force Vectors

We can use a simple diagram to visualize how forces are exerted on objects. Because we are using the particle model, in which objects are treated as points, the process of drawing a force vector is straightforward. Here is how it goes:

TACTICS BOX 5.1 Drawing force vectors MP

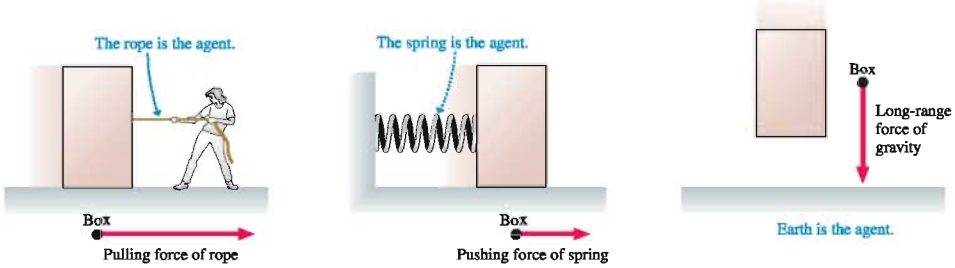
1. Represent the object as a particle.
2. Place the *tail* of the force vector on the particle.
3. Draw the force vector as an arrow pointing in the proper direction and with a length proportional to the size of the force.
4. Give the vector an appropriate label.



Step 2 may seem contrary to what a "push" should do, but recall that moving a vector does not change it as long as the length and angle do not change. The vector $\vec{F}$ is the same regardless of whether the tail or the tip is placed on the particle. Our reason for using the tail will become clear when we consider how to combine several forces.

FIGURE 5.1 shows three examples of force vectors. One is a push, one a pull, and one a long-range force, but in all three the *tail* of the force vector is placed on the particle representing the object.

FIGURE 5.1 Three examples of forces and their vector representations



Combining Forces

FIGURE 5.2a shows a box being pulled by two ropes, each exerting a force on the box. How will the box respond? Experimentally, we find that when several forces $\vec{F}_1$, $\vec{F}_2$, $\vec{F}_3$, . . . are exerted on an object, they combine to form a **net force** given by the *vector* sum of *all* the forces:

$$\vec{F}_{\text{net}} \equiv \sum_{i=1}^{N} \vec{F}_i = \vec{F}_1 + \vec{F}_2 + \cdots + \vec{F}_N \qquad (5.1)$$

Recall that $\equiv$ is the symbol meaning "is defined as." Mathematically, this summation is called a **superposition of forces.** The net force is sometimes called the *resultant force.* FIGURE 5.2b shows the net force on the box.

STOP TO THINK 5.1 Two of the three forces exerted on an object are shown. The net force points to the left. Which is the missing third force?

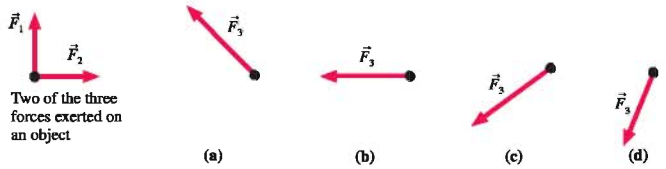
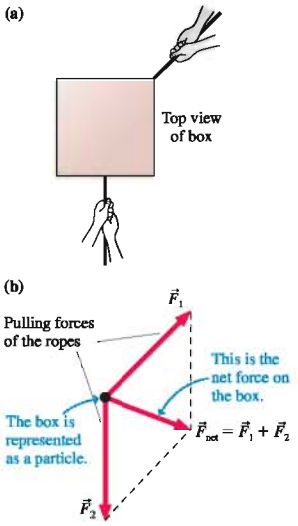


FIGURE 5.2 Two forces applied to a box.



5.2 A Short Catalog of Forces

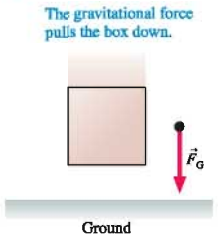
There are many forces we will deal with over and over. This section will introduce you to some of them. Many of these forces have special symbols. As you learn the major forces, be sure to learn the symbol for each.

Gravity

A falling rock is pulled toward the earth by the long-range force of gravity. Gravity—the only long-range force we will encounter in the next few chapters—keeps you in your chair, keeps the planets in their orbits around the sun, and shapes the large-scale structure of the universe. We'll have a thorough look at gravity in Chapter 13. For now we'll concentrate on objects on or near the surface of the earth (or other planet).

The gravitational pull of a planet on an object on or near the surface is called the **gravitational force.** The agent for the gravitational force is the *entire planet* pulling on the object. Gravity acts on *all* objects, whether moving or at rest. The symbol for gravitational force is $\vec{F}_G$. The gravitational force vector always points vertically downward, as shown in FIGURE 5.3.

FIGURE 5.3 Gravity.



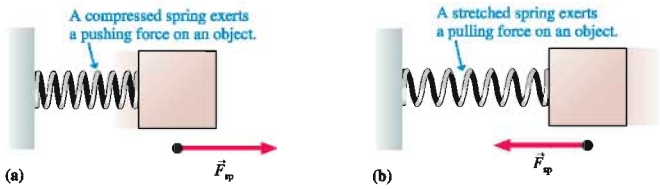
NOTE ▶ We often refer to "the weight" of an object. For an object at rest on the surface of a planet, its weight is simply the magnitude F_G of the gravitational force. However, weight and gravitational force are not the same thing, nor is weight the same as mass. We will briefly examine mass later in the chapter, and we'll explore the rather subtle connections among gravity, weight, and mass in Chapter 6. ◀

Spring Force

Springs exert one of the most common contact forces. A spring can either push (when compressed) or pull (when stretched). FIGURE 5.4 on the next page shows the **spring force,** for which we use the symbol $\vec{F}_{\text{sp}}$. In both cases, pushing and pulling, the tail of the force vector is placed on the particle in the force diagram.

A stretched spring exerts a force on an object.

FIGURE 5.4 The spring force.



Although you may think of a spring as a metal coil that can be stretched or compressed, this is only one type of spring. Hold a ruler, or any other thin piece of wood or metal, by the ends and bend it slightly. It flexes. When you let go, it "springs" back to its original shape. This is just as much a spring as is a metal coil.

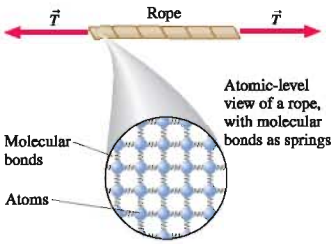
Tension Force

FIGURE 5.5 Tension.

When a string or rope or wire pulls on an object, it exerts a contact force that we call the **tension force,** represented by a capital $\vec{T}$. The direction of the tension force is always in the direction of the string or rope, as you can see in FIGURE 5.5. The commonplace reference to "the tension" in a string is an informal expression for T, the size or magnitude of the tension force.

NOTE ▶ Tension is represented by the symbol T. This is logical, but there's a risk of confusing the tension T with the identical symbol T for the period of a particle in circular motion. The number of symbols used in science and engineering far exceeds the number of letters in the English alphabet. Even after borrowing from the Greek alphabet, scientists inevitably use some letters several times to represent entirely different quantities. The use of T is the first time we've run into this problem, but it won't be the last. You must be alert to the *context* of a symbol's use to deduce its meaning. ◀

FIGURE 5.6 An atomic model of tension.



If you were to use a very powerful microscope to look inside a rope, you would "see" that it is made of *atoms* joined together by *molecular bonds.* Molecular bonds are not rigid connections between the atoms. They are more accurately thought of as tiny *springs* holding the atoms together, as in FIGURE 5.6. Pulling on the ends of a string or rope stretches the molecular springs ever so slightly. The tension within a rope and the tension force experienced by an object at the end of the rope are really the net spring force being exerted by billions and billions of microscopic springs.

This atomic-level view of tension introduces a new idea: a microscopic **atomic model** for understanding the behavior and properties of macroscopic objects. It is a *model* because atoms and molecular bonds aren't really little balls and springs. We're using macroscopic concepts—balls and springs—to understand atomic-scale phenomena that we cannot directly see or sense. This is a good model for explaining the elastic properties of materials, but it would not necessarily be a good model for explaining other phenomena. We will frequently use atomic models to obtain a deeper understanding of our observations.

Normal Force

If you sit on a bed, the springs in the mattress compress and, as a consequence of the compression, exert an upward force on you. Stiffer springs would show less compression but still exert an upward force. The compression of extremely stiff springs might be measurable only by sensitive instruments. Nonetheless, the springs would compress ever so slightly and exert an upward spring force on you.

FIGURE 5.7 shows an object resting on top of a sturdy table. The table may not visibly flex or sag, but—just as you do to the bed—the object compresses the molecular springs in the table. The size of the compression is very small, but it is not zero. As a consequence, the compressed molecular springs *push upward* on the object. We say that "the table" exerts the upward force, but it is important to understand that the pushing is *really* done by molecular springs. Similarly, an object resting on the ground compresses the molecular springs holding the ground together and, as a consequence, the ground pushes up on the object.

We can extend this idea. Suppose you place your hand on a wall and lean against it, as shown in FIGURE 5.8. Does the wall exert a force on your hand? As you lean, you compress the molecular springs in the wall and, as a consequence, they push outward against your hand. So the answer is yes, the wall does exert a force on you.

The force the table surface exerts is vertical; the force the wall exerts is horizontal. In all cases, the force exerted on an object that is pressing against a surface is in a direction *perpendicular* to the surface. Mathematicians refer to a line that is perpendicular to a surface as being *normal* to the surface. In keeping with this terminology, we define the **normal force** as the force exerted by a surface (the agent) against an object that is pressing against the surface. The symbol for the normal force is $\vec{n}$.

We're not using the word *normal* to imply that the force is an "ordinary" force or to distinguish it from an "abnormal force." A surface exerts a force *perpendicular* (i.e., normal) to itself as the molecular springs press *outward*. FIGURE 5.9 shows an object on an inclined surface, a common situation. Notice how the normal force $\vec{n}$ is perpendicular to the surface.

We have spent a lot of time describing the normal force because many people have a difficult time understanding it. The normal force is a very real force arising from the very real compression of molecular bonds. It is in essence just a spring force, but one exerted by a vast number of microscopic springs acting at once. The normal force is responsible for the "solidness" of solids. It is what prevents you from passing right through the chair you are sitting in and what causes the pain and the lump if you bang your head into a door. Your head can then tell you that the force exerted on it by the door was very real!

FIGURE 5.7 An atomic model of the force exerted by a table.

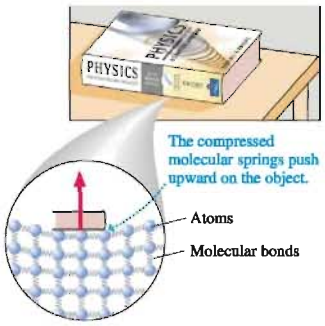


FIGURE 5.8 The wall pushes outward.

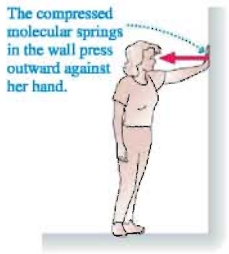
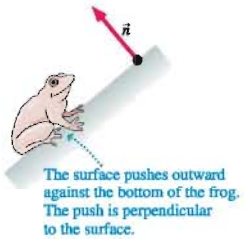


FIGURE 5.9 The normal force.



Friction

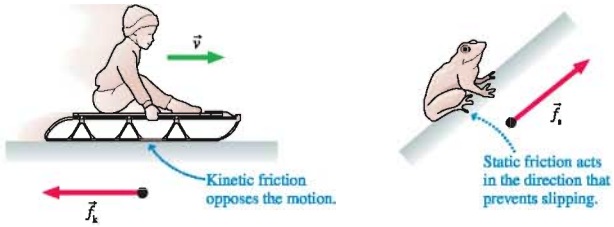
You've certainly observed that a rolling or sliding object, if not pushed or propelled, slows down and eventually stops. You've probably discovered that you can slide better across a sheet of ice than across asphalt. And you also know that most objects stay in place on a table without sliding off even if the table isn't absolutely level. The force responsible for these sorts of behavior is **friction.** The symbol for friction is a lowercase $\vec{f}$.

Friction, like the normal force, is exerted by a surface. But whereas the normal force is perpendicular to the surface, the friction force is always *tangent* to the surface. On a microscopic level, friction arises as atoms from the object and atoms on the surface run into each other. The rougher the surface is, the more these atoms are forced into close proximity and, as a result, the larger the friction force. We will develop a simple model of friction in the next chapter that will be sufficient for our needs. For now, it is useful to distinguish between two kinds of friction:

- *Kinetic friction,* denoted $\vec{f}_k$, appears as an object slides across a surface. This is a force that "opposes the motion," meaning that the friction force vector $\vec{f}_k$ points in a direction opposite the velocity vector $\vec{v}$ (i.e., "the motion").
- *Static friction,* denoted $\vec{f}_s$, is the force that keeps an object "stuck" on a surface and prevents its motion. Finding the direction of $\vec{f}_s$ is a little trickier than finding it for $\vec{f}_k$. Static friction points opposite the direction in which the object *would* move if there were no friction. That is, it points in the direction necessary to *prevent* motion.

FIGURE 5.10 on the next page shows examples of kinetic and static friction.

FIGURE 5.10 Kinetic and static friction.



NOTE ▶ A surface exerts a kinetic friction force when an object moves *relative to* the surface. A package on a conveyor belt is in motion, but it does not experience a kinetic friction force because it is not moving relative to the belt. So to be precise, we should say that the kinetic friction force points opposite to an object's motion *relative to* a surface. ◀

Drag

FIGURE 5.11 Air resistance is an example of drag.

Friction at a surface is one example of a *resistive force*, a force that opposes or resists motion. Resistive forces are also experienced by objects moving through fluids—gases and liquids. The resistive force of a fluid is called **drag** and is symbolized as $\vec{D}$. Drag, like kinetic friction, points opposite the direction of motion. FIGURE 5.11 shows an example of drag.

Drag can be a large force for objects moving at high speeds or in dense fluids. Hold your arm out the window as you ride in a car and feel how the air resistance against it increases rapidly as the car's speed increases. Drop a lightweight object into a beaker of water and watch how slowly it settles to the bottom. In both cases the drag force is very significant.

For objects that are heavy and compact, that move in air, and whose speed is not too great, the drag force of air resistance is fairly small. To keep things as simple as possible, **you can neglect air resistance in all problems unless a problem explicitly asks you to include it.** The error introduced into calculations by this approximation is generally pretty small. This textbook will not consider objects moving in liquids.

Thrust

FIGURE 5.12 Thrust force on a rocket.

A jet airplane obviously has a force that propels it forward during takeoff. Likewise for the rocket being launched in FIGURE 5.12. This force, called **thrust,** occurs when a jet or rocket engine expels gas molecules at high speed. Thrust is a contact force, with the exhaust gas being the agent that pushes on the engine. The process by which thrust is generated is rather subtle, and we will postpone a full discussion until we study Newton's third law in Chapter 7. For now, we will treat thrust as a force opposite the direction in which the exhaust gas is expelled. There's no special symbol for thrust, so we will call it $\vec{F}_{\text{thrust}}$.

Electric and Magnetic Forces

Electricity and magnetism, like gravity, exert long-range forces. The forces of electricity and magnetism act on charged particles. We will study electric and magnetic forces in detail in Part VI of this textbook. For now, it is worth noting that the forces holding molecules together—the molecular bonds—are not actually tiny springs. Atoms and molecules are made of charged particles—electrons and protons—and what we call a molecular bond is really an electric force between these particles. So when we say that the normal force and the tension force are due to "molecular springs," or that friction is due to atoms running into each other, what we're really saying is that these forces, at the most fundamental level, are actually electric forces between the charged particles in the atoms.

FIGURE 5.10 Kinetic and static friction.

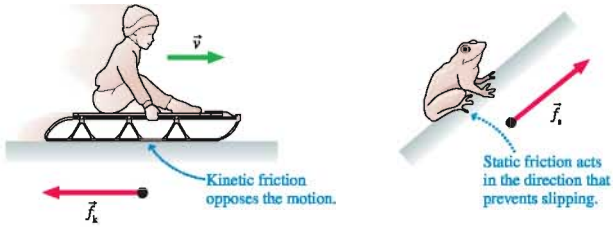


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FIGURE 5.11 Air resistance is an example of drag.

Friction at a surface is one example of a *resistive force*, a force that opposes or resists motion. Resistive forces are also experienced by objects moving through fluids—gases and liquids. The resistive force of a fluid is called **drag** and is symbolized as $\vec{D}$. Drag, like kinetic friction, points opposite the direction of motion. FIGURE 5.11 shows an example of drag.

Drag can be a large force for objects moving at high speeds or in dense fluids. Hold your arm out the window as you ride in a car and feel how the air resistance against it increases rapidly as the car's speed increases. Drop a lightweight object into a beaker of water and watch how slowly it settles to the bottom. In both cases the drag force is very significant.

For objects that are heavy and compact, that move in air, and whose speed is not too great, the drag force of air resistance is fairly small. To keep things as simple as possible, **you can neglect air resistance in all problems unless a problem explicitly asks you to include it.** The error introduced into calculations by this approximation is generally pretty small. This textbook will not consider objects moving in liquids.

Thrust

FIGURE 5.12 Thrust force on a rocket.

A jet airplane obviously has a force that propels it forward during takeoff. Likewise for the rocket being launched in FIGURE 5.12. This force, called **thrust,** occurs when a jet or rocket engine expels gas molecules at high speed. Thrust is a contact force, with the exhaust gas being the agent that pushes on the engine. The process by which thrust is generated is rather subtle, and we will postpone a full discussion until we study Newton's third law in Chapter 7. For now, we will treat thrust as a force opposite the direction in which the exhaust gas is expelled. There's no special symbol for thrust, so we will call it $\vec{F}_{\text{thrust}}$.

Electric and Magnetic Forces

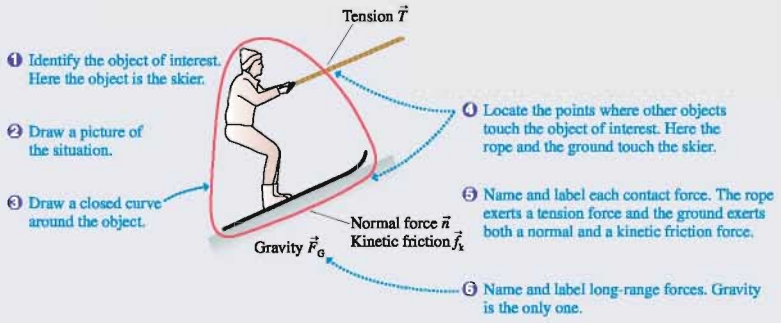
Electricity and magnetism, like gravity, exert long-range forces. The forces of electricity and magnetism act on charged particles. We will study electric and magnetic forces in detail in Part VI of this textbook. For now, it is worth noting that the forces holding molecules together—the molecular bonds—are not actually tiny springs. Atoms and molecules are made of charged particles—electrons and protons—and what we call a molecular bond is really an electric force between these particles. So when we say that the normal force and the tension force are due to "molecular springs," or that friction is due to atoms running into each other, what we're really saying is that these forces, at the most fundamental level, are actually electric forces between the charged particles in the atoms.

EXAMPLE 5.2 Forces on a skier

A skier is being towed up a snow-covered hill by a tow rope. What forces are being exerted on the skier?

VISUALIZE

FIGURE 5.14 Forces on a skier.



NOTE ▶ You might have expected two friction forces and two normal forces in Example 5.2, one on each ski. Keep in mind, however, that we're working within the particle model, which represents the skier by a single point. A particle has only one contact with the ground, so there is a single normal force and a single friction force. The particle model is valid if we want to analyze the translational motion of the skier as a whole, but we would have to go beyond the particle model to find out what happens to each ski. ◀

Now that you're getting the hang of this, the next example is meant to look much more like a sketch you should make when asked to identify forces in a homework problem.

EXAMPLE 5.3 Forces on a rocket

A rocket is being launched to place a new satellite in orbit. Air resistance is not negligible. What forces are being exerted on the rocket?

VISUALIZE

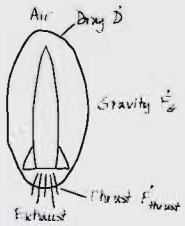


FIGURE 5.15 Forces on a rocket.

STOP TO THINK 5.2 You've just kicked a rock, and it is now sliding across the ground about 2 meters in front of you. Which of these forces act on the rock? List all that apply.

a. Gravity, acting downward.
b. The normal force, acting upward.
c. The force of the kick, acting in the direction of motion.
d. Friction, acting opposite the direction of motion.
e. Air resistance, acting opposite the direction of motion.

5.4 What Do Forces Do? A Virtual Experiment

The fundamental question is: How does an object move when a force is exerted on it? The only way to answer this question is to do experiments. To do experiments, however, we need a way to reproduce the same amount of force again and again.

Let's conduct a "virtual experiment," one you can easily visualize. Imagine using your fingers to stretch a rubber band to a certain length—say 10 centimeters—that you can measure with a ruler. We'll call this the *standard length*. FIGURE 5.16 shows the idea. You know that a stretched rubber band exerts a force because your fingers *feel* the pull. Furthermore, this is a reproducible force. The rubber band exerts the same force every time you stretch it to the standard length. We'll call this the *standard force F*.

Not surprisingly, two identical rubber bands, each stretched to the standard length, exert twice the pull of one rubber band: $F_{net} = 2F$. N side-by-side rubber bands, each pulled to the standard length, will exert N times the standard force: $F_{net} = NF$.

Now we're ready to start the virtual experiment. Imagine an object to which you can attach rubber bands, such as a block of wood with a hook. If you attach a rubber band and stretch it to the standard length, the object experiences the same force F as did your finger. N rubber bands attached to the object will exert N times the force of one rubber band. The rubber bands give us a way of applying a known and reproducible force to an object.

Our task is to measure the object's motion in response to these forces. Imagine using the rubber bands to pull the object across a horizontal table. Friction between the object and the surface might affect our results, so let's just eliminate friction. This is, after all, a virtual experiment! (In practice you could nearly eliminate friction by pulling a smooth block over a smooth sheet of ice or by supporting the object on a cushion of air.)

If you stretch the rubber band and then release the object, it moves toward your hand. But as it does so, the rubber band gets shorter and the pulling force decreases. To keep the pulling force constant, you must *move your hand* at just the right speed to keep the length of the rubber band from changing! FIGURE 5.17 shows the experiment being carried out. Once the motion is complete, you can use motion diagrams (made from movie frames from a camera) and kinematics to analyze the object's motion.

FIGURE 5.16 A reproducible force.

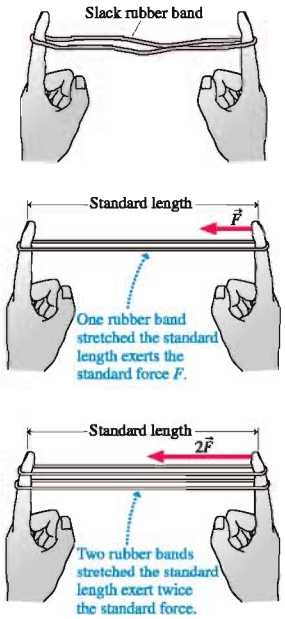


FIGURE 5.17 Measuring the motion of an object that is pulled with a constant force.

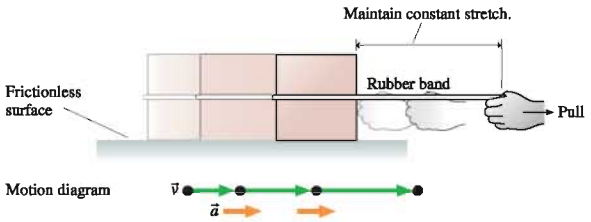


TABLE 5.1 Acceleration due to an increasing force

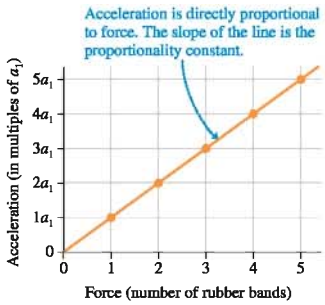
Rubber bands	Force	Acceleration
1	F	a_1
2	$2F$	$a_2 = 2a_1$
3	$3F$	$a_3 = 3a_1$
⋮	⋮	⋮
N	NF	$a_N = Na_1$

The first important finding of this experiment is that **an object pulled with a constant force moves with a constant acceleration.** This finding could not have been anticipated in advance. It's conceivable that the object would speed up for a while, then move with a steady speed. Or that it would continue to speed up, but that the *rate* of increase, the acceleration, would steadily decline. These are conceivable motions, but they're not what happens. Instead, the object accelerates *with a constant acceleration* for as long as you pull it with a constant force.

What happens if you increase the force by using several rubber bands? To find out, use 2 rubber bands. Stretch both to the standard length to double the force, then measure the acceleration. Then measure the acceleration due to 3 rubber bands, then 4, and so on. Table 5.1 shows the results of this experiment. You can see that doubling the force causes twice the acceleration, tripling the force causes three times the acceleration, and so on.

FIGURE 5.18 is a graph of the data. Force is the independent variable, the one you can control, so we've placed force on the horizontal axis to make an acceleration-versus-

FIGURE 5.18 Graph of acceleration versus force.



force graph. The graph reveals our second important finding, that **the acceleration is directly proportional to the force.** This result can be written

$$a = cF \tag{5.2}$$

where c, called the *proportionality constant,* is the slope of the graph.

MATHEMATICAL ASIDE **Proportionality and proportional reasoning**

The concept of **proportionality** arises frequently in physics. A quantity symbolized by u is *proportional* to another quantity symbolized by v if

$$u = cv$$

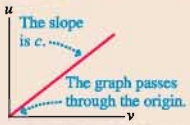
where c (which might have units) is called the **proportionality constant.** This relationship between u and v is often written

$$u \propto v$$

where the symbol $\propto$ means "is proportional to."

If v is doubled to $2v$, then u is doubled to $c(2v) = 2(cv) = 2u$. In general, if v is changed by any factor f, then u changes by the same factor. This is the essence of what we *mean* by proportionality.

A graph of u versus v is a straight line *passing through the origin* (i.e., the y-intercept is zero) with slope $= c$. Notice that proportionality is a much more specific relationship between u and v than mere linearity. The linear equation $u = cv + b$ has a straight-line graph, but it doesn't pass through the origin (unless b happens to be zero) and doubling v does not double u.



u is proportional to v.

If $u \propto v$, then $u_1 = cv_1$ and $u_2 = cv_2$. Dividing the second equation by the first, we find

$$\frac{u_2}{u_1} = \frac{v_2}{v_1}$$

By working with *ratios,* we can deduce information about u without needing to know the value of c. (This would not be true if the relationship were merely linear.) This is called **proportional reasoning.**

Proportionality is not limited to being linearly proportional. The graph on the left below shows that u is clearly not proportional to w. But a graph of u versus $1/w^2$ *is* a straight line passing through the origin, thus, in this case, u is proportional to $1/w^2$, or $u \propto 1/w^2$. We would say that "u is proportional to the inverse square of w."

u is proportional to the inverse square of w.

EXAMPLE u is proportional to the inverse square of w. By what factor does u change if w is tripled?

SOLUTION This is an opportunity for proportional reasoning; we don't need to know the proportionality constant. If u is proportional to $1/w^2$, then

$$\frac{u_2}{u_1} = \frac{1/w_2^2}{1/w_1^2} = \frac{1}{w_2^2/w_1^2} = \left(\frac{1}{w_2/w_1}\right)^2$$

Tripling w, for which $w_2/w_1 = 3$, changes u to

$$u_2 = \left(\frac{1}{w_2/w_1}\right)^2 u_1 = \left(\frac{1}{3}\right)^2 u_1 = \frac{1}{9}u_1$$

Tripling w causes u to become $\frac{1}{9}$ of its original value.

Many *Student Workbook* and end-of-chapter homework questions will require proportional reasoning. It's an important skill to learn.

TABLE 5.2 Acceleration with different numbers of objects

Number of objects	Acceleration
1	a_1
2	$a_2 = \frac{1}{2}a_1$
3	$a_3 = \frac{1}{3}a_1$
⋮	⋮
N	$a_N = \frac{1}{N}a_1$

The final question for our virtual experiment is: How does the acceleration depend on the size of the object? (The "size" of an object is somewhat ambiguous. We'll be more precise below.) To find out, glue the original object and an identical copy together, and then, applying the *same force* as you applied to the original, single object, measure the acceleration of this new object. Doing several such experiments, applying the same force to each object, would give you the results shown in Table 5.2. An object twice the size of the original has only half the acceleration of the original object when both are subjected to the same force.

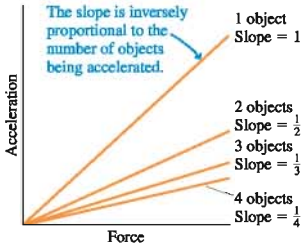
FIGURE 5.19 adds these results to the graph of Figure 5.18. You can see that the proportionality constant c between acceleration and force—the slope of the line—changes with the size of the object. The graph for an object twice the size of the original is a line with half the slope. It may seem surprising that larger objects have smaller slopes, so you'll want to think about this carefully.

Mass

Now, "twice the size" is a little vague; we could mean the object's external dimensions or some other measure. Although *mass* is a common word, we've avoided the term so far because we first need to define what mass is. Because we made the larger objects in our experiment from the same material as the original object, an object twice the size has twice as many atoms—twice the amount of matter—as the original. Thus it should come as no surprise that it has twice the mass as the original. Loosely speaking, **an object's mass is a measure of the amount of matter it contains.** This is certainly our everyday meaning of *mass,* but it is not yet a precise definition.

Figure 5.19 shows that an object with twice the amount of matter as the original accelerates only half as quickly if both experience the same force. An object with N times as much matter has only $\frac{1}{N}$ of the original acceleration. The more matter an object has, the more it *resists* accelerating in response to a force. You're familiar with this idea: Your car is much harder to push than your bicycle. The tendency of an object to resist a *change* in its velocity (i.e., to resist acceleration) is called **inertia.** Figure 5.19 tells us that larger objects have more inertia than smaller objects of the same material.

FIGURE 5.19 Acceleration-versus-force graphs for objects of different sizes.



We can make this idea precise by defining the **inertial mass** m of an object to be

$$m \equiv \frac{1}{\text{slope of the acceleration-versus-force graph}} = \frac{F}{a}$$

(Notice that this is another *operational definition.*) We usually refer to the inertial mass as simply "the mass." Mass is an *intrinsic* property of an object. It is the property that determines how an object accelerates in response to an applied force.

We can now answer the question with which we started: How does an object move when a force is exerted on it? Figure 5.18 showed that the acceleration is directly proportional to the force, a conclusion that we wrote in Equation 5.2 with the unspecified proportionality constant c. Now we see that c, the slope of the acceleration-versus-force graph, is the inverse of the inertial mass m. Thus we've found that a force of magnitude F causes an object of mass m to accelerate with

$$a = \frac{F}{m} \tag{5.3}$$

A force causes an object to *accelerate!* Furthermore, the size of the acceleration is directly proportional to the size of the force and inversely proportional to the object's mass.

STOP TO THINK 5.3 Two rubber bands stretched to the standard length cause an object to accelerate at 2 m/s^2. Suppose another object with twice the mass is pulled by four rubber bands stretched to the standard length. The acceleration of this second object is

a. 1 m/s^2 b. 2 m/s^2 c. 4 m/s^2 d. 8 m/s^2 e. 16 m/s^2

Hint: Use proportional reasoning.

5.5 Newton's Second Law

Equation 5.3 is an important finding, but our experiment was limited to looking at an object's response to a single applied force. Realistically, an object is likely to be subjected to several distinct forces $\vec{F}_1, \vec{F}_2, \vec{F}_3, \ldots$ that may point in different directions. What happens then? In that case, it is found experimentally that the acceleration is determined by the *net* force.

Newton was the first to recognize the connection between force and motion. This relationship is known today as Newton's second law.

Newton's second law An object of mass m subjected to forces $\vec{F}_1$, $\vec{F}_2$, $\vec{F}_3$, . . . will undergo an acceleration $\vec{a}$ given by

$$\vec{a} = \frac{\vec{F}_{net}}{m} \tag{5.4}$$

where the net force $\vec{F}_{net} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \cdots$ is the vector sum of all forces acting on the object. The acceleration vector $\vec{a}$ points in the same direction as the net force vector $\vec{F}_{net}$.

The significance of Newton's second law cannot be overstated. There was no reason to suspect that there should be any simple relationship between force and acceleration. Yet there it is, a simple but exceedingly powerful equation relating the two.

The critical idea is that **an object accelerates in the direction of the net force vector $\vec{F}_{net}$.** It's also worth noting that the object responds only to the forces it feels *at this instant.* The object has no will or intent to act on its own, nor does it have any memory of forces that may have been exerted at earlier times.

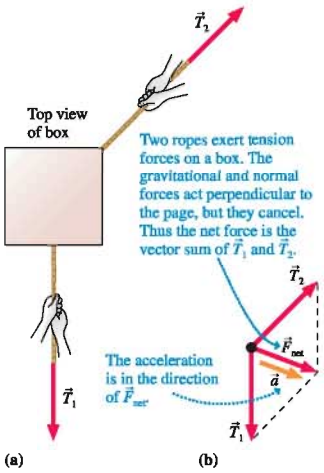
We can rewrite Newton's second law in the form

$$\vec{F}_{net} = m\vec{a} \tag{5.5}$$

which is how you'll see it presented in many textbooks. Equations 5.3 and 5.4 are mathematically equivalent, but Equation 5.4 better describes the central idea of Newtonian mechanics: A force applied to an object causes the object to accelerate.

NOTE ▶ Be careful not to think that one force "overcomes" the others to determine the motion. Forces are not in competition with each other! It is $\vec{F}_{net}$, the sum of *all* the forces, that determines the acceleration $\vec{a}$. ◀

FIGURE 5.20 Acceleration of a pulled box.



As an example, FIGURE 5.20a shows a box being pulled by two ropes. The ropes exert tension forces $\vec{T}_1$ and $\vec{T}_2$ on the box. FIGURE 5.20b represents the box as a particle, shows the forces acting on the box, and adds them graphically to find the net force $\vec{F}_{net}$. The box will accelerate in the direction of $\vec{F}_{net}$ with an acceleration of magnitude

$$\vec{a} = \frac{\vec{F}_{net}}{m} = \frac{\vec{T}_1 + \vec{T}_2}{m}$$

NOTE ▶ The acceleration is *not* $(T_1 + T_2)/m$. You must add the forces as *vectors,* not merely add their magnitudes as scalars. ◀

Units of Force

Because $\vec{F}_{net} = m\vec{a}$, the units of force must be mass units multiplied by acceleration units. We've previously specified the SI unit of mass as the kilogram. We can now define the basic unit of force as "the force that causes a 1 kg mass to accelerate at 1 m/s^2." From the second law, this force is

$$1 \text{ basic unit of force} \equiv 1 \text{ kg} \times 1\,\frac{\text{m}}{\text{s}^2} = 1\,\frac{\text{kg m}}{\text{s}^2}$$

This basic unit of force is called a newton:

One **newton** is the force that causes a 1 kg mass to accelerate at 1 m/s^2. The abbreviation for newton is N. Mathematically, 1 N = 1 kg m/s^2.

The newton is a *secondary unit,* meaning that it is defined in terms of the *primary units* of kilograms, meters, and seconds. We will introduce other secondary units as needed.

It is important to develop a feeling for what the sizes of forces should be. Table 5.3 shows some typical forces. As you can see, "typical" forces on "typical" objects are likely to be in the range 0.01–10,000 N. Forces less than 0.01 N are too small to consider unless you are dealing with very small objects. Forces greater than 10,000 N would make sense only if applied to very massive objects.

TABLE 5.3 Approximate magnitude of some typical forces

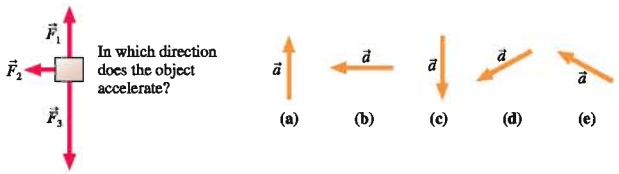
Force	Approximate magnitude (newtons)
Weight of a U.S. quarter	0.05
Weight of a 1 pound object	5
Weight of a 110 pound person	500
Propulsion force of a car	5,000
Thrust force of a rocket motor	5,000,000

Forces Are Interactions

There's one more important aspect of forces. If you push against a door (the object) to close it, the door pushes back against your hand (the agent). If a tow rope pulls on a car (the object), the car pulls back on the rope (the agent). In general, if an agent exerts a force on an object, the object exerts a force on the agent. We really need to think of a force as an *interaction* between two objects. This idea is captured in Newton's third law—that for every action there is an equal but opposite reaction.

Although the interaction perspective is a more exact way to view forces, it adds complications that we would like to avoid for now. Our approach will be to start by focusing on how a single object responds to forces exerted on it. Then, in Chapter 7, we'll return to Newton's third law and the larger issue of how two or more objects interact with each other.

STOP TO THINK 5.4 Three forces act on an object. In which direction does the object accelerate?



5.6 Newton's First Law

As we remarked earlier, Aristotle and his contemporaries in the world of ancient Greece were very interested in motion. One question they asked was: What is the "natural state" of an object if left to itself? It does not take an expensive research program to see that every moving object on earth, if left to itself, eventually comes to rest. Aristotle concluded that the natural state of an earthly object is to be at rest. An object at rest requires no explanation; it is doing precisely what comes naturally to it. A moving object, though, is not in its natural state and thus requires an explanation: Why is this object moving? What keeps it going and prevents it from being in its natural state?

Galileo reopened the question of the "natural state" of objects. He suggested focusing on the *limiting case* in which resistance to the motion (e.g., friction or air resistance) is zero. Many careful experiments in which he minimized the influence of friction led Galileo to a conclusion that was in sharp contrast to Aristotle's belief that rest is an object's natural state.

Galileo found that an external influence (i.e., a force) is needed to make an object accelerate—to *change* its velocity. In particular, a force is needed to put an object in motion. But, in the absence of friction or air resistance, a moving object continues to

move along a straight line forever with no loss of speed. In other words, the natural state of an object—its behavior if free of external influences—is *uniform motion* with constant velocity! This does not happen in practice because friction or air resistance prevents the object from being left alone. "At rest" has no special significance in Galileo's view of motion; it is simply uniform motion that happens to have $\vec{v} = \vec{0}$.

It was left to Newton to generalize this result, and today we call it Newton's first law of motion.

Newton's first law An object that is at rest will remain at rest, or an object that is moving will continue to move in a straight line with constant velocity, if and only if the net force acting on the object is zero.

Newton's first law is also known as the *law of inertia*. If an object is at rest, it has a tendency to stay at rest. If it is moving, it has a tendency to continue moving with the *same velocity*.

NOTE ▶ The first law refers to *net* force. An object can remain at rest, or can move in a straight line with constant velocity, even though forces are exerted on it as long as the *net* force is zero. ◀

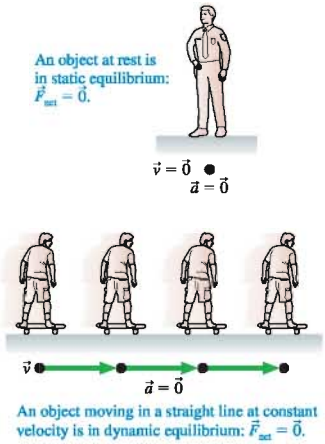
Notice the "if and only if" aspect of Newton's first law. If an object is at rest or moves with constant velocity, then we can conclude that there is no net force acting on it. Conversely, if no net force is acting on it, we can conclude that the object will have constant velocity, not just constant speed. The direction remains constant, too!

An object on which the net force is zero, $\vec{F}_{\text{net}} = \vec{0}$, is said to be in **mechanical equilibrium.** According to Newton's first law, there are two distinct forms of mechanical equilibrium:

1. The object is at rest. This is **static equilibrium.**
2. The object is moving in a straight line with constant velocity. This is **dynamic equilibrium.**

Two examples of mechanical equilibrium are shown in FIGURE 5.21. Both share the common feature that the acceleration is zero: $\vec{a} = \vec{0}$.

FIGURE 5.21 Two examples of mechanical equilibrium.



An object moving in a straight line at constant velocity is in dynamic equilibrium: $\vec{F}_{\text{net}} = \vec{0}$.

What Good Is Newton's First Law?

The first law completes our definition of force. It answers the question: What is a force? If an "influence" on an object causes the object's velocity to change, the influence is a force.

Newton's first law changes the question the ancient Greeks were trying to answer: What causes an object to move? Newton's first law says **no cause is needed for an object to move!** Uniform motion is the object's natural state. Nothing at all is required for it to remain in that state. The proper question, according to Newton, is: What causes an object to *change* its velocity? Newton, with Galileo's help, also gave us the answer. **A *force* is what causes an object to change its velocity.**

The preceding paragraph contains the essence of Newtonian mechanics. This new perspective on motion, however, is often contrary to our common experience. We all know perfectly well that you must keep pushing an object—exerting a force on it—to keep it moving. Newton is asking us to change our point of view and to consider motion *from the object's perspective* rather than from our personal perspective. As far as the object is concerned, our push is just one of several forces acting on it. Others might include friction, air resistance, or gravity. Only by knowing the *net* force can we determine the object's motion.

Newton's first law may seem to be merely a special case of Newton's second law. After all, the equation $\vec{F}_{\text{net}} = m\vec{a}$ tells us that an object moving with constant velocity ($\vec{a} = \vec{0}$) has $\vec{F}_{\text{net}} = \vec{0}$. The difficulty is that the second law assumes that we already

know what force is. The purpose of the first law is to *identify* a force as something that disturbs a state of equilibrium. The second law then describes how the object responds to this force. Thus from a *logical* perspective, the first law really is a separate statement that must precede the second law. But this is a rather formal distinction. From a pedagogical perspective it is better—as we have done—to use a commonsense understanding of force and start with Newton's second law.

Inertial Reference Frames

If a car stops suddenly, you may be "thrown" into the windshield if you're not wearing your seat belt. You have a very real forward acceleration *relative to the car,* but is there a force pushing you forward? A force is a push or a pull caused by an identifiable agent in contact with the object. Although you *seem* to be pushed forward, there's no agent to do the pushing.

The difficulty—an acceleration without an apparent force—comes from using an inappropriate reference frame. Your acceleration measured in a reference frame attached to the car is not the same as your acceleration measured in a reference frame attached to the ground. Newton's second law says $\vec{F}_{net} = m\vec{a}$. But which $\vec{a}$? Measured in which reference frame?

We define an **inertial reference frame** as a reference frame in which Newton's laws are valid. The first law provides a convenient way to test whether a reference frame is inertial. If $\vec{a} = \vec{0}$ (an object is at rest or moving with constant velocity) only when $\vec{F}_{net} = \vec{0}$, then the reference frame in which $\vec{a}$ is measured is an inertial reference frame.

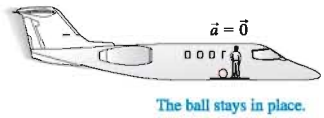
This guy thinks there's a force hurling him into the windshield. What a dummy!

Not all reference frames are inertial reference frames. FIGURE 5.22a shows a physics student cruising at constant velocity in an airplane. If the student places a ball on the floor, it stays there. There are no horizontal forces, and the ball remains at rest relative to the airplane. That is, $\vec{a} = \vec{0}$ in the airplane's reference frame when $\vec{F}_{net} = \vec{0}$. Newton's first law is satisfied, so this airplane is an inertial reference frame.

The physics student in FIGURE 5.22b conducts the same experiment during takeoff. She carefully places the ball on the floor just as the airplane starts to accelerate down the runway. You can imagine what happens. The ball rolls to the back of the plane as the passengers are being pressed back into their seats. Nothing exerts a horizontal contact force on the ball, yet the ball accelerates *in the plane's reference frame.* This violates Newton's first law, so the plane is *not* an inertial reference frame during takeoff.

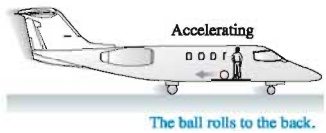
FIGURE 5.22 Reference frames.

(a)



A ball with no horizontal forces stays at rest in an airplane cruising at constant velocity. The airplane is an inertial reference frame.

(b)



The ball rolls to the back of the plane during takeoff. An accelerating plane is not an inertial reference frame.

In the first example, the plane is traveling with constant velocity. In the second, the plane is accelerating. **Accelerating reference frames are not inertial reference frames.** Consequently, Newton's laws are not valid in a reference frame attached to an accelerating object.

The earth is not exactly an inertial reference frame because the earth rotates on its axis and orbits the sun. However, the earth's acceleration is so small that violations of Newton's laws can be measured only in high-precision experiments. We will treat the earth and laboratories attached to the earth as inertial reference frames, an approximation that is exceedingly well justified.

In Chapter 4 we defined inertial reference frames to be those reference frames moving with constant velocity. These are the reference frames in which Newton's laws are valid. Because the earth is (very nearly) an inertial reference frame, the airplane of Figure 5.22a, which is moving at constant velocity relative to the earth, is also an inertial reference frame. But a car braking to a stop is not, so you *cannot* use Newton's laws in the car's reference frame.

To understand the motion of objects in the car, such as the passengers, you need to measure velocities and accelerations *relative to the ground.* From the perspective of an observer on the ground, the body of a passenger in a braking car tries to continue moving forward with constant velocity, exactly as we would expect on the basis of Newton's first law, while his immediate surroundings are decelerating. The passenger is not "thrown" into the windshield. Instead, the windshield runs into the passenger!

Common Misconceptions About Force

It is important to identify correctly all the forces acting on an object. It is equally important not to include forces that do not really exist. We have established a number of criteria for identifying forces; the two critical ones are:

- A force has an agent. Something tangible and identifiable causes the force.
- Forces exist at the point of contact between the agent and the object experiencing the force (except for the few special cases of long-range forces).

There's no "force of motion" or any other forward force on this arrow. It continues to move because of inertia.

We all have had many experiences suggesting that a force is necessary to keep something moving. Consider a bowling ball rolling along on a smooth floor. It is very tempting to think that a horizontal "force of motion" keeps it moving in the forward direction. But if we draw a closed curve around the ball, *nothing contacts it* except the floor. No agent is giving the ball a forward push. According to our definition, then, there is *no* forward "force of motion" acting on the ball. So what keeps it going? Recall our discussion of the first law: *No* cause is needed to keep an object moving at constant velocity. It continues to move forward simply because of its inertia.

One reason for wanting to include a "force of motion" is that we tend to view the problem from our perspective as one of the agents of force. You certainly have to keep pushing to shove a box across the floor at constant velocity. If you stop, it stops. Newton's laws, though, require that we adopt the object's perspective. The box experiences your pushing force in one direction *and* a friction force in the opposite direction. The box moves at constant velocity if the *net* force is zero. This will be true as long as your pushing force exactly balances the friction force. When you stop pushing, the friction force causes an acceleration that slows and stops the box.

A related problem occurs if you throw a ball. A pushing force was indeed required to accelerate the ball *as it was thrown.* But that force disappears the instant the ball loses contact with your hand. The force does not stick with the ball as the ball travels through the air. Once the ball has acquired a velocity, *nothing* is needed to keep it moving with that velocity.

A final difficulty worth noting is the force due to air pressure. You may have learned in an earlier science class that air, like any fluid, exerts forces on objects. Perhaps you learned this idea as "the air presses down with a weight of 15 pounds on every square inch." There is only one error here, but it is a serious one: the word *down.* Air pressure, at sea level, does indeed exert a force of 15 pounds per square inch, but in *all* directions. It presses down on the top of an object, but also inward on the sides and upward on the bottom. For most purposes, the *net* force due to air pressure is zero! The only way to experience an air pressure force is to form a seal around one side of the object and then remove the air, creating a *vacuum.* When you press a suction cup against the wall, you press the air out and the rubber forms a seal that prevents the air from returning. Now the air pressure does hold the suction cup in place! We do not need to be concerned with air pressure until Part III of this book.

5.7 Free-Body Diagrams

Having discussed at length what is and is not a force, we are ready to assemble our knowledge about force and motion into a single diagram called a *free-body diagram.* You will learn in the next chapter how to write the equations of motion directly from the free-body diagram. Solution of the equations is a mathematical exercise—possibly a difficult one, but nonetheless an exercise that could be done by a computer. The *physics* of the problem, as distinct from the purely calculational aspects, are the steps that lead to the free-body diagram.

A **free-body diagram,** part of the *pictorial representation* of a problem, represents the object as a particle and shows *all* of the forces acting on the object.

TACTICS BOX 5.3 Drawing a free-body diagram MP

1. **Identify all forces acting on the object.** This step was described in Tactics Box 5.2.
2. **Draw a coordinate system.** Use the axes defined in your pictorial representation. If those axes are tilted, for motion along an incline, then the axes of the free-body diagram should be similarly tilted.
3. **Represent the object as a dot at the origin of the coordinate axes.** This is the particle model.
4. **Draw vectors representing each of the identified forces.** This was described in Tactics Box 5.1. Be sure to label each force vector.
5. **Draw and label the *net force* vector $\vec{F}_{net}$.** Draw this vector beside the diagram, not on the particle. Or, if appropriate, write $\vec{F}_{net} = \vec{0}$. Then check that $\vec{F}_{net}$ points in the same direction as the acceleration vector $\vec{a}$ on your motion diagram.

Exercises 24–29

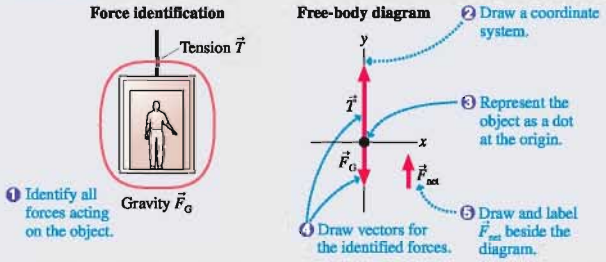
EXAMPLE 5.4 An elevator accelerates upward

An elevator, suspended by a cable, speeds up as it moves upward from the ground floor. Identify the forces and draw a free-body diagram of the elevator.

MODEL Treat the elevator as a particle.

VISUALIZE

FIGURE 5.23 Free-body diagram of an elevator accelerating upward.



ASSESS The coordinate axes, with a vertical y-axis, are the ones we would use in a pictorial representation of the motion. The elevator is accelerating upward, so $\vec{F}_{net}$ must point upward. For this to be true, the magnitude of $\vec{T}$ must be larger than the magnitude of $\vec{F}_G$. The diagram has been drawn accordingly.

EXAMPLE 5.5 An ice block shoots across a frozen lake

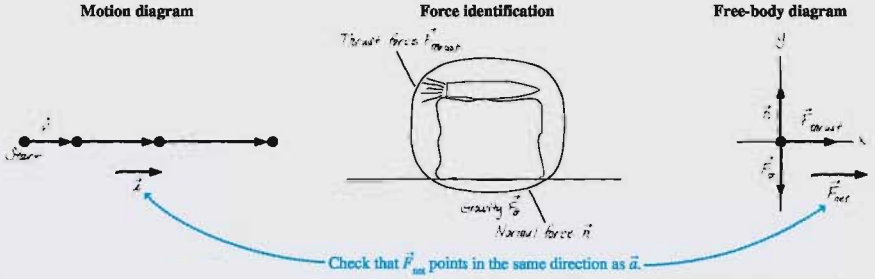
Bobby straps a small model rocket to a block of ice and shoots it across the smooth surface of a frozen lake. Friction is negligible. Draw a pictorial representation of the block of ice.

MODEL Treat the block of ice as a particle. The pictorial representation consists of a motion diagram to determine $\vec{a}$, a force-identification picture, and a free-body diagram. The statement of the situation implies that friction is negligible.

Continued

VISUALIZE

FIGURE 5.24 Pictorial representation for a block of ice shooting across a frictionless frozen lake.



ASSESS The motion diagram tells us that the acceleration is in the positive x-direction. According to the rules of vector addition, this can be true only if the upward-pointing $\vec{n}$ and the downward-pointing $\vec{F}_G$ are equal in magnitude and thus cancel each other $\left((F_G)_y = -n_y\right)$. The vectors have been drawn accordingly, and this leaves the net force vector pointing toward the right, in agreement with $\vec{a}$ from the motion diagram.

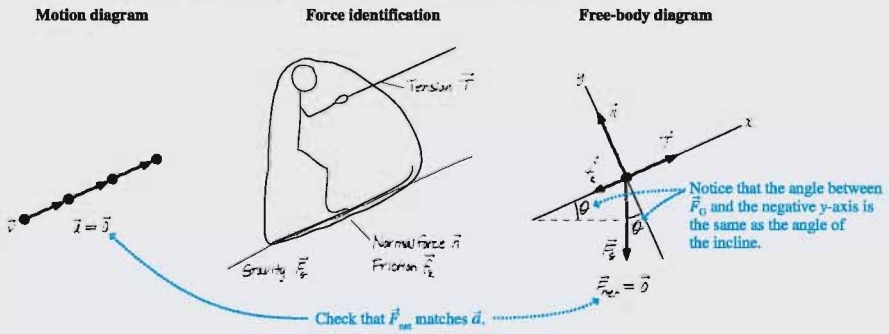
EXAMPLE 5.6 A skier is pulled up a hill

A tow rope pulls a skier up a snow-covered hill at a constant speed. Draw a pictorial representation of the skier.

MODEL This is Example 5.2 again with the additional information that the skier is moving at constant speed. The skier will be treated as a particle in *dynamic equilibrium*. If we were doing a kinematics problem, the pictorial representation would use a tilted coordinate system with the x-axis parallel to the slope, so we use these same tilted coordinate axes for the free-body diagram.

VISUALIZE

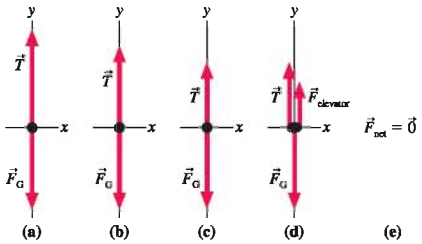
FIGURE 5.25 Pictorial representation for a skier being towed at a constant speed.



ASSESS We have shown $\vec{T}$ pulling parallel to the slope and $\vec{f}_k$, which opposes the direction of motion, pointing down the slope. $\vec{n}$ is perpendicular to the surface and thus along the y-axis. Finally, and this is important, the gravitational force $\vec{F}_G$ is vertically downward, not along the negative y-axis. In fact, you should convince yourself from the geometry that the angle θ between the $\vec{F}_G$ vector and the negative y-axis is the same as the angle θ of the incline above the horizontal. The skier moves in a straight line with constant speed, so $\vec{a} = \vec{0}$ and, from Newton's first law, $\vec{F}_{net} = \vec{0}$. Thus we have drawn the vectors such that the y-component of $\vec{F}_G$ is equal in magnitude to $\vec{n}$. Similarly, $\vec{T}$ must be large enough to match the negative x-components of both $\vec{f}_k$ and $\vec{F}_G$.

Free-body diagrams will be our major tool for the next several chapters. Careful practice with the workbook exercises and homework in this chapter will pay immediate benefits in the next chapter. Indeed, it is not too much to assert that a problem is half solved, or even more, when you complete the free-body diagram.

STOP TO THINK 5.5 An elevator suspended by a cable is moving upward and slowing to a stop. Which free-body diagram is correct?



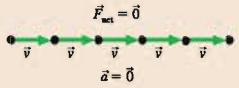
SUMMARY

The goal of Chapter 5 has been to learn how force and motion are connected.

General Principles

Newton's First Law

An object at rest will remain at rest, or an object that is moving will continue to move in a straight line with constant velocity, if and only if the net force on the object is zero.



The first law tells us that no "cause" is needed for motion. Uniform motion is the "natural state" of an object.

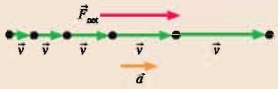
Newton's laws are valid only in inertial reference frames.

Newton's Second Law

An object with mass m will undergo acceleration

$$\vec{a} = \frac{1}{m}\vec{F}_{net}$$

where $\vec{F}_{net} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \cdots$ is the vector sum of all the individual forces acting on the object.



The second law tells us that a net force causes an object to accelerate. This is the connection between force and motion that we are seeking.

Important Concepts

Acceleration is the link to kinematics.

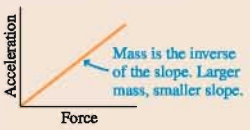
From $\vec{F}_{net}$, find $\vec{a}$.
From a, find v and x.

$\vec{a} = \vec{0}$ is the condition for **equilibrium.**

Static equilibrium if $\vec{v} = \vec{0}$.
Dynamic equilibrium if $\vec{v}$ = constant.

Equilibrium occurs if and only if $\vec{F}_{net} = \vec{0}$.

Mass is the resistance of an object to acceleration. It is an intrinsic property of an object.



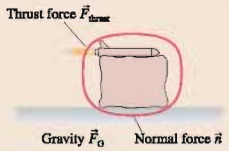
Force is a push or a pull on an object.

- Force is a vector, with a magnitude and a direction.
- Force requires an agent.
- Force is either a contact force or a long-range force.

Key Skills

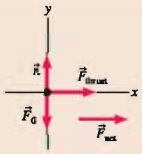
Identifying Forces

Forces are identified by locating the points where other objects touch the object of interest. These are points where contact forces are exerted. In addition, objects with mass feel a long-range gravitational force.



Free-Body Diagrams

A free-body diagram represents the object as a particle at the origin of a coordinate system. Force vectors are drawn with their tails on the particle. The net force vector is drawn beside the diagram.



Terms and Notation

dynamics
mechanics
force, $\vec{F}$
agent
contact force
long-range force
net force, $\vec{F}_{net}$
superposition of forces
gravitational force, $\vec{F}_G$
spring force, $\vec{F}_{sp}$
tension force, $\vec{T}$
atomic model
normal force, $\vec{n}$
friction, $\vec{f}_k$ or $\vec{f}_s$
drag, $\vec{D}$
thrust, $\vec{F}_{thrust}$
proportionality
proportionality constant
proportional reasoning
inertia
inertial mass, m
Newton's second law
newton, N
Newton's first law
mechanical equilibrium
static equilibrium
dynamic equilibrium
inertial reference frame
free-body diagram

For homework assigned on MasteringPhysics, go to www.masteringphysics.com

Problem difficulty is labeled as I (straightforward) to III (challenging).

CONCEPTUAL QUESTIONS

1. An elevator suspended by a cable is descending at constant velocity. How many force vectors would be shown on a free-body diagram? List them.
2. A compressed spring is pushing a block across a rough horizontal table. How many force vectors would be shown on a free-body diagram? List them.
3. A brick is falling from the roof of a three-story building. How many force vectors would be shown on a free-body diagram? List them.
4. In FIGURE Q5.4, block B is falling and dragging block A across a table. How many force vectors would be shown on a free-body diagram of block A? List them.

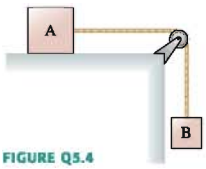


FIGURE Q5.4

5. You toss a ball straight up in the air. Immediately after you let go of it, what forces are acting on the ball? For each force you list, (a) state whether it is a contact force or a long-range force and (b) identify the agent of the force.
6. A constant force applied to A causes A to accelerate at 5 m/s^2. The same force applied to B causes an acceleration of 3 m/s^2. Applied to C, it causes an acceleration of 8 m/s^2.
   a. Which object has the largest mass? Explain.
   b. Which object has the smallest mass?
   c. What is the ratio m_A/m_B of the mass of A to the mass of B?
7. An object experiencing a constant force accelerates at 10 m/s^2. What will the acceleration of this object be if
   a. The force is doubled? Explain.
   b. The mass is doubled?
   c. The force is doubled *and* the mass is doubled?
8. An object experiencing a constant force accelerates at 8 m/s^2. What will the acceleration of this object be if
   a. The force is halved? Explain.
   b. The mass is halved?
   c. The force is halved *and* the mass is halved?
9. If an object is at rest, can you conclude that there are no forces acting on it? Explain.
10. If a force is exerted on an object, is it possible for that object to be moving with constant velocity? Explain.
11. Is the statement "An object always moves in the direction of the net force acting on it" true or false? Explain.
12. Newton's second law says $\vec{F}_{net} = m\vec{a}$. So is $m\vec{a}$ a force? Explain.
13. Is it possible for the friction force on an object to be in the direction of motion? If so, give an example. If not, why not?
14. Suppose you press your physics book against a wall hard enough to keep it from moving. Does the friction force on the book point (a) into the wall, (b) out of the wall, (c) up, (d) down, or (e) is there no friction force? Explain.
15. FIGURE Q5.15 shows a hollow tube forming three-quarters of a circle. It is lying flat on a table. A ball is shot through the tube at high speed. As the ball emerges from the other end, does it follow path A, path B, or path C? Explain.

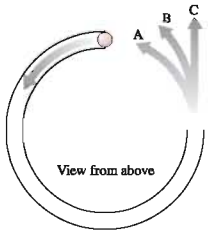


FIGURE Q5.15

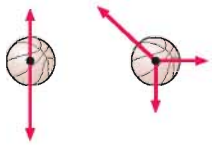


FIGURE Q5.16

16. Which, if either, of the basketballs in FIGURE Q5.16 are in equilibrium? Explain.
17. Which of the following are inertial reference frames? Explain.
   a. A car driving at steady speed on a straight and level road.
   b. A car driving at steady speed up a 10° incline.
   c. A car speeding up after leaving a stop sign.
   d. A car driving at steady speed around a curve.

EXERCISES AND PROBLEMS

Exercises

Section 5.3 Identifying Forces

1. I A mountain climber is hanging from a rope in the middle of a crevasse. The rope is vertical. Identify the forces on the mountain climber.
2. I A car is parked on a steep hill. Identify the forces on the car.
3. I A baseball player is sliding into second base. Identify the forces on the baseball player.
4. II A jet plane is speeding down the runway during takeoff. Air resistance is not negligible. Identify the forces on the jet.
5. II An arrow has just been shot from a bow and is now traveling horizontally. Air resistance is not negligible. Identify the forces on the arrow.

Section 5.4 What Do Forces Do? A Virtual Experiment

6. | Two rubber bands pulling on an object cause it to accelerate at 1.2 m/s^2.
   a. What will be the object's acceleration if it is pulled by four rubber bands?
   b. What will be the acceleration of two of these objects glued together if they are pulled by two rubber bands?
7. | Two rubber bands cause an object to accelerate with acceleration a. How many rubber bands are needed to cause an object with half the mass to accelerate three times as quickly?
8. || FIGURE EX5.8 shows an acceleration-versus-force graph for three objects pulled by rubber bands. The mass of object 2 is 0.20 kg. What are the masses of objects 1 and 3? Explain your reasoning.

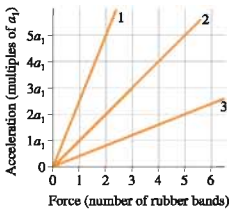


FIGURE EX5.8

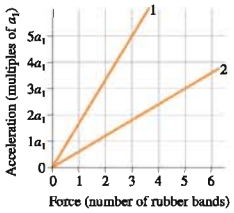


FIGURE EX5.9

9. || FIGURE EX5.9 shows acceleration-versus-force graphs for two objects pulled by rubber bands. What is the mass ratio m_1/m_2?
10. | For an object starting from rest and accelerating with constant acceleration, distance traveled is proportional to the square of the time. If an object travels 2.0 furlongs in the first 2.0 s, how far will it travel in the first 4.0 s?
11. || The period of a pendulum is proportional to the square root of its length. A 2.0-m-long pendulum has a period of 3.0 s. What is the period of a 3.0-m-long pendulum?

Section 5.5 Newton's Second Law

12. | Write a one-paragraph essay on the topic "Force and Motion." Explain in your own words the connection between force and motion. Where possible, cite *evidence* supporting your statements.
13. | FIGURE EX5.13 shows an acceleration-versus-force graph for a 500 g object. What acceleration values go in the blanks on the vertical scale?

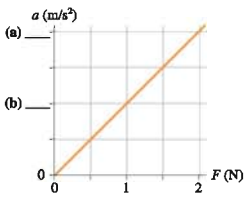


FIGURE EX5.13

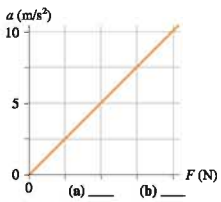


FIGURE EX5.14

14. | FIGURE EX5.14 shows an acceleration-versus-force graph for a 200 g object. What force values go in the blanks on the horizontal scale?
15. | FIGURE EX5.15 shows an object's acceleration-versus-force graph. What is the object's mass?

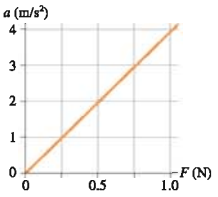


FIGURE EX5.15

16. | Based on the information in Table 5.3, estimate
   a. The weight of a laptop computer.
   b. The propulsion force of a bicycle.
17. | Based on the information in Table 5.3, estimate
   a. The weight of a pencil.
   b. The propulsion force of a sprinter.

Section 5.6 Newton's First Law

Exercises 18 through 20 show two of the three forces acting on an object in equilibrium. Redraw the diagram, showing all three forces. Label the third force $\vec{F}_3$.

18. |

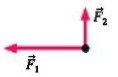


FIGURE EX5.18

19. |

FIGURE EX5.19

20. |

FIGURE EX5.20

Section 5.7 Free-Body Diagrams

Exercises 21 through 23 show a free-body diagram. For each:
   a. Redraw the free-body diagram.
   b. Write a short description of a real object for which this is the correct free-body diagram. Use Examples 5.4, 5.5, and 5.6 as models of what a description should be like.

21. |

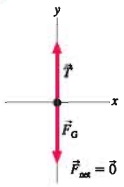


FIGURE EX5.21

22. |

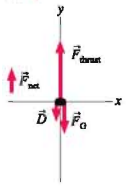


FIGURE EX5.22

23. |

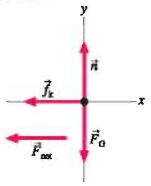


FIGURE EX5.23

Exercises 24 through 27 describe a situation. For each, identify all forces acting on the object and draw a free-body diagram of the object.

24. | You are sitting on a bench in the park.
25. | An ice hockey puck glides across frictionless ice.
26. | A steel beam is being lifted straight up at steady speed by a crane.
27. | Your physics textbook is sliding across the table.

Problems

28. | Redraw the two motion diagrams shown in FIGURE P5.28, then draw a vector beside each one to show the direction of the net force acting on the object. Explain your reasoning.

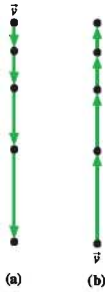


FIGURE P5.28

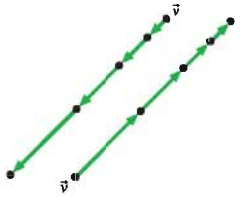


FIGURE P5.29

29. | Redraw the two motion diagrams shown in FIGURE P5.29, then draw a vector beside each one to show the direction of the net force acting on the object. Explain your reasoning.
30. | A single force with x-component F_x acts on a 2.0 kg object as it moves along the x-axis. The object's acceleration graph (a_x versus t) is shown in FIGURE P5.30. Draw a graph of F_x versus t.

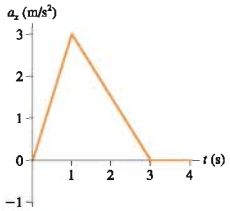


FIGURE P5.30

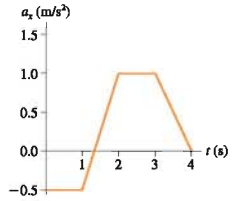


FIGURE P5.31

31. | A single force with x-component F_x acts on a 500 g object as it moves along the x-axis. The object's acceleration graph (a_x versus t) is shown in FIGURE P5.31. Draw a graph of F_x versus t.
32. | A single force with x-component F_x acts on a 2.0 kg object as it moves along the x-axis. A graph of F_x versus t is shown in FIGURE P5.32. Draw an acceleration graph (a_x versus t) for this object.

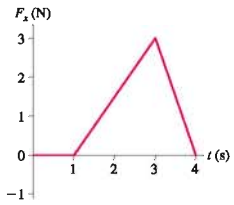


FIGURE P5.32

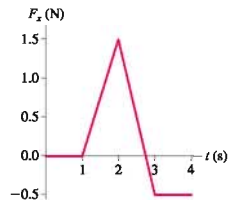


FIGURE P5.33

33. | A single force with x-component F_x acts on a 500 g object as it moves along the x-axis. A graph of F_x versus t is shown in FIGURE P5.33. Draw an acceleration graph (a_x versus t) for this object.
34. | A constant force is applied to an object, causing the object to accelerate at 10 m/s^2. What will the acceleration be if
    a. The force is halved?
    b. The object's mass is halved?
    c. The force and the object's mass are both halved?
    d. The force is halved and the object's mass is doubled?
35. | A constant force is applied to an object, causing the object to accelerate at 8.0 m/s^2. What will the acceleration be if
    a. The force is doubled?
    b. The object's mass is doubled?
    c. The force and the object's mass are both doubled?
    d. The force is doubled and the object's mass is halved?

Problems 36 through 42 show a free-body diagram. For each:
a. Redraw the diagram.
b. Identify the direction of the acceleration vector $\vec{a}$ and show it as a vector next to your diagram. Or, if appropriate, write $\vec{a} = \vec{0}$.
c. If possible, identify the direction of the velocity vector $\vec{v}$ and show it as a labeled vector.
d. Write a short description of a real object for which this is the correct free-body diagram. Use Examples 5.4, 5.5, and 5.6 as models of what a description should be like.

36. |

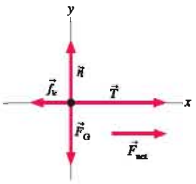


FIGURE P5.36

37. |

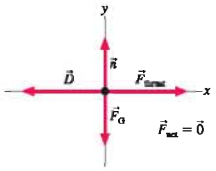


FIGURE P5.37

38. |

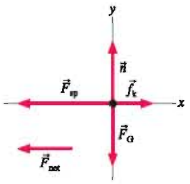


FIGURE P5.38

39. |

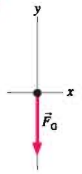


FIGURE P5.39

40. |

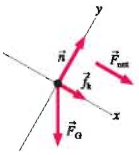


FIGURE P5.40

41. |

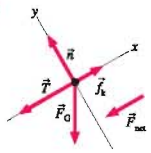


FIGURE P5.41

42. |

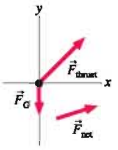


FIGURE P5.42

Problems 43 through 52 describe a situation. For each, draw a motion diagram, a force-identification diagram, and a free-body diagram.

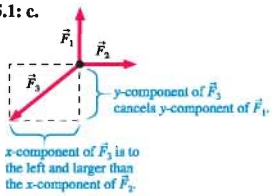
43. | An elevator, suspended by a single cable, has just left the tenth floor and is speeding up as it descends toward the ground floor.
44. | A rocket is being launched straight up. Air resistance is not negligible.
45. | A jet plane is speeding down the runway during takeoff. Air resistance is not negligible.
46. | You've slammed on the brakes and your car is skidding to a stop while going down a 20° hill.
47. | A skier is going down a 20° slope. A *horizontal* headwind is blowing in the skier's face. Friction is small, but not zero.
48. | You've just kicked a rock on the sidewalk and it is now sliding along the concrete.
49. | A styrofoam ball has just been shot straight up. Air resistance is not negligible.
50. | A spring-loaded gun shoots a plastic ball. The trigger has just been pulled and the ball is starting to move down the barrel. The barrel is horizontal.
51. | A person on a bridge throws a rock straight down toward the water. The rock has just been released.
52. | A gymnast has just landed on a trampoline. She's still moving downward as the trampoline stretches.

Challenge Problems

53. A heavy box is in the back of a truck. The truck is accelerating to the right. Draw a motion diagram, a force-identification diagram, and a free-body diagram for the box.
54. A bag of groceries is on the seat of your car as you stop for a stop light. The bag does not slide. Draw a motion diagram, a force-identification diagram, and a free-body diagram for the bag.
55. A rubber ball bounces. We'd like to understand *how* the ball bounces.
    a. A rubber ball has been dropped and is bouncing off the floor. Draw a motion diagram of the ball during the brief time interval that it is in contact with the floor. Show 4 or 5 frames as the ball compresses, then another 4 or 5 frames as it expands. What is the direction of $\vec{a}$ during each of these parts of the motion?
    b. Draw a picture of the ball in contact with the floor and identify all forces acting on the ball.
    c. Draw a free-body diagram of the ball during its contact with the ground. Is there a net force acting on the ball? If so, in which direction?
    d. Write a paragraph in which you describe what you learned from parts a to c and in which you answer the question: How does a ball bounce?
56. If a car stops suddenly, you feel "thrown forward." We'd like to understand what happens to the passengers as a car stops. Imagine yourself sitting on a *very* slippery bench inside a car. This bench has no friction, no seat back, and there's nothing for you to hold to.
    a. Draw a picture and identify all of the forces acting on you as the car travels at a perfectly steady speed on level ground.
    b. Draw your free-body diagram. Is there a net force on you? If so, in which direction?
    c. Repeat parts a and b with the car slowing down.
    d. Describe what happens to you as the car slows down.
    e. Use Newton's laws to explain why you seem to be "thrown forward" as the car stops. Is there really a force pushing you forward?
    f. Suppose now that the bench is not slippery. As the car slows down, you stay on the bench and don't slide off. What force is responsible for your deceleration? In which direction does this force point? Include a free-body diagram as part of your answer.

STOP TO THINK ANSWERS

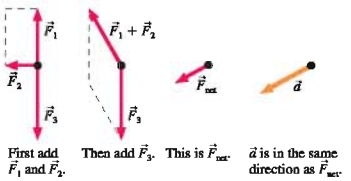
Stop to Think 5.1: c.



Stop to Think 5.2: a, b, and d. Friction and the normal force are the only contact forces. Nothing is touching the rock to provide a "force of the kick." We've agreed to ignore air resistance unless a problem specifically calls for it.

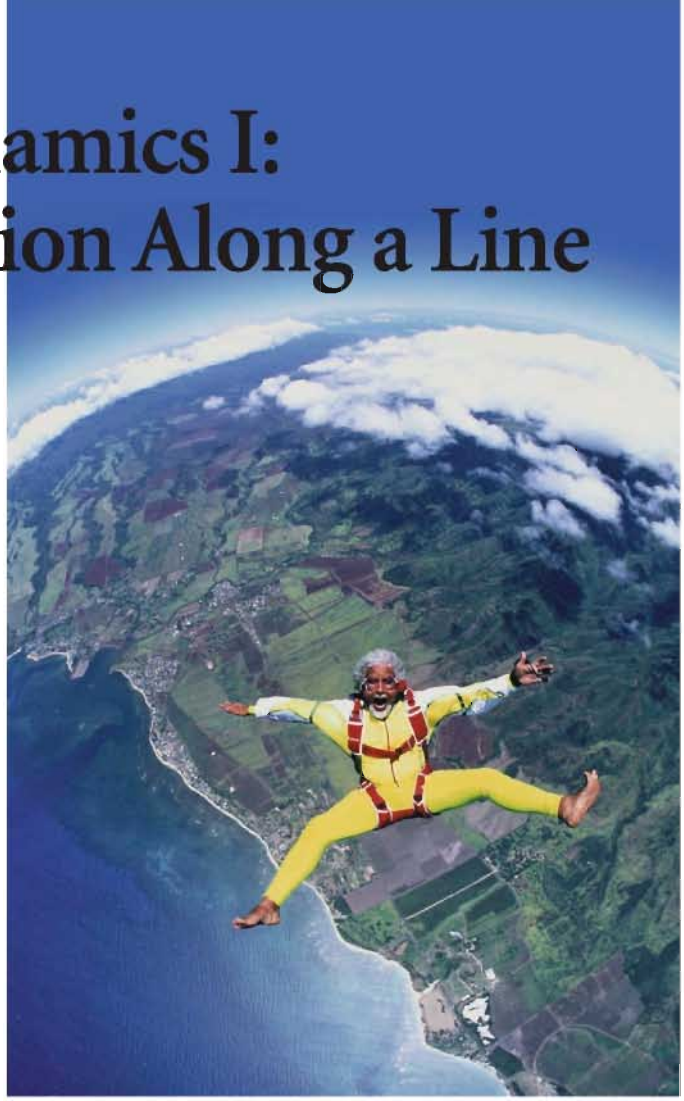
Stop to Think 5.3: b. Acceleration is proportional to force, so doubling the number of rubber bands doubles the acceleration of the original object from 2 m/s^2 to 4 m/s^2. But acceleration is also inversely proportional to mass. Doubling the mass cuts the acceleration in half, back to 2 m/s^2.

Stop to Think 5.4: d.



Stop to Think 5.5: c. The acceleration vector points downward as the elevator slows. $\vec{F}_{net}$ points in the same direction as $\vec{a}$, so $\vec{F}_{net}$ also points down. This will be true if the tension is less than the gravitational force: $T < F_G$.

6 Dynamics I: Motion Along a Line



This skydiver may not know it, but he is testing Newton's second law as he plunges toward the ground below.

▶ Looking Ahead

The goal of Chapter 6 is to learn how to solve problems about motion in a straight line. In this chapter you will learn to:

- Solve static and dynamic equilibrium problems by applying a Newton's-first-law strategy.
- Solve dynamics problems by applying a Newton's-second-law strategy.
- Understand how mass and weight differ.
- Use simple models of friction and drag.

◀ Looking Back

This chapter pulls together many strands of thought from Chapters 1–5. Please review:

- Sections 2.4–2.6 Constant-acceleration kinematics, including free fall.
- Sections 3.3–3.4 Working with vectors and vector components.
- Sections 5.2, 5.3, and 5.7 Identifying forces and drawing free-body diagrams.

A skydiver accelerates until reaching a *terminal speed* of about 140 mph. To understand the skydiver's motion, we need to look closely at the forces exerted on him. We also need to understand how those forces determine his motion.

In Chapter 5 we learned what a force is and is not. We also discovered the fundamental relationship between force and motion: Newton's second law. Chapter 6 begins to develop a *strategy* for solving force and motion problems. Our strategy is to learn a set of *procedures*, not to memorize a set of equations.

This chapter focuses on objects that move in a straight line, such as runners, bicycles, cars, planes, and rockets. Gravitational, tension, thrust, friction, and drag forces will be essential to our understanding. Two-dimensional and circular motion will be the topics of Chapter 8.

The concept of equilibrium is essential for the engineering analysis of stationary objects such as bridges.

6.1 Equilibrium

An object on which the net force is zero is said to be in *equilibrium*. The object might be at rest in *static equilibrium*, or it might be moving along a straight line with constant velocity in *dynamic equilibrium*. Both are identical from a Newtonian perspective because $\vec{F}_{net} = \vec{0}$ and $\vec{a} = \vec{0}$.

Newton's first law is the basis for a four-step *strategy* for solving equilibrium problems.

PROBLEM-SOLVING STRATEGY 6.1 Equilibrium problems MP

MODEL Make simplifying assumptions. When appropriate, represent the object as a particle.

VISUALIZE

- Establish a coordinate system, define symbols, and identify what the problem is asking you to find. This is the process of translating words into symbols.
- Identify all forces acting on the object and show them on a free-body diagram.
- These elements form the **pictorial representation** of the problem.

SOLVE The mathematical representation is based on Newton's first law:

$$\vec{F}_{net} = \sum_i \vec{F}_i = \vec{0}$$

The vector sum of the forces is found directly from the free-body diagram.

ASSESS Check that your result has the correct units, is reasonable, and answers the question.

Newton's laws are *vector equations*. Recall from Chapter 3 that the vector equation in the step labeled Solve is a shorthand way of writing two simultaneous equations:

$$\begin{aligned} (F_{net})_x &= \sum_i (F_i)_x = 0 \\ (F_{net})_y &= \sum_i (F_i)_y = 0 \end{aligned} \qquad (6.1)$$

In other words, each component of $\vec{F}_{net}$ must simultaneously be zero. Although real-world situations often have forces pointing in three dimensions, thus requiring a third equation for the z-component of $\vec{F}_{net}$, we will restrict ourselves for now to problems that can be analyzed in two dimensions.

NOTE ▶ The equilibrium condition of Equations 6.1 applies only to particles, which cannot rotate. Equilibrium of an extended object, which can rotate, requres an additional condition. We will study the equilibrium of extended objects in Chapter 12.◀

Equilibrium problems occur frequently, especially in engineering applications. Let's look at a couple of examples.

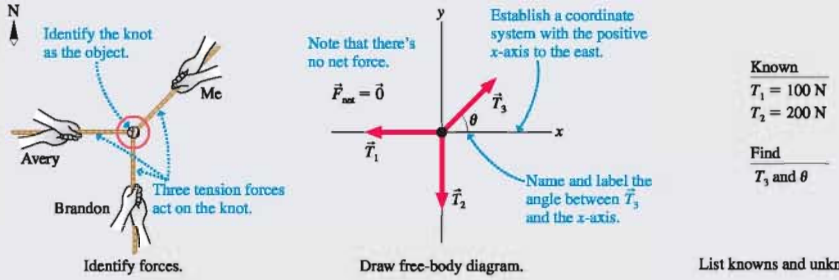
Static Equilibrium

EXAMPLE 6.1 Three-way tug-of-war

You and two friends find three ropes tied together with a single knot and decide to have a three-way tug-of-war. Avery pulls to the west with 100 N of force, while Brandon pulls to the south with 200 N. How hard, and in which direction, should you pull to keep the knot from moving?

MODEL We'll treat the *knot* in the rope as a particle in static equilibrium.

FIGURE 6.1 Pictorial representation for a knot in static equilibrium.



VISUALIZE FIGURE 6.1 shows how to draw a pictorial representation. We've chosen the y-axis to point north, and we've labeled the three forces $\vec{T}_1$, $\vec{T}_2$, and $\vec{T}_3$. Notice that we've *defined* angle θ to indicate the direction of your pull.

SOLVE The free-body diagram shows the tension forces $\vec{T}_1$, $\vec{T}_2$, and $\vec{T}_3$ acting on the knot. Newton's first law, written in component form, is

$$(F_{net})_x = \sum_i (F_i)_x = T_{1x} + T_{2x} + T_{3x} = 0$$

$$(F_{net})_y = \sum_i (F_i)_y = T_{1y} + T_{2y} + T_{3y} = 0$$

NOTE ▶ You might have been tempted to write $-T_{1x}$ in the first equation because $\vec{T}_1$ points in the negative x-direction. But the net force, by definition, is the *sum* of all the individual forces. The fact that $\vec{T}_1$ points to the left will be taken into account when we *evaluate* the components. ◀

The components of the force vectors can be evaluated directly from the free-body diagram:

$$T_{1x} = -T_1 \qquad T_{1y} = 0$$
$$T_{2x} = 0 \qquad T_{2y} = -T_2$$
$$T_{3x} = +T_3\cos\theta \qquad T_{3y} = +T_3\sin\theta$$

This is where the signs enter, with T_{1x} being assigned a negative value because $\vec{T}_1$ points to the left. Similarly, $T_{2y} = -T_2$. With these components, Newton's first law becomes

$$-T_1 + T_3\cos\theta = 0$$
$$-T_2 + T_3\sin\theta = 0$$

These are two simultaneous equations for the two unknowns T_3 and θ. We will encounter equations of this form on many occasions, so make a note of the method of solution. First, rewrite the two equations as

$$T_1 = T_3\cos\theta$$
$$T_2 = T_3\sin\theta$$

Next, divide the second equation by the first to eliminate T_3:

$$\frac{T_2}{T_1} = \frac{T_3\sin\theta}{T_3\cos\theta} = \tan\theta$$

Then solve for θ:

$$\theta = \tan^{-1}\left(\frac{T_2}{T_1}\right) = \tan^{-1}\left(\frac{200\text{ N}}{100\text{ N}}\right) = 63.4°$$

Finally, use θ to find T_3:

$$T_3 = \frac{T_1}{\cos\theta} = \frac{100\text{ N}}{\cos 63.4°} = 224\text{ N}$$

The force that maintains equilibrium and prevents the knot from moving is thus

$$\vec{T}_3 = (224\text{ N}, 63.4° \text{ north of east})$$

ASSESS Is this result reasonable? Because your friends pulled west and south, you expected to pull in a generally northeast direction. You also expected to pull harder than either of them but, because they didn't pull in the same direction, less than the sum of their pulls. The result for $\vec{T}_3$ meets these expectations.

Dynamic Equilibrium

EXAMPLE 6.2 Towing a car up a hill

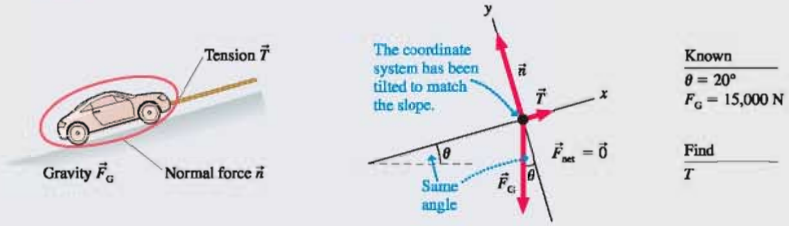
A car with a weight of 15,000 N is being towed up a 20° slope at constant velocity. Friction is negligible. The tow rope is rated at 6000 N maximum tension. Will it break?

MODEL We'll treat the car as a particle in dynamic equilibrium. We'll ignore friction.

VISUALIZE This problem asks for a yes or no answer, not a number, but we still need a quantitative analysis. Part of our analysis of the problem statement is to determine which quantity or quantities allow us to answer the question. In this case the answer is clear: We need to calculate the tension in the rope. FIGURE 6.2 shows the pictorial representation. Note the similarities to Examples 5.2 and 5.6 in Chapter 5, which you may want to review.

We noted in Chapter 5 that the weight of an object at rest is the magnitude F_G of the gravitational force acting on it, and that information has been listed as known. We'll examine weight more closely later in the chapter.

FIGURE 6.2 Pictorial representation of a car being towed up a hill.



SOLVE The free-body diagram shows forces $\vec{T}$, $\vec{n}$, and $\vec{F}_G$ acting on the car. Newton's first law is

$$(F_{net})_x = \sum F_x = T_x + n_x + (F_G)_x = 0$$

$$(F_{net})_y = \sum F_y = T_y + n_y + (F_G)_y = 0$$

Notice that we dropped the label i from the sum. From here on, we'll use $\sum F_x$ and $\sum F_y$ as a simple shorthand notation to indicate that we're adding all the x-components and all the y-components of the forces.

We can deduce the components directly from the free-body diagram:

$$T_x = T \qquad T_y = 0$$

$$n_x = 0 \qquad n_y = n$$

$$(F_G)_x = -F_G \sin\theta \qquad (F_G)_y = -F_G \cos\theta$$

NOTE ▶ The gravitational force has both x- and y-components in this coordinate system, both of which are negative due to the direction of the vector $\vec{F}_G$. You'll see this situation often, so be sure you understand where $(F_G)_x$ and $(F_G)_y$ come from. ◀

With these components, the first law becomes

$$T - F_G \sin\theta = 0$$

$$n - F_G \cos\theta = 0$$

The first of these can be rewritten as

$$T = F_G \sin\theta$$

$$= (15{,}000 \text{ N}) \sin 20° = 5100 \text{ N}$$

Because $T < 6000$ N, we conclude that the rope will *not* break. It turned out that we did not need the y-component equation in this problem.

ASSESS Because there's no friction, it would not take *any* tension force to keep the car rolling along a horizontal surface ($\theta = 0°$). At the other extreme, $\theta = 90°$, the tension force would need to equal the car's weight ($T = 15{,}000$ N) to lift the car straight up at constant velocity. The tension force for a 20° slope should be somewhere in between, and 5100 N is a little less than half the weight of the car. That our result is reasonable doesn't prove it's right, but we have at least ruled out careless errors that give unreasonable results.

6.2 Using Newton's Second Law

2.1, 2.2, 2.3, 2.4 Activ ONLINE Physics

Equilibrium is important, but it is a special case of motion. Newton's second law is a more general link between force and motion. We now need a strategy for using Newton's second law to solve dynamics problems.

The essence of Newtonian mechanics can be expressed in two steps:

- The forces on an object determine its acceleration $\vec{a} = \vec{F}_{net}/m$.
- The object's trajectory can be determined by using $\vec{a}$ in the equations of kinematics.

These two ideas are the basis of a strategy for solving dynamics problems.

PROBLEM-SOLVING STRATEGY 6.2 Dynamics problems MP

MODEL Make simplifying assumptions.

VISUALIZE Draw a **pictorial representation.**

- Show important points in the motion with a sketch, establish a coordinate system, define symbols, and identify what the problem is trying to find. This is the process of translating words into symbols.
- Use a motion diagram to determine the object's acceleration vector $\vec{a}$.
- Identify all forces acting on the object and show them on a free-body diagram.
- It's OK to go back and forth between these steps as you visualize the situation.

SOLVE The mathematical representation is based on Newton's second law:

$$\vec{F}_{net} = \sum_i \vec{F}_i = m\vec{a}$$

The vector sum of the forces is found directly from the free-body diagram. Depending on the problem, either

- Solve for the acceleration, then use kinematics to find velocities and positions; or
- Use kinematics to determine the acceleration, then solve for unknown forces.

ASSESS Check that your result has the correct units, is reasonable, and answers the question.

Newton's second law is a vector equation. To apply the step labeled Solve, you must write the second law as two simultaneous equations:

$$\begin{aligned}(F_{net})_x &= \sum F_x = ma_x \\ (F_{net})_y &= \sum F_y = ma_y\end{aligned} \qquad (6.2)$$

The primary goal of this chapter is to illustrate the use of this strategy. Let's start with some examples.

EXAMPLE 6.3 Speed of a towed car

A 1500 kg car is pulled by a tow truck. The tension in the tow rope is 2500 N, and a 200 N friction force opposes the motion. If the car starts from rest, what is its speed after 5.0 seconds?

MODEL We'll treat the car as an accelerating particle. We'll assume, as part of our *interpretation* of the problem, that the road is horizontal and that the direction of motion is to the right.

VISUALIZE FIGURE 6.3 on the next page shows the pictorial representation. We've established a coordinate system and defined symbols to represent kinematic quantities. We've identified the speed v_1, rather than the velocity v_{1x}, as what we're trying to find.

SOLVE We begin with Newton's second law:

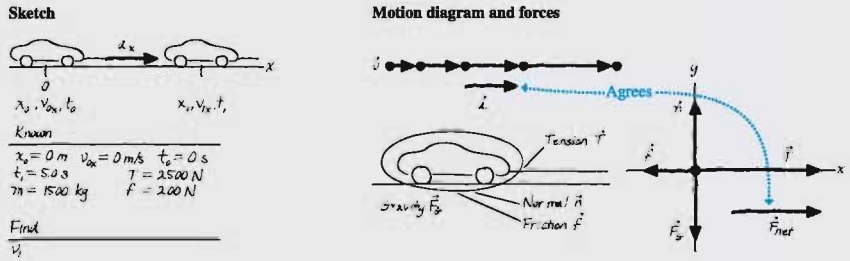
$$(F_{net})_x = \sum F_x = T_x + f_x + n_x + (F_G)_x = ma_x$$
$$(F_{net})_y = \sum F_y = T_y + f_y + n_y + (F_G)_y = ma_y$$

All four forces acting on the car have been included in the vector sum. The equations are perfectly general, with + signs everywhere, because the four vectors are *added* to give $\vec{F}_{net}$. We can now "read" the vector components from the free-body diagram:

$$\begin{aligned} T_x &= +T & T_y &= 0 \\ n_x &= 0 & n_y &= +n \\ f_x &= -f & f_y &= 0 \\ (F_G)_x &= 0 & (F_G)_y &= -F_G \end{aligned}$$

Continued

FIGURE 6.3 Pictorial representation of a car being towed.



The signs of the components depend on which way the vectors point. Substituting these into the second-law equations and dividing by m give

$$a_x = \frac{1}{m}(T - f)$$

$$= \frac{1}{1500\text{ kg}}(2500\text{ N} - 200\text{ N}) = 1.53\text{ m/s}^2$$

$$a_y = \frac{1}{m}(n - F_G)$$

NOTE ▶ Newton's second law has allowed us to determine a_x exactly but has given only an algebraic expression for a_y. However, we know *from the motion diagram* that $a_y = 0$! That is, the motion is purely along the x-axis, so there is *no* acceleration along the y-axis. The requirement $a_y = 0$ allows us to conclude that $n = F_G$. Although we do not need n for this problem, it will be important in many future problems. ◀

We can finish by using constant-acceleration kinematics to find the velocity:

$$v_{1x} = v_{0x} + a_x \Delta t$$
$$= 0 + (1.53\text{ m/s}^2)(5.0\text{ s})$$
$$= 7.7\text{ m/s}$$

The problem asked for the *speed* after 5.0 s, which is $v_1 = 7.7$ m/s.

ASSESS 7.7 m/s ≈ 15 mph, a reasonable speed after 5 s of acceleration.

EXAMPLE 6.4 Altitude of a rocket

A 500 g model rocket with a gravitational force of 4.90 N is launched straight up. The small rocket motor burns for 5.00 s and has a steady thrust of 20.0 N. What maximum altitude does the rocket reach? Assume that the mass loss of the burned fuel is negligible.

MODEL We'll treat the rocket as an accelerating particle. Air resistance will be neglected.

VISUALIZE The pictorial representation of **FIGURE 6.4** finds that this is a two-part problem. First, the rocket accelerates straight up. Second, the rocket continues going up as it slows down, a free-fall situation. The maximum altitude is at the end of the second part of the motion.

SOLVE We now know what the problem is asking, have established relevant symbols and coordinates, and know what the forces are. We begin the mathematical representation by writing Newton's second law, in component form, as the rocket accelerates upward. The free-body diagram shows two forces, so

$$(F_{net})_x = \sum F_x = (F_{thrust})_x + (F_G)_x = ma_{0x}$$
$$(F_{net})_y = \sum F_y = (F_{thrust})_y + (F_G)_y = ma_{0y}$$

The fact that vector $\vec{F}_G$ points downward—and which might have tempted you to use a minus sign in the y-equation—will be taken into account when we *evaluate* the components. None of the vectors in this problem has an x-component, so only the y-component of the second law is needed. We can use the free-body diagram to see that

$$(F_{thrust})_y = +F_{thrust}$$
$$(F_G)_y = -F_G$$

This is the point at which the directional information about the force vectors enters. The y-component of the second law is then

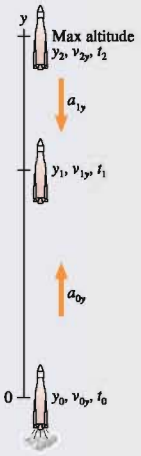
$$a_{0y} = \frac{1}{m}(F_{thrust} - F_G)$$

$$= \frac{20.0\text{ N} - 4.90\text{ N}}{0.500\text{ kg}} = 30.2\text{ m/s}^2$$

Notice that we converted the mass to SI units of kilograms before doing any calculations and that, because of the definition of the newton, the division of newtons by kilograms automatically gives the correct SI units of acceleration.

FIGURE 6.4 Pictorial representation of a rocket launch.

Sketch



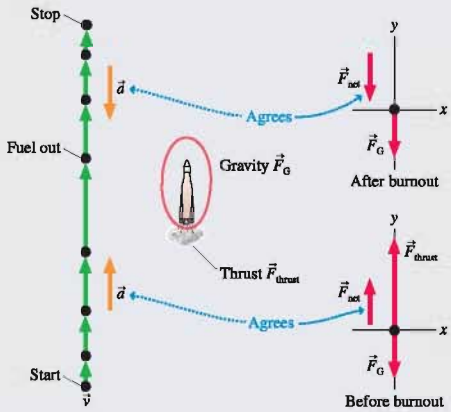
Known

$y_0 = 0$ m	$F_{thrust} = 20.0$ N
$v_{0y} = 0$ m/s	$F_G = 4.90$ N
$t_0 = 0$ s	
$t_1 = 5.00$ s	$a_{1y} = -9.80$ m/s^2
$v_{2y} = 0$ (top)	$m = 500$ g $= 0.500$ kg

Find

y_2

Motion diagram and forces



The acceleration of the rocket is constant until it runs out of fuel, so we can use constant-acceleration kinematics to find the altitude and velocity at burnout ($\Delta t = t_1 = 5.00$ s):

$$y_1 = y_0 + v_{0y}\Delta t + \tfrac{1}{2}a_{0y}(\Delta t)^2$$
$$= \tfrac{1}{2}a_{0y}(\Delta t)^2 = 377 \text{ m}$$
$$v_{1y} = v_{0y} + a_{0y}\Delta t = a_{0y}\Delta t = 151 \text{ m/s}$$

The only force on the rocket after burnout is gravity, so the second part of the motion is free fall with $a_{1y} = -g$. We do not know how long it takes to reach the top, but we do know that the final velocity is $v_{2y} = 0$.

We can use free-fall kinematics to find the maximum altitude:

$$v_{2y}^2 = 0 = v_{1y}^2 - 2g\Delta y = v_{1y}^2 - 2g(y_2 - y_1)$$

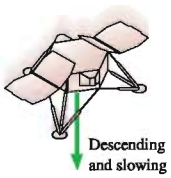
which we can solve to find

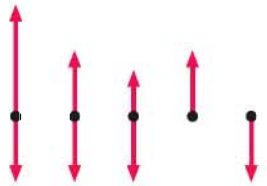
$$y_2 = y_1 + \frac{v_{1y}^2}{2g} = 377 \text{ m} + \frac{(151 \text{ m/s})^2}{2(9.80 \text{ m/s}^2)}$$
$$= 1540 \text{ m} = 1.54 \text{ km}$$

ASSESS The maximum altitude reached by this rocket is 1.54 km, or just slightly under one mile. While this does not seem unreasonable for a high-acceleration rocket, the neglect of air resistance was probably not a terribly realistic assumption.

These first examples have shown all the details. Our purpose has been to show how the problem-solving strategy is put into practice. Future examples will be briefer, but the basic *procedure* will remain the same.

STOP TO THINK 6.1 A Martian lander is approaching the surface. It is slowing its descent by firing its rocket motor. Which is the correct free-body diagram for the lander?





6.3 Mass, Weight, and Gravity

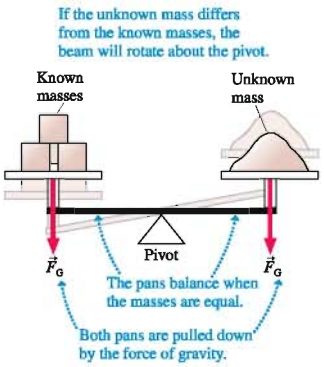
We often do not make a large distinction between mass, weight, and gravity in our ordinary use of language. However, these are separate and distinct concepts in science and engineering, and we need careful definitions if we're going to think clearly about force and motion.

Mass: An Intrinsic Property

Mass, you'll recall from Chapter 5, is a scalar quantity that describes an object's inertia. Loosely speaking, it also describes the amount of matter in an object. **Mass is an intrinsic property of an object.** It tells us something about the object, regardless of where the object is, what it's doing, or whatever forces may be acting on it.

A *pan balance,* shown in FIGURE 6.5, is a device for measuring mass. An unknown mass is placed in one pan, then known masses are added to the other until the pans balance. Although a pan balance requires gravity to function, it does not depend on the strength of gravity. Consequently, the pan balance would give the same result on another planet.

FIGURE 6.5 A pan balance measures mass.



Gravity: A Force

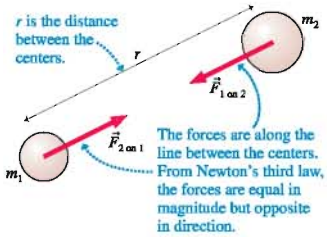
The idea of gravity has a long and interesting history intertwined with our evolving ideas about earth and the solar system. It was Newton who—along with discovering his three laws of motion—first recognized that **gravity is an attractive, long-range force between *any* two objects.** Somewhat more loosely, gravity is a force that acts on mass.

FIGURE 6.6 shows two objects with masses m_1 and m_2 separated by distance r. Each object pulls on the other with a force given by *Newton's law of gravity:*

$$F_{1\text{ on }2} = F_{2\text{ on }1} = \frac{Gm_1m_2}{r^2} \qquad \text{(Newton's law of gravity)} \qquad (6.3)$$

where $G = 6.67 \times 10^{-11}\ \text{N}\,\text{m}^2/\text{kg}^2$, called the *gravitational constant,* is one of the basic constants of nature. You can see from the units of G that the force will be in newtons if the masses are in kilograms and the distance is in meters, all standard SI units. Note that the gravitational force is a vector, with a direction; Equation 6.3 gives only the magnitude of the force. The force gets weaker as the distance between the objects increases.

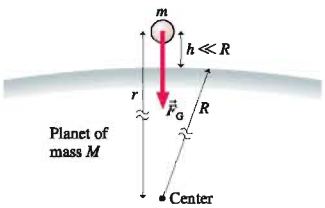
FIGURE 6.6 Newton's law of gravity.



Because G is a very small number, the force between two human-sized objects is minuscule, completely insignificant in comparison with other forces. That's why you're not aware of being tugged toward everything around you. Only when one or both objects is planet-size or larger does gravity become an important force. Indeed, Chapter 13 will explore in detail the application of Newton's law of gravity to the orbits of satellites and planets.

Newton's law of gravity, with its inverse-square dependence on distance, is a rather complicated force. It would be very useful to have a simpler version of the gravitational force for objects on or very near the surface of a planet, things like balls and cars and planes. FIGURE 6.7 shows an object of mass m at height h above the surface of a planet of mass M and radius R. Distance $r = R + h$ is the separation between the center of the planet and the center of the object. If $h \ll R$ (i.e., the height above the surface is very small in comparison with the size of the planet), then there's virtually no difference between the true separation r and the planet's radius R. Consequently, a very good approximation for the gravitational force of the planet on mass m is simply

FIGURE 6.7 Gravity near the surface of a planet.



$$\vec{F}_G = \vec{F}_{\text{planet on }m} = \left(\frac{GMm}{R^2}, \text{straight down}\right) \qquad (6.4)$$

On earth, for example, the approximation GMm/R^2 differs from the exact GMm/r^2 by only 0.3% at a height of 10 km (33,000 ft), the height at which jet planes fly.

For motions whose vertical and horizontal extents are less than roughly 10 km, we can approximate the earth as a flat surface (i.e., the earth's curvature is irrelevant over these distances) that pulls on objects with a gravitational force given by Equation 6.4. This **flat-earth approximation** is another model, but one we've shown is well justified for small-scale motions on or near the surface of the earth, the types of motion we'll be studying in the next few chapters.

Notice that the gravitational force of Equation 6.4 is directly proportional to the object's mass m. We can write the gravitational force even more simply as

$$\vec{F}_G = (mg, \text{straight down}) \qquad \text{(gravitational force)} \tag{6.5}$$

where the quantity g is defined to be

$$g = \frac{GM}{R^2} \tag{6.6}$$

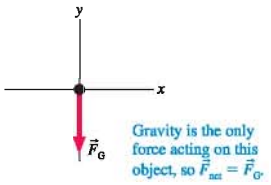
In fact, the direction of the gravitational force defines what we *mean* by "straight down."

The quantity g—sometimes called the *gravitational field* of a planet—is a property of the planet, depending only on the planet's mass and size. Once we've calculated g, we can use it to find the gravitational force on any object near the planet. (We will develop the *field model* of long-range forces when we study electricity and magnetism in Part VI.)

But why did we choose to call it g, a symbol we've already used as the free-fall acceleration? To see the connection, FIGURE 6.8 shows the free-body diagram of an object in free fall near the surface of a planet. With $\vec{F}_{net} = \vec{F}_G$, Newton's second law predicts that the acceleration of an object in free fall is

$$\vec{a}_{\text{free fall}} = \frac{\vec{F}_{net}}{m} = \frac{\vec{F}_G}{m} = (g, \text{straight down}) \tag{6.7}$$

FIGURE 6.8 The free-body diagram of an object in free fall.



Interestingly, the magnitude $a_{\text{free fall}}$ of the free-fall acceleration is simply g, the magnitude of the gravitational field. Because g is a property of the planet, independent of the object, **all objects on the same planet, regardless of mass, have the same free-fall acceleration.** We introduced this idea in Chapter 2 as an experimental discovery of Galileo, but now we see that the mass independence of $\vec{a}_{\text{free fall}}$ is a theoretical prediction of Newton's law of gravity.

The last thing to check is whether Newton's law predicts the correct value, which we know from experiment (at least at midlatitudes on the earth's surface) to be $g = |a_{\text{free fall}}| = 9.80\ \text{m/s}^2$. Things are a bit complicated by the fact that the earth isn't a perfect sphere, but we can use the average radius ($R_{\text{earth}} = 6.37 \times 10^6$ m) and mass ($M_{\text{earth}} = 5.98 \times 10^{24}$ kg) of the earth to calculate

$$g_{\text{earth}} = \frac{GM_{\text{earth}}}{(R_{\text{earth}})^2} = \frac{(6.67 \times 10^{-11}\ \text{N m}^2/\text{kg}^2)(5.98 \times 10^{24}\ \text{kg})}{(6.37 \times 10^6\ \text{m})^2} = 9.83\ \text{N/kg}$$

You should convince yourself that N/kg is equivalent to m/s^2, so $g_{\text{earth}} = 9.83\ \text{m/s}^2$. (Data for other astronomical objects, which you may need for homework, are located inside the back cover of the book.)

Newton's prediction is very close, but it's not quite right. The free-fall acceleration *would* be 9.83 m/s^2 on a stationary earth, but, in reality, the earth is rotating on its axis. The "missing" 0.03 m/s^2 is due to the earth's rotation, a claim we'll justify when we study circular motion in Chapter 8. Because we're on the outside of a rotating sphere, rather like being on the outside edge of a merry-go-round, the effect of rotation is to "weaken" gravity.

Our goal is to analyze motion from within our own reference frame, a reference frame attached to the earth. Strictly speaking, Newton's laws of motion are not valid in our reference frame because it is rotating and thus is not an inertial reference frame.

Fortunately, we can use Newton's laws to analyze motion near the earth's surface, and we can use $F_G = mg$ for the gravitational force *if* we use $g = |a_{\text{free fall}}| = 9.80\ \text{m/s}^2$ rather than $g = g_{\text{earth}}$. (This assertion is proved in more advanced classes.) In our rotating reference frame, $\vec{F}_G$ is the *effective gravitational force,* the true gravitational force given by Newton's law of gravity plus a small correction due to our rotation. This is the force to show on free-body diagrams and use in calculations.

NOTE ▶ You may be familiar with the idea that future space stations will generate "artificial gravity" by rotating. From *within* a rotating reference frame, the effects of rotation can't be distinguished from the effects of gravity. On our rotating earth, we can measure only the combined influence of gravity and rotation, hence the idea of the *effective* gravitational force. Because the rotational correction is very small, the term "gravitational force" will, unless noted otherwise, mean the effective gravitational force we actually experience on our rotating planet. ◀

A spring scale, such as the familiar bathroom scale, measures weight, not mass.

Weight: A Measurement

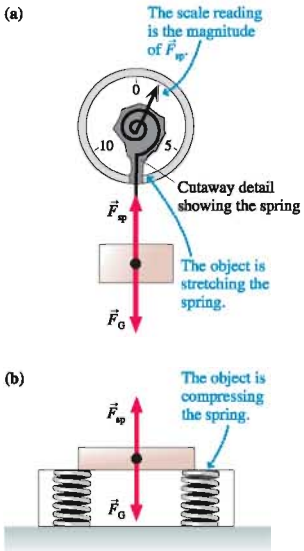
When you weigh yourself, you stand on a *spring scale* and compress a spring. You weigh apples in the grocery store by placing them in a spring scale and stretching a spring. The reading of a spring scale, such as the two shown in FIGURE 6.9, is F_{sp}, the magnitude of the force the spring is exerting.

With that in mind, let's define the **weight** of an object as the reading F_{sp} of a calibrated spring scale on which the object is stationary. That is, **weight is a measurement, the result of "weighing" an object.** Because F_{sp} is a force, weight is measured in newtons.

Suppose the scales in Figure 6.9 are at rest relative to the earth. Then the object being weighed is in static equilibrium, with $\vec{F}_{net} = \vec{0}$. The stretched spring *pulls* up, the compressed spring *pushes* up, but in both cases $\vec{F}_{net} = \vec{0}$ only if the upward spring force exactly balances the downward gravitational force:

$$F_{sp} = F_G = mg$$

FIGURE 6.9 A spring scale measures weight.



Because we defined weight as the reading F_{sp} of a spring scale, the weight of a stationary object is

$$w = mg \qquad \text{(weight of a stationary object)} \qquad (6.8)$$

The scale does not "know" the weight of the object. All it can do is to measure how much its spring is stretched or compressed. On earth, a student with a mass of 70 kg has weight $w = (70\ \text{kg})(9.80\ \text{m/s}^2) = 686\ \text{N}$ *because* he compresses a spring until the spring pushes upward with 686 N. On a different planet, with a different value for g, the expansion or compression of the spring would be different and the student's weight would be different.

NOTE ▶ Mass and weight are not the same thing. Mass, in kg, is an intrinsic property of an object; its value is unique and always the same. Weight, in N, does depend on the object's mass, but it also depends on the situation—the strength of gravity and, as we will see, whether or not the object is accelerating. Weight is *not* a property of the object, and thus weight does not have a unique value. ◀

The unit of force in the English system is the *pound,* which is defined as $1\ \text{lb} \equiv 4.45\ \text{N}$. An object whose weight (on earth) is $w = mg = 4.45\ \text{N}$ has mass

$$m = \frac{w}{g} = \frac{4.45\ \text{N}}{9.80\ \text{m/s}^2} = 0.454\ \text{kg} = 454\ \text{g}$$

You may have learned in previous science classes that "1 pound = 454 grams" or, equivalently, that "1 kg = 2.2 lb." Strictly speaking, these well-known "conversion factors" are not true. They are comparing a weight (pounds) to a mass (kilograms). The correct statement is: "A mass of 1 kg has a weight *on earth* of 2.2 pounds." On another planet, the weight of a 1 kg mass would be something other than 2.2 pounds.

EXAMPLE 6.5 Mass and weight on Jupiter

What is the kilograms-to-pounds conversion factor on Jupiter, where the free-fall acceleration is 25.9 m/s^2?

SOLVE Consider an object with a mass of 1 kg. Its weight on Jupiter is

$$w_{\text{Jupiter}} = mg_{\text{Jupiter}} = (1\text{ kg})(25.9\text{ m/s})^2$$
$$= 25.9\text{ N} \times \frac{1\text{ lb}}{4.45\text{ N}} = 5.82\text{ lb}$$

If you had gone to school on Jupiter, you would have learned that 1 kg = 5.82 lb.

You may never have thought about it, but you cannot directly feel or sense gravity. Your *sensation*—how heavy you feel—is due to contact forces pressing against you, forces that touch you and activate nerve endings in your skin. As you read this, your sensation of weight is due to the normal force exerted on you by the chair in which you are sitting. When you stand, you feel the contact force of the floor pushing against your feet.

But recall the sensations you feel while accelerating. You feel "heavy" when an elevator suddenly accelerates upward, but this sensation vanishes as soon as the elevator reaches a steady speed. Your stomach seems to rise a little and you feel lighter than normal as the upward-moving elevator brakes to a halt or a roller coaster goes over the top. Has your weight actually changed?

To answer this question, FIGURE 6.10 shows a man weighing himself on a spring scale in an accelerating elevator. The only forces acting on the man are the upward spring force of the scale and the downward gravitational force. This seems to be the same situation as Figure 6.9b, but there's one big difference: The man is accelerating, hence there must be a net force on the man in the direction of $\vec{a}$.

For the net force $\vec{F}_{\text{net}}$ to point upward, the magnitude of the spring force must be *greater* than the magnitude of the gravitational force. That is, $F_{\text{sp}} > mg$. Looking at the free-body diagram in Figure 6.10, we see that the y-component of Newton's second law is

$$(F_{\text{net}})_y = (F_{\text{sp}})_y + (F_{\text{G}})_y = F_{\text{sp}} - mg = ma_y \qquad (6.9)$$

where m is the man's mass.

We defined weight as the reading F_{sp} of a calibrated spring scale *on which the object is stationary.* That is the case here as the scale and man accelerate upward together. Thus the man's weight as he accelerates vertically is

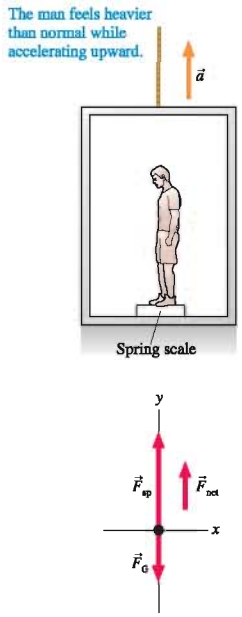
$$w = \text{scale reading } F_{\text{sp}} = mg + ma_y = mg\left(1 + \frac{a_y}{g}\right) \qquad (6.10)$$

If an object is either at rest or moving with constant velocity, then $a_y = 0$ and $w = mg$. That is, the weight of an object at rest is the magnitude of the (effective) gravitational force acting on it. But its weight differs if it has a vertical acceleration.

You *do* weigh more as an elevator accelerates upward ($a_y > 0$) because the reading of a scale—a weighing—increases. Similarly, your weight is less when the acceleration vector $\vec{a}$ points downward ($a_y < 0$) because the scale reading goes down. Weight, as we've defined it, corresponds to your sensation of heaviness or lightness.*

We found Equation 6.10 by considering a person in an accelerating elevator, but it applies to any object with a vertical acceleration. Further, an object doesn't really have

FIGURE 6.10 A man weighing himself in an accelerating elevator.



*Surprisingly, there is no universally agreed-upon definition of *weight.* Some textbooks define weight as the gravitational force on an object, $\vec{w} = (mg, \text{down})$. In that case, the scale reading of an accelerating object, and your sensation of weight, is often called *apparent weight.* This textbook prefers the operational definition of *weight* as being what a scale reads, the result of a weighing measurement. You should be aware of these differences if you refer to other textbooks.

to be on a scale to have a weight; an object's weight is the magnitude of the contact force supporting it. It makes no difference whether this is the spring force of the scale or simply the normal force of the floor.

NOTE ▶ Informally, we sometimes say "This object weighs such and such" or "The weight of this object is. . . ." Strictly speaking, we should use the term "mass" because we're talking about a property of the object. Nonetheless, these expressions are widely used, and we'll interpret them as meaning mg, the weight of an object of mass m at rest ($a_y = 0$) on the surface of the earth or some other astronomical body. ◀

Astronauts are weightless as they orbit the earth.

Weightlessness

Suppose the elevator cable breaks and the elevator, along with the man and his scale, plunges straight down in free fall! What will the scale read? When the free-fall acceleration $a_y = -g$ is used in Equation 6.10, we find $w = 0$. In other words, *the man has no weight!*

Think about this carefully. Suppose, as the elevator falls, the man inside releases a ball from his hand. In the absence of air resistance, as Galileo discovered, both the man and the ball would fall at the same rate. From the man's perspective, the ball would appear to "float" beside him. Similarly, the scale would float beneath him and not press against his feet. He is what we call *weightless*. Gravity is still pulling down on him—that's why he's falling—but he has no *sensation* of weight as everything floats around him in free fall.

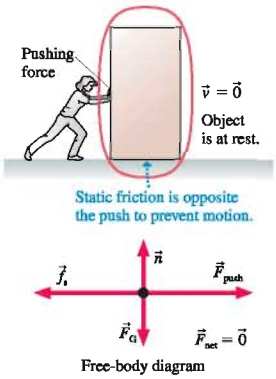
But isn't this exactly what happens to astronauts orbiting the earth? You've seen films of astronauts and various objects floating inside the space shuttle. If an astronaut tries to stand on a scale, it does not exert any force against her feet and reads zero. She is said to be weightless. But if the criterion to be weightless is to be in free fall, and if astronauts orbiting the earth are weightless, does this mean that they are in free fall? This is a very interesting question to which we shall return in Chapter 8.

STOP TO THINK 6.2 An elevator that has descended from the 50th floor is coming to a halt at the 1st floor. As it does, your weight is

a. More than mg. b. Less than mg. c. Equal to mg. d. Zero.

6.4 Friction

FIGURE 6.11 Static friction keeps an object from slipping.



Friction is absolutely essential for many things we do. Without friction you could not walk, drive, or even sit down (you would slide right off the chair!). It is sometimes useful to think about idealized frictionless situations, but it is equally necessary to understand a real world where friction is present. Although friction is a complicated force, many aspects of friction can be described with a simple model.

Static Friction

Chapter 5 defined *static friction* $\vec{f}_s$ as the force on an object that keeps it from slipping. FIGURE 6.11 shows a person pushing on a box with horizontal force $\vec{F}_{push}$. If the box remains at rest, "stuck" to the floor, it must be because of a static friction force pushing back to the left. The box is in static equilibrium, so the static friction must exactly balance the pushing force:

$$f_s = F_{push} \qquad (6.11)$$

To determine the direction of $\vec{f}_s$, decide which way the object would move if there were no friction. The static friction force $\vec{f}_s$ points in the *opposite* direction to prevent the motion.

Unlike the gravitational force, which has the precise and unambiguous magnitude $F_G = mg$, the size of the static friction force depends on how hard you push. The harder the person in Figure 6.11 pushes, the harder the floor pushes back. Reduce the pushing force, and the static friction force will automatically be reduced to match. Static friction acts in *response* to an applied force. FIGURE 6.12 illustrates this idea.

But there's clearly a limit to how big f_s can get. If you push hard enough, the object slips and starts to move. In other words, the static friction force has a *maximum* possible size $f_{s\,max}$.

- An object remains at rest as long as $f_s < f_{s\,max}$.
- The object slips when $f_s = f_{s\,max}$.
- A static friction force $f_s > f_{s\,max}$ is not physically possible.

Experiments with friction (first done by Leonardo da Vinci) show that $f_{s\,max}$ is proportional to the magnitude of the normal force. That is,

$$f_{s\,max} = \mu_s n \tag{6.12}$$

where the proportionality constant μ_s is called the **coefficient of static friction.** The coefficient is a dimensionless number that depends on the materials of which the object and the surface are made. Table 6.1 shows some typical coefficients of friction. It is to be emphasized that these are only approximate. The exact value of the coefficient depends on the roughness, cleanliness, and dryness of the surfaces.

NOTE ▶ Equation 6.12 does *not* say $f_s = \mu_s n$. The value of f_s depends on the force or forces that static friction has to balance to keep the object from moving. It can have any value from 0 up to, but not exceeding, $\mu_s n$. ◀

Kinetic Friction

Once the box starts to slide, in FIGURE 6.13, the static friction force is replaced by a kinetic friction force $\vec{f}_k$. Experiments show that kinetic friction, unlike static friction, has a nearly *constant* magnitude. Furthermore, the size of the kinetic friction force is *less* than the maximum static friction, $f_k < f_{s\,max}$, which explains why it is easier to keep the box moving than it was to start it moving. The direction of $\vec{f}_k$ is always opposite to the direction in which an object slides across the surface.

The kinetic friction force is also proportional to the magnitude of the normal force:

$$f_k = \mu_k n \tag{6.13}$$

where μ_k is called the **coefficient of kinetic friction.** Table 6.1 includes typical values of μ_k. You can see that $\mu_k < \mu_s$, causing the kinetic friction to be less than the maximum static friction.

Rolling Friction

If you slam on the brakes hard enough, your car tires slide against the road surface and leave skid marks. This is kinetic friction. A wheel *rolling* on a surface also experiences friction, but not kinetic friction. The portion of the wheel that contacts the surface is stationary with respect to the surface, not sliding. To see this, roll a wheel slowly and watch how it touches the ground.

Textbooks draw wheels as circles, but no wheel is perfectly round. The weight of the wheel, and of any object supported by the wheel, causes the bottom of the wheel to flatten where it touches the surface, as FIGURE 6.14 on the next page shows. The contact area between a car tire and the road is fairly large. The contact area between a steel locomotive wheel and a steel rail is much less, but it's not zero.

Molecular bonds are quickly established where the wheel presses against the surface. These bonds have to be broken as the wheel rolls forward, and the effort needed to break them causes **rolling friction.** (Think how it is to walk with a wad of chewing

FIGURE 6.12 Static friction acts in *response* to an applied force.

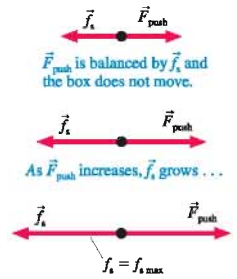
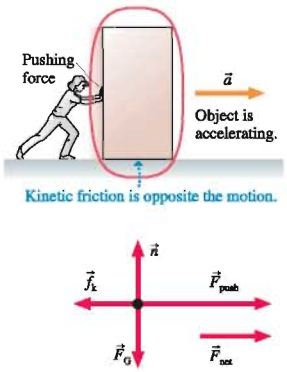


TABLE 6.1 Coefficients of friction

Materials	Static μ_s	Kinetic μ_k	Rolling μ_r
Rubber on concrete	1.00	0.80	0.02
Steel on steel (dry)	0.80	0.60	0.002
Steel on steel (lubricated)	0.10	0.05	
Wood on wood	0.50	0.20	
Wood on snow	0.12	0.06	
Ice on ice	0.10	0.03	

FIGURE 6.13 The kinetic friction force is opposite the direction of motion.



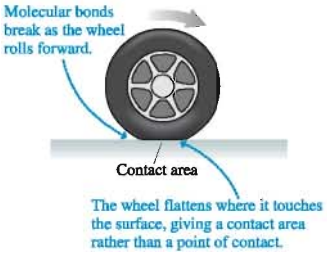


FIGURE 6.14 Rolling friction is due to the contact area between a wheel and the surface.

gum stuck to the sole of your shoe!) The force of rolling friction can be calculated in terms of a **coefficient of rolling friction** μ_r:

$$f_r = \mu_r n \tag{6.14}$$

Rolling friction acts very much like kinetic friction, but values of μ_r (see Table 6.1) are much lower than values of μ_k. This is why it is easier to roll an object on wheels than to slide it.

A Model of Friction

These ideas can be summarized in a *model* of friction:

$$\begin{aligned} &\text{Static: } \vec{f}_s \le (\mu_s n, \text{ direction as necessary to prevent motion}) \\ &\text{Kinetic: } \vec{f}_k = (\mu_k n, \text{ direction opposite the motion}) \\ &\text{Rolling: } \vec{f}_r = (\mu_r n, \text{ direction opposite the motion}) \end{aligned} \tag{6.15}$$

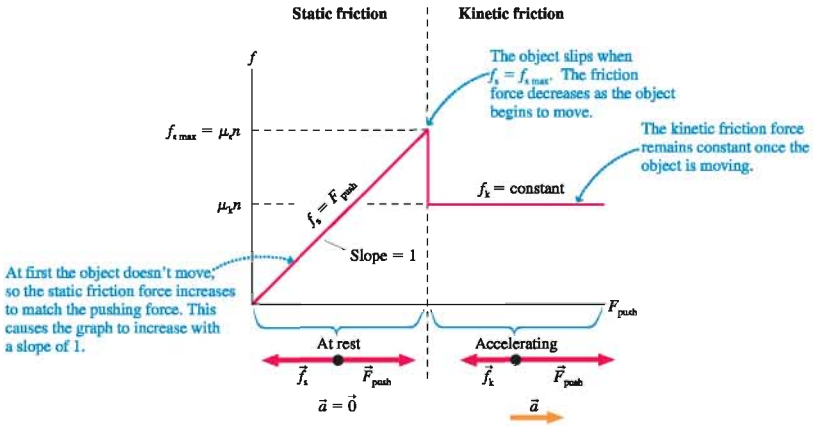
2.5, 2.6 Activ ONLINE Physics

Here "motion" means "motion relative to the surface." The maximum value of static friction $f_{s\,max} = \mu_s n$ occurs at the point where the object slips and begins to move.

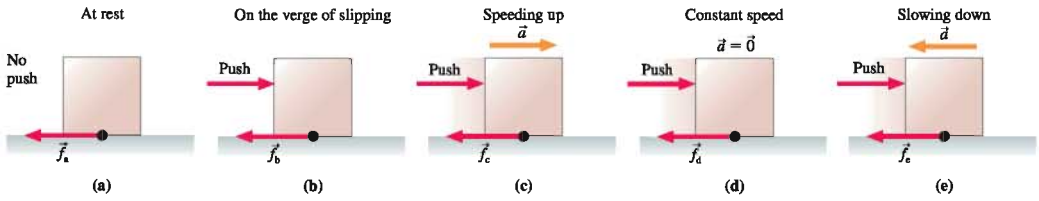
NOTE ▶ Equations 6.15 are a "model" of friction, not a "law" of friction. These equations provide a reasonably accurate, but not perfect, description of how friction forces act. For example, we've ignored the surface area of the object because surface area has little effect. Likewise, our model assumes that the kinetic friction force is independent of the object's speed. This is a fairly good, but not perfect, approximation. Equations 6.15 are a simplification of reality that works reasonably well, which is what we mean by a "model." They are not a "law of nature" on a level with Newton's laws. ◀

FIGURE 6.15 summarizes these ideas graphically by showing how the friction force changes as the magnitude of an applied force $\vec{F}_{push}$ increases.

FIGURE 6.15 The friction force response to an increasing applied force.



STOP TO THINK 6.3 Rank in order, from largest to smallest, the sizes of the friction forces $\vec{f}_a$ to $\vec{f}_e$ in these 5 different situations. The box and the floor are made of the same materials in all situations.



EXAMPLE 6.6 How far does a box slide?

Carol pushes a 50 kg wood box across a wood floor at a steady speed of 2.0 m/s. How much force does Carol exert on the box? If she stops pushing, how far will the box slide before coming to rest?

MODEL We model the box as a particle and we describe the friction forces with the model of static and kinetic friction. This is a two-part problem: first while Carol is pushing the box, then as it slides after she releases it.

VISUALIZE This is a fairly complex situation, one that calls for careful visualization. FIGURE 6.16 shows the pictorial representation both while Carol pushes, when $\vec{a} = \vec{0}$, and after she stops. We've placed $x = 0$ at the point where she stops pushing because this is the point where the kinematics calculation for "How far?" will begin. Notice that each part of the motion needs its own free-body diagram. The box is moving until the very instant that the problem ends, so only kinetic friction is relevant.

SOLVE We'll start by finding how hard Carol has to push to keep the box moving at a steady speed. The box is in dynamic equilibrium ($\vec{a} = \vec{0}$), and Newton's first law is

$$\sum F_x = F_{push} - f_k = 0$$
$$\sum F_y = n - F_G = n - mg = 0$$

where we've used $F_G = mg$ for the gravitational force. The negative sign occurs in the first equation because $\vec{f}_k$ points to the left and thus the *component* is negative: $(f_k)_x = -f_k$. Similarly, $(F_G)_y = -F_G$ because the gravitational force vector points down. In addition to Newton's laws, we also have our model of kinetic friction:

$$f_k = \mu_k n$$

Altogether we have three simultaneous equations in the three unknowns F_{push}, f_k, and n. Fortunately, these equations are easy to solve. The y-component of Newton's law tells us that $n = mg$. We can then find the friction force to be

$$f_k = \mu_k mg$$

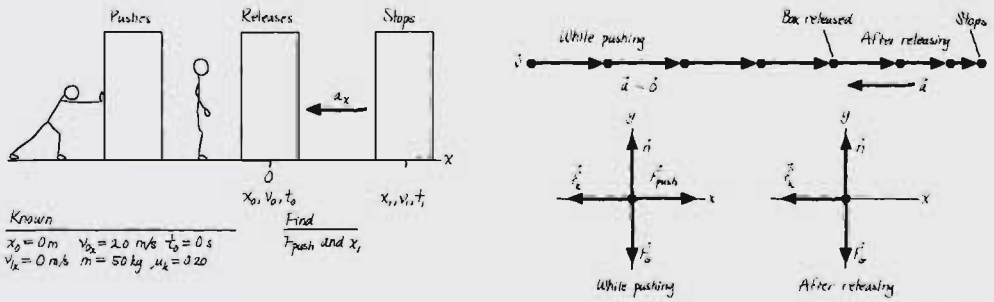
We substitute this into the x-component of the first law, giving

$$F_{push} = f_k = \mu_k mg$$
$$= (0.20)(50\text{ kg})(9.80\text{ m/s}^2) = 98\text{ N}$$

where μ_k for wood on wood was taken from Table 6.1. This is how hard Carol pushes to keep the box moving at a steady speed.

The box is not in equilibrium after Carol stops pushing it. Our strategy for the second half of the problem is to use Newton's second law to find the acceleration, then use kinematics to find how far the box moves before stopping. We know from the

FIGURE 6.16 Pictorial representation of a box sliding across a floor.



Continued

motion diagram that $a_y = 0$. Newton's second law, applied to the second free-body diagram of Figure 6.16, is

$$\sum F_x = -f_k = ma_x$$
$$\sum F_y = n - mg = ma_y = 0$$

We also have our model of friction,

$$f_k = \mu_k n$$

We see from the y-component equation that $n = mg$, and thus $f_k = \mu_k mg$. Using this in the x-component equation gives

$$ma_x = -f_k = -\mu_k mg$$

This is easily solved to find the box's acceleration:

$$a_x = -\mu_k g = -(0.20)(9.80\ \text{m/s}^2) = -1.96\ \text{m/s}^2$$

The acceleration component a_x is negative because the acceleration vector $\vec{a}$ points to the left, as we see from the motion diagram.

Now we are left with a problem of constant-acceleration kinematics. We are interested in a distance, rather than a time interval, so the easiest way to proceed is

$$v_{1x}^2 = 0 = v_{0x}^2 + 2a_x \Delta x = v_{0x}^2 + 2a_x x_1$$

from which the distance that the box slides is

$$x_1 = \frac{-v_{0x}^2}{2a_x} = \frac{-(2.0\ \text{m/s})^2}{2(-1.96\ \text{m/s}^2)} = 1.0\ \text{m}$$

We get a positive answer because the two negative signs cancel.

ASSESS Carol was pushing at 2 m/s ≈ 4 mph, which is fairly fast. The box slides 1.0 m, which is slightly over 3 feet. That sounds reasonable.

NOTE ▶ We needed both the horizontal and the vertical components of the second law even though the motion was entirely horizontal. This need is typical when friction is involved because we must find the normal force before we can evaluate the friction force. ◀

EXAMPLE 6.7 Dumping a file cabinet

A 50 kg steel file cabinet is in the back of a dump truck. The truck's bed, also made of steel, is slowly tilted. What is the size of the static friction force on the cabinet when the bed is tilted 20°? At what angle will the file cabinet begin to slide?

MODEL We'll model the file cabinet as a particle. We'll also use the model of static friction. The file cabinet will slip when the static friction force reaches its maximum value $f_{s\,max}$.

VISUALIZE FIGURE 6.17 shows the pictorial representation when the truck bed is tilted at angle θ. We can make the analysis easier if we tilt the coordinate system to match the bed of the truck. To prevent the file cabinet from slipping, the static friction force must point *up* the slope.

SOLVE The file cabinet is in static equilibrium. Newton's first law is

$$(F_{net})_x = \sum F_x = n_x + (F_G)_x + (f_s)_x = 0$$
$$(F_{net})_y = \sum F_y = n_y + (F_G)_y + (f_s)_y = 0$$

From the free-body diagram we see that f_s has only a *negative* x-component and that n has only a positive y-component. The gravitational force vector can be written $\vec{F}_G = +F_G\sin\theta\,\hat{i} - F_G\cos\theta\,\hat{j}$, so $\vec{F}_G$ has both x- and y-components in this coordinate system. Thus the first law becomes

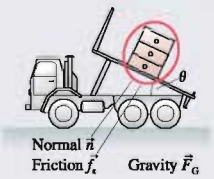
$$\sum F_x = F_G\sin\theta - f_s = mg\sin\theta - f_s = 0$$
$$\sum F_y = n - F_G\cos\theta = n - mg\cos\theta = 0$$

where we've used $F_G = mg$. The x-component equation allows us to determine the size of the static friction force when $\theta = 20°$:

$$f_s = mg\sin\theta = (50\ \text{kg})(9.80\ \text{m/s}^2)\sin 20°$$
$$= 170\ \text{N}$$

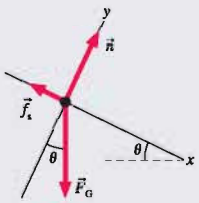
This value does not require knowing μ_s. We simply have to find the size of the friction force that will balance the component of $\vec{F}_G$ that points down the slope. The coefficient of static friction enters only when we want to find the angle at which the file cabinet slips.

FIGURE 6.17 The pictorial representation of a file cabinet in a tilted dump truck.



Known
$\mu_s = 0.80$ $m = 50$ kg
$\mu_k = 0.60$

Find
f_s where $\theta = 20°$
θ where cabinet slips



Slipping occurs when the static friction reaches its maximum value

$$f_s = f_{s\,max} = \mu_s n$$

From the y-component of Newton's law we see that $n = mg\cos\theta$. Consequently,

$$f_{s\,max} = \mu_s mg\cos\theta$$

Substituting this into the x-component of the first law gives

$$mg\sin\theta - \mu_s mg\cos\theta = 0$$

The mg in both terms cancels, and we find

$$\frac{\sin\theta}{\cos\theta} = \tan\theta = \mu_s$$

$$\theta = \tan^{-1}\mu_s = \tan^{-1}(0.80) = 39°$$

ASSESS Steel doesn't slide all that well on unlubricated steel, so a fairly large angle is not surprising. The answer seems reasonable. It is worth noting that $n = mg\cos\theta$ in this example. A common error is to use simply $n = mg$. Be sure to evaluate the normal force within the context of each specific problem.

The angle at which slipping begins is called the *angle of repose*. FIGURE 6.18 shows that knowing the angle of repose can be very important because it is the angle at which loose materials (gravel, sand, snow, etc.) begin to slide on a mountainside, leading to landslides and avalanches.

FIGURE 6.18 The angle of repose is the angle at which loose materials, such as gravel or snow, begin to slide.

Causes of Friction

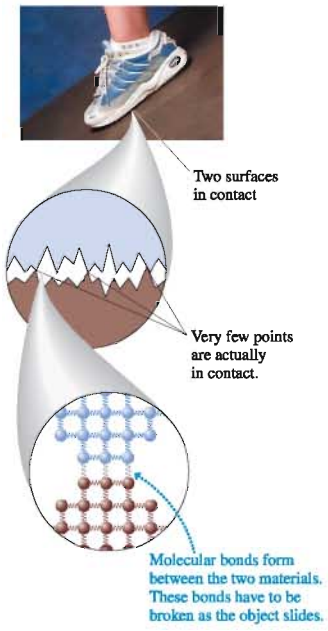
It is worth a brief pause to look at the *causes* of friction. All surfaces, even those quite smooth to the touch, are very rough on a microscopic scale. When two objects are placed in contact, they do not make a smooth fit. Instead, as FIGURE 6.19 shows, the high points on one surface become jammed against the high points on the other surface, while the low points are not in contact at all. Only a very small fraction (typically 10^{-4}) of the surface area is in actual contact. The amount of contact depends on how hard the surfaces are pushed together, which is why friction forces are proportional to n.

At the points of actual contact, the atoms in the two materials are pressed closely together and molecular bonds are established between them. These bonds are the "cause" of the static friction force. For an object to slip, you must push it hard enough to break these molecular bonds between the surfaces. Once they are broken, and the two surfaces are sliding against each other, there are still attractive forces between the atoms on the opposing surfaces as the high points of the materials push past each other. However, the atoms move past each other so quickly that they do not have time to establish the tight bonds of static friction. That is why the kinetic friction force is smaller.

Occasionally, in the course of sliding, two high points will be forced together so closely that they do form a tight bond. As the motion continues, it is not this surface bond that breaks but weaker bonds at the *base* of one of the high points. When this happens, a small piece of the object is left behind "embedded" in the surface. This is what we call *abrasion*. Abrasion causes materials to wear out as a result of friction, be they the piston rings in your car or the seat of your pants. In machines, abrasion is minimized with lubrication, a very thin film of liquid between the surfaces that allows them to "float" past each other with many fewer points in actual contact.

Friction, at the atomic level, is a very complex phenomenon. A detailed understanding of friction is at the forefront of engineering research today, where it is especially important for designing highly miniaturized machines and nanostructures.

FIGURE 6.19 An atomic-level view of friction.



6.5 Drag

The air exerts a drag force on objects as they move through the air. You experience drag forces every day as you jog, bicycle, ski, or drive your car. The drag force is especially important for the skydiver at the beginning of the chapter.

FIGURE 6.20 The drag force on a high-speed motorcyclist is significant.

The drag force $\vec{D}$

- Is opposite in direction to $\vec{v}$.
- Increases in magnitude as the object's speed increases.

FIGURE 6.20 illustrates the drag force.

Drag is a more complex force than ordinary friction because drag depends on the object's speed. Drag also depends on the object's shape and on the density of the medium through which it moves. Fortunately, we can use a fairly simple *model* of drag if the following three conditions are met:

- The object's size (diameter) is between a few millimeters and a few meters.
- The object's speed is less than a few hundred meters per second.
- The object is moving through the air near the earth's surface.

These conditions are usually satisfied for balls, people, cars, and many other objects of the everyday world. Under these conditions, the drag force can be written

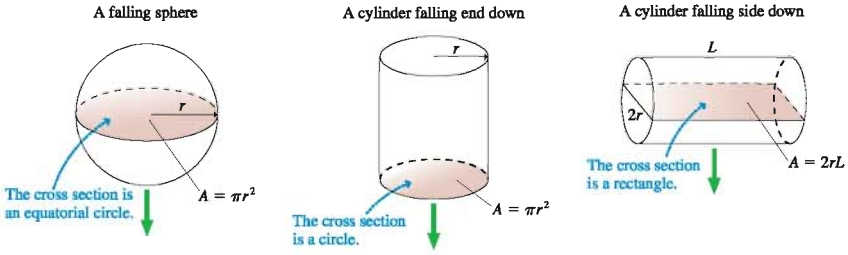
$$\vec{D} \approx (\tfrac{1}{4}Av^2, \text{ direction opposite the motion}) \qquad (6.16)$$

where A is the cross-section area of the object. The size of the drag force is proportional to the *square* of the object's speed. This model of drag fails for objects that are very small (such as dust particles), very fast (such as jet planes), or that move in other media (such as water). We'll leave those situations to more advanced textbooks.

NOTE ▶ Let's look at this model more closely. You may have noticed that an area multiplied by a speed squared does not give units of force. Unlike the $\frac{1}{2}$ in $\Delta x = \frac{1}{2}a(\Delta t)^2$, which is a "pure" number, the $\frac{1}{4}$ in the expression for $\vec{D}$ has units. This number depends on the air's density, and it's actually $\frac{1}{4}$ kg/m^3. We've suppressed the units in Equation 6.16, but doing so gives us an expression that works *only* if A is in m^2 and v is in m/s. Equation 6.16 cannot be converted to other units. And the number is not exactly $\frac{1}{4}$, which is why Equation 6.16 has an $\approx$ sign rather than an $=$ sign, but it's close enough to allow Equation 6.16 to be a reasonable yet simple model of drag. ◀

The area in Equation 6.16 is the cross section of the object as it "faces into the wind." FIGURE 6.21 shows how to calculate the cross-section area for objects of different shape. It's interesting to note that the magnitude of the drag force, $\frac{1}{4}Av^2$, depends on the object's *size and shape* but not on its *mass*. We will see shortly that this mass independence has important consequences.

FIGURE 6.21 Cross-section areas for objects of different shape.



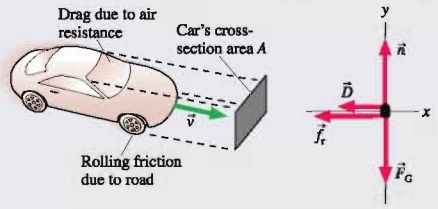
EXAMPLE 6.8 Air resistance compared to rolling friction

The profile of a typical 1500 kg passenger car, as seen from the front, is 1.6 m wide and 1.4 m high. At what speed does the magnitude of the drag equal the magnitude of the rolling friction?

MODEL Treat the car as a particle. Use the models of rolling friction and drag.

VISUALIZE FIGURE 6.22 shows the car and a free-body diagram. A full pictorial representation is not needed because we won't be doing any kinematics calculations.

FIGURE 6.22 A car experiences both rolling friction and drag.



SOLVE Drag is less than friction at low speeds, where air resistance is negligible. But drag increases as v increases, so there will be a speed at which the two forces are equal in size. Above this speed, drag is more important than rolling friction.

The magnitudes of the forces are $D \approx \frac{1}{4}Av^2$ and $f_r = \mu_r n$. There's no motion and no acceleration in the vertical direction, so we can see from the free-body diagram that $n = F_G = mg$. Thus $f_r = \mu_r mg$. Equating friction and drag, we have

$$\tfrac{1}{4}Av^2 = \mu_r mg$$

Solving for v, we find

$$v = \sqrt{\frac{4\mu_r mg}{A}} = \sqrt{\frac{4(0.02)(1500\text{ kg})(9.8\text{ m/s}^2)}{(1.4\text{ m})(1.6\text{ m})}} = 23\text{ m/s}$$

where the value of μ_r for rubber on concrete was taken from Table 6.1.

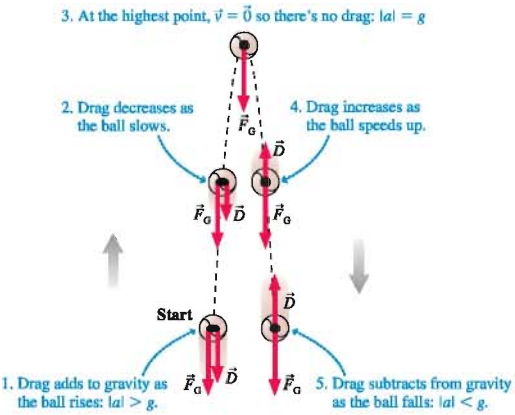
ASSESS 23 m/s is approximately 50 mph, a reasonable result. This calculation shows that our assumption that we can ignore air resistance is really quite good for car speeds less than 30 or 40 mph. Calculations that neglect drag will be increasingly inaccurate as speeds go above 50 mph.

FIGURE 6.23 shows a ball moving up and down vertically. If there were no air resistance, the ball would be in free fall with $a_{\text{free fall}} = -g$ throughout its flight. Let's see how drag changes this.

Referring to Figure 6.23:

1. The drag force $\vec{D}$ points down as the ball rises. This *increases* the net force on the ball and causes the ball to slow down *more quickly* than it would in a vacuum. The magnitude of the acceleration, which we'll calculate below, is $|a| > g$.
2. The drag force decreases as the ball slows.
3. $\vec{v} = \vec{0}$ at the highest point in the ball's motion, so there's no drag and the acceleration is simply $a_{\text{free fall}} = -g$.
4. The drag force increases as the ball speeds up.
5. The drag force $\vec{D}$ points up as the ball falls. This *decreases* the net force on the ball and causes the ball to speed up *less quickly* than it would in a vacuum. The magnitude of the acceleration is $|a| < g$.

FIGURE 6.23 Drag force on a ball moving vertically.



We can use Newton's second law to find the ball's acceleration $a_\uparrow$ as it rises. You can see from the forces in Figure 6.23 that

$$a_\uparrow = \frac{(F_{\text{net}})_y}{m} = \frac{-mg - D}{m} = -\left(g + \frac{D}{m}\right) \qquad (6.17)$$

The magnitude of $a_\uparrow$, which is the ball's deceleration as it rises, is $g + D/m$. Air resistance causes the ball to slow down *more quickly* than it would in a vacuum. But Equation 6.17 tells us more. Because D depends on the object's size but not on its mass, drag has a larger *effect* (larger acceleration) on a less massive ball than on a more massive ball of the same size.

A Ping-Pong ball and a golf ball are about the same size, but it's harder to throw the Ping-Pong ball than the golf ball. We can now give an *explanation:*

- The drag force has the same magnitude for two objects of equal size.
- According to Newton's second law, the acceleration (the *effect* of the force) depends inversely on the mass.
- Therefore, the effect of the drag force is larger on a less massive ball than on a more massive ball of equal size.

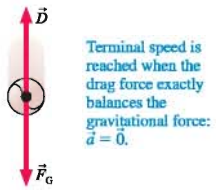
As the ball in Figure 6.23 falls, its acceleration $a_\downarrow$ is

$$a_\downarrow = \frac{(F_{net})_y}{m} = \frac{-mg + D}{m} = -\left(g - \frac{D}{m}\right) \tag{6.18}$$

The magnitude of $a_\downarrow$ is $g - D/m$, so the ball speeds up *less quickly* than it would in a vacuum. Once again, the effect is larger for a less massive ball than for a more massive ball of equal size.

Terminal Speed

FIGURE 6.24 An object falling at terminal speed.



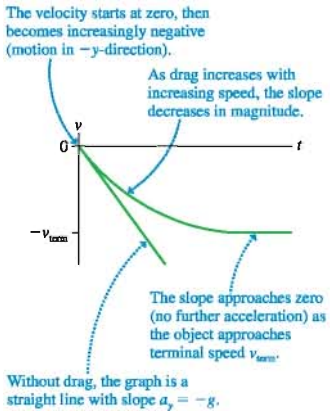
The drag force increases as an object falls and gains speed. If the object falls far enough, it will eventually reach a speed, shown in FIGURE 6.24, at which $D = F_G$. That is, the drag force will be equal and opposite to the gravitational force. The net force at this speed is $\vec{F}_{net} = \vec{0}$, so there is no further acceleration and the object falls with a *constant* speed. The speed at which the exact balance between the upward drag force and the downward gravitational force causes an object to fall without acceleration is called the **terminal speed** v_{term}. Once an object has reached terminal speed, it will continue falling at that speed until it hits the ground.

It's not hard to compute the terminal speed. It is the speed, by definition, at which $D = F_G$ or, equivalently, $\frac{1}{4}Av^2 \approx mg$. This speed is

$$v_{term} \approx \sqrt{\frac{4mg}{A}} \tag{6.19}$$

A more massive object has a larger terminal speed than a less massive object of equal size. A 10-cm-diameter lead ball, with a mass of 6 kg, has a terminal speed of 170 m/s, while a 10-cm-diameter Styrofoam ball, with a mass of 50 g, has a terminal speed of only 15 m/s.

FIGURE 6.25 The velocity-versus-time graph of a falling object with and without drag.



A popular use of Equation 6.19 is to find the terminal speed of a skydiver. A skydiver is rather like the cylinder of Figure 6.21 falling "side down." A typical skydiver is 1.8 m long and 0.40 m wide ($A = 0.72\ m^2$) and has a mass of 75 kg. His terminal speed is

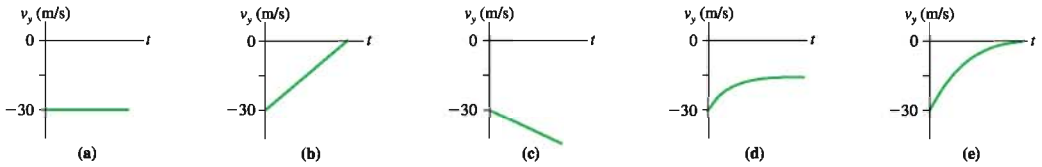
$$v_{term} \approx \sqrt{\frac{4mg}{A}} = \sqrt{\frac{4(75\ kg)(9.8\ m/s^2)}{0.72\ m^2}} = 64\ m/s$$

This is roughly 140 mph. A higher speed can be reached by falling feet first or head first, which reduces the area A.

FIGURE 6.25 shows the results of a more detailed calculation for a falling object. Without drag, the velocity graph is a straight line with slope $= a_y = -g$. When drag is included, the slope steadily decreases in magnitude and approaches zero (no further acceleration) as the object reaches terminal speed.

Although we've focused our analysis on objects moving vertically, the same ideas apply to objects moving horizontally. If an object is thrown or shot horizontally, $\vec{D}$ causes the object to slow down. An airplane reaches its maximum speed, which is analogous to the terminal speed, when the drag is equal and opposite to the thrust: $D = F_{thrust}$. The net force is then zero and the plane cannot go any faster. The maximum speed of a passenger jet is about 550 mph.

STOP TO THINK 6.4 The terminal speed of a Styrofoam ball is 15 m/s. Suppose a Styrofoam ball is shot straight down with an initial speed of 30 m/s. Which velocity graph is correct?



6.6 More Examples of Newton's Second Law

We will finish this chapter with three additional examples in which we use the problem-solving strategy in more complex scenarios.

Activ Physics ONLINE 2.7, 2.8, 2.9

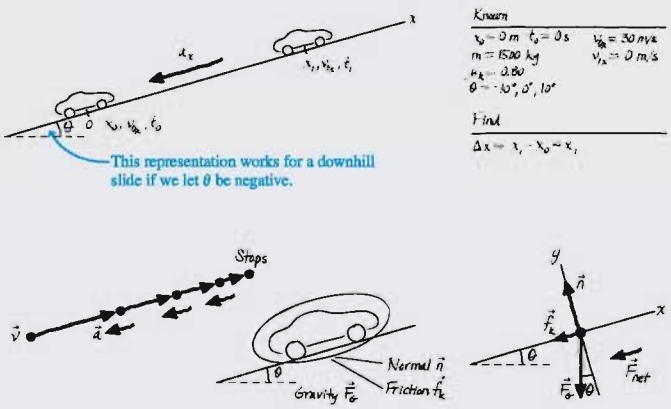
EXAMPLE 6.9 Stopping distances

A 1500 kg car is traveling at a speed of 30 m/s when the driver slams on the brakes and skids to a halt. Determine the stopping distance if the car is traveling up a 10° slope, down a 10° slope, or on a level road.

MODEL We'll represent the car as a particle and we'll use the model of kinetic friction. We want to solve the problem only once, not three separate times, so we'll leave the slope angle θ unspecified until the end.

VISUALIZE FIGURE 6.26 shows the pictorial representation. We've shown the car sliding uphill, but these representations work equally well for a level or downhill slide if we let θ be zero or negative, respectively. We've used a tilted coordinate system so that the motion is along one of the axes. We've *assumed* that the car is traveling to the right, although the problem didn't state this. You could equally well make the opposite assumption, but you would have to be careful with negative values of x and v_x. The car *skids* to a halt, so we've taken the coefficient of *kinetic* friction for rubber on concrete from Table 6.1.

FIGURE 6.26 Pictorial representation of a skidding car.



Continued

SOLVE Newton's second law and the model of kinetic friction are

$$\sum F_x = n_x + (F_G)_x + (f_k)_x$$
$$= -mg\sin\theta - f_k = ma_x$$
$$\sum F_y = n_y + (F_G)_y + (f_k)_y$$
$$= n - mg\cos\theta = ma_y = 0$$
$$f_k = \mu_k n$$

We've written these equations by "reading" the motion diagram and the free-body diagram. Notice that both components of the gravitational force vector $\vec{F}_G$ are negative. $a_y = 0$ because the motion is entirely along the x-axis.

The second equation gives $n = mg\cos\theta$. Using this in the friction model, we find $f_k = \mu_k mg\cos\theta$. Inserting this result back into the first equation then gives

$$ma_x = -mg\sin\theta - \mu_k mg\cos\theta$$
$$= -mg(\sin\theta + \mu_k\cos\theta)$$
$$a_x = -g(\sin\theta + \mu_k\cos\theta)$$

This is a constant acceleration. Constant-acceleration kinematics gives

$$v_{1x}^2 = 0 = v_{0x}^2 + 2a_x(x_1 - x_0) = v_{0x}^2 + 2a_x x_1$$

which we can solve for the stopping distance x_1:

$$x_1 = -\frac{v_{0x}^2}{2a_x} = \frac{v_{0x}^2}{2g(\sin\theta + \mu_k\cos\theta)}$$

Notice how the minus sign in the expression for a_x canceled the minus sign in the expression for x_1. Evaluating our result at the three different angles gives the stopping distances:

$$x_1 = \begin{cases} 48\text{ m} & \theta = 10^\circ & \text{uphill} \\ 57\text{ m} & \theta = 0^\circ & \text{level} \\ 75\text{ m} & \theta = -10^\circ & \text{downhill} \end{cases}$$

The implications are clear about the danger of driving downhill too fast!

ASSESS 30 m/s ≈ 60 mph and 57 m ≈ 180 feet on a level surface. This is similar to the stopping distances you learned when you got your driver's license, so the results seem reasonable. Additional confirmation comes from noting that the expression for a_x becomes $-g\sin\theta$ if $\mu_k = 0$. This is what you learned in Chapter 2 for the acceleration on a frictionless inclined plane.

This is a good example for pointing out the advantages of working problems *algebraically*. If you had started plugging in numbers early, you would not have found that the mass eventually cancels out and you would have done several needless calculations. In addition, it is now easy to calculate the stopping distance for different angles. Had you been computing numbers, rather than algebraic expressions, you would have had to go all the way back to the beginning for each angle.

EXAMPLE 6.10 A dog sled race

It's dog sled race day in Alaska! A wooden sled, with rider and supplies, has a mass of 200 kg. When the starting gun sounds, it takes the dogs 15 m to reach their "cruising speed" of 5.0 m/s across the snow. Two ropes are attached to the sled, one on each side of the dogs. The ropes pull upward at 10°. What are the tensions in the ropes at the start of the race?

MODEL We'll represent the sled as a particle and we'll use the model of kinetic friction. We interpret the question as asking for the *magnitude* T of the tension forces. We'll assume that the tensions in the two ropes are equal and that the acceleration is constant during the first 15 m.

VISUALIZE FIGURE 6.27 shows the pictorial representation. Notice that the tension forces $\vec{T}_1$ and $\vec{T}_2$ are tilted up, but the net force is directly to the right in order to match the acceleration $\vec{a}$ of the motion diagram.

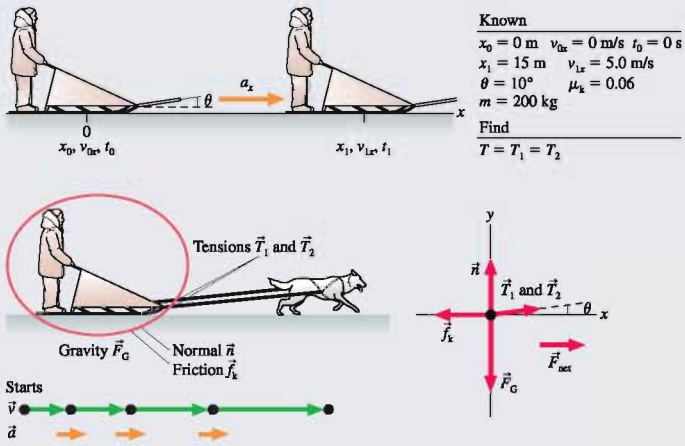
SOLVE We have enough information to calculate the acceleration. We can then use $\vec{a}$ to find the tension. From kinematics,

$$v_{1x}^2 = v_{0x}^2 + 2a_x(x_1 - x_0) = 2a_x x_1$$
$$a_x = \frac{v_{1x}^2}{2x_1} = \frac{(5.0\text{ m/s})^2}{2(15\text{ m})} = 0.833\text{ m/s}^2$$

Newton's second law can be written by "reading" the free-body diagram:

$$\sum F_x = n_x + T_{1x} + T_{2x} + (F_G)_x + (f_k)_x$$
$$= 2T\cos\theta - f_k = ma_x$$
$$\sum F_y = n_y + T_{1y} + T_{2y} + (F_G)_y + (f_k)_y$$
$$= n + 2T\sin\theta - mg = ma_y = 0$$

FIGURE 6.27 Pictorial representation of an accelerating dog sled.



Make sure you understand where all the terms come from, including their signs. We've used $F_G = mg$ and our knowledge that $\vec{a}$ has only an x-component. The tensions $\vec{T}_1$ and $\vec{T}_2$ have both x- and y-components. The assumption of equal tensions allows us to write $T_1 = T_2 = T$, and this introduces the factors of 2.

In addition, we have the model of kinetic friction

$$f_k = \mu_k n$$

From the y-equation and the friction equation,

$$n = mg - 2T\sin\theta$$

$$f_k = \mu_k n = \mu_k mg - 2\mu_k T\sin\theta$$

Notice that n is *not* equal to mg. The y-components of the tension forces support part of the weight, so the ground does not press against the bottom of the sled as hard as it would otherwise.

Substituting the friction back into the x-equation gives

$$2T\cos\theta - (\mu_k mg - 2\mu_k T\sin\theta)$$
$$= 2T(\cos\theta + \mu_k\sin\theta) - \mu_k mg = ma_x$$

$$T = \frac{1}{2}\frac{m(a_x + \mu_k g)}{\cos\theta + \mu_k\sin\theta}$$

Using $a_x = 0.833$ m/s^2 from above with $m = 200$ kg and $\theta = 10°$, we find that the tension is

$$T = 140 \text{ N}$$

ASSESS It's a bit hard to assess this result. We do know that the weight of the sled is $mg \approx 2000$ N. We also know that the dogs can drag a sled over snow (small μ_k) but probably can't lift the sled straight up, so we anticipate that $T \ll 2000$ N. Our calculation agrees.

EXAMPLE 6.11 **Make sure the cargo doesn't slide**

A 100 kg box of dimensions 50 cm × 50 cm × 50 cm is in the back of a flatbed truck. The coefficients of friction between the box and the bed of the truck are $\mu_s = 0.40$ and $\mu_k = 0.20$. What is the maximum acceleration the truck can have without the box slipping?

MODEL This is a somewhat different problem from any we have looked at thus far. Let the box, which we'll model as a particle, be the object of interest. It contacts other objects only where it touches the truck bed, so only the truck can exert contact forces on the box. If the box does *not* slip, then there is no motion of the box *relative to the truck* and the box must accelerate *with the truck:* $a_{box} = a_{truck}$. As the box accelerates, it must, according to Newton's second law, have a net force acting on it. But from what?

Imagine, for a moment, that the truck bed is frictionless. The box would slide backward (as seen in the truck's reference frame) as the truck accelerates. The force that prevents sliding is *static friction,* so the truck must exert a static friction force on the box to "pull" the box along with it and prevent the box from sliding *relative to the truck.*

Continued

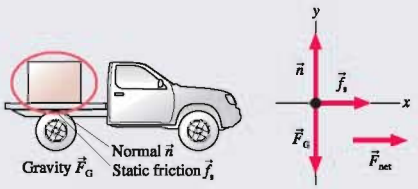
FIGURE 6.28 Pictorial representation for the box in a flatbed truck.

Known

$m = 100$ kg
Box dimensions 50 cm × 50 cm × 50 cm
$\mu_s = 0.40$ $\mu_k = 0.20$

Find

Acceleration at which box slips



VISUALIZE This situation is shown in **FIGURE 6.28**. There is only one horizontal force on the box, $\vec{f}_s$, and it points in the *forward* direction to accelerate the box. Notice that we're solving the problem with the ground as our reference frame. Newton's laws are not valid in the accelerating truck because it is not an inertial reference frame.

SOLVE Newton's second law, which we can "read" from the free-body diagram, is

$$\sum F_x = f_s = ma_x$$

$$\sum F_y = n - F_G = n - mg = ma_y = 0$$

Now, static friction, you will recall, can be *any* value between 0 and $f_{s\,max}$. If the truck accelerates slowly, so that the box doesn't slip, then $f_s < f_{s\,max}$. However, we're interested in the acceleration a_{max} at which the box begins to slip. This is the acceleration at which f_s reaches its maximum possible value

$$f_s = f_{s\,max} = \mu_s n$$

The y-equation of the second law and the friction model combine to give $f_{s\,max} = \mu_s mg$. Substituting this into the x-equation, and noting that a_x is now a_{max}, we find

$$a_{max} = \frac{f_{s\,max}}{m} = \mu_s g = 3.9 \text{ m/s}^2$$

The truck must keep its acceleration less than 3.9 m/s^2 if slipping is to be avoided.

ASSESS 3.9 m/s^2 is about one-third of g. You may have noticed that items in a car or truck are likely to *tip over* when you start or stop, but they slide only if you really floor it and accelerate very quickly. So this answer seems reasonable. Notice that the dimensions of the crate were not needed. Real-world situations rarely have exactly the information you need, no more and no less. Many problems in this textbook will require you to assess the information in the problem statement in order to learn which is relevant to the solution.

The mathematical representation of this last example was quite straightforward. The challenge was in the analysis that preceded the mathematics—that is, in the *physics* of the problem rather than the mathematics. It is here that our analysis tools—motion diagrams, force identification, and free-body diagrams—prove their value.

SUMMARY

The goal of Chapter 6 has been to learn how to solve problems about motion in a straight line.

General Strategy

All examples in this chapter follow a four-part strategy. You'll become a better problem solver if you adhere to it as you do the homework problems. The *Dynamics Worksheets* in the *Student Workbook* will help you structure your work in this way.

Equilibrium Problems

Object at rest or moving with constant velocity.

MODEL Make simplifying assumptions.

VISUALIZE

- Translate words into symbols.
- Identify forces.
- Draw a free-body diagram.

SOLVE Use Newton's first law:

$$\vec{F}_{\text{net}} = \sum_i \vec{F}_i = \vec{0}$$

"Read" the vectors from the free-body diagram.

ASSESS Is the result reasonable?

Go back and forth between these steps as needed.

Dynamics Problems

Object accelerating.

MODEL Make simplifying assumptions.

VISUALIZE

- Translate words into symbols.
- Draw a sketch to define the situation.
- Draw a motion diagram.
- Identify forces.
- Draw a free-body diagram.

SOLVE Use Newton's second law:

$$\vec{F}_{\text{net}} = \sum_i \vec{F}_i = m\vec{a}$$

"Read" the vectors from the free-body diagram. Use kinematics to find velocities and positions.

ASSESS Is the result reasonable?

Important Concepts

Specific information about three important forces:

Gravity $\vec{F}_G = (mg, \text{downward})$

Friction $\vec{f}_s = (0 \text{ to } \mu_s n, \text{ direction as necessary to prevent motion})$

$\vec{f}_k = (\mu_k n, \text{ direction opposite the motion})$

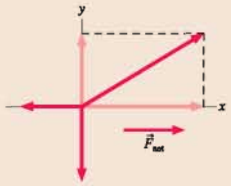
$\vec{f}_r = (\mu_r n, \text{ direction opposite the motion})$

Drag $\vec{D} \approx \left(\frac{1}{4}Av^2, \text{ direction opposite the motion}\right)$

Newton's laws are vector expressions. You must write them out by components:

$$(F_{\text{net}})_x = \sum F_x = ma_x \text{ or } 0$$

$$(F_{\text{net}})_y = \sum F_y = ma_y \text{ or } 0$$



Applications

The **weight** of an object is the reading of a calibrated spring scale on which the object is stationary. Weight is the result of weighing. The weight of an object with vertical acceleration a_y is

$$w = mg\left(1 + \frac{a_y}{g}\right)$$

A falling object reaches **terminal speed**

$$v_{\text{term}} \approx \sqrt{\frac{4mg}{A}}$$

Terminal speed is reached when the drag force exactly balances the gravitational force: $\vec{a} = \vec{0}$.

Terms and Notation

flat-earth approximation
weight
coefficient of static friction, μ_s
coefficient of kinetic friction, μ_k
rolling friction
coefficient of rolling friction, μ_r
terminal speed, v_{term}

For homework assigned on MasteringPhysics, go to www.masteringphysics.com

Problem difficulty is labeled as | (straightforward) to ||| (challenging).

Problems labeled can be done on a Dynamics Worksheet.

Problems labeled integrate significant material from earlier chapters.

CONCEPTUAL QUESTIONS

1. Are the objects described here in static equilibrium, dynamic equilibrium, or not in equilibrium at all? Explain.
   a. A 200 pound barbell is held over your head.
   b. A girder is lifted at constant speed by a crane.
   c. A girder is being lowered into place. It is slowing down.
   d. A jet plane has reached its cruising speed and altitude.
   e. A box in the back of a truck doesn't slide as the truck stops.
2. A ball tossed straight up has $v = 0$ at its highest point. Is it in equilibrium? Explain.
3. Kat, Matt, and Nat are arguing about why a physics book on a table doesn't fall. According to Kat, "Gravity pulls down on it, but the table is in the way so it can't fall." "Nonsense," says Matt. "There are all kinds of forces acting on the book, but the upward forces overcome the downward forces to prevent it from falling." "But what about Newton's first law?" counters Nat. "It's not moving, so there can't be any forces acting on it." None of the statements is exactly correct. Who comes closest, and how would you change his or her statement to make it correct?
4. "Forces cause an object to move." Do you agree or disagree with this statement? Explain.
5. If you know all of the forces acting on a moving object, can you tell the direction the object is moving? If yes, explain how. If no, give an example.
6. An elevator, hanging from a single cable, moves upward at constant speed. Friction and air resistance are negligible. Is the tension in the cable greater than, less than, or equal to the gravitational force on the elevator? Explain. Include a free-body diagram as part of your explanation.
7. An elevator, hanging from a single cable, moves downward and is slowing. Friction and air resistance are negligible. Is the tension in the cable greater than, less than, or equal to the gravitational force on the elevator? Explain. Include a free-body diagram as part of your explanation.
8. The three arrows in FIGURE Q6.8 have left the bow and are traveling parallel to the ground. Air resistance is negligible. Rank in order, from largest to smallest, the magnitudes F_a, F_b, and F_c of the *horizontal* forces acting on the arrows. Some may be equal. Give your answer in the form a > b = c and explain your ranking.

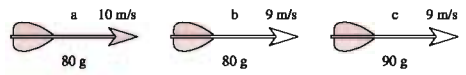


FIGURE Q6.8

9. Are the following statements true or false? Explain.
   a. The mass of an object depends on its location.
   b. The weight of an object depends on its location.
   c. Mass and weight describe the same thing in different units.
10. An astronaut takes his bathroom scale to the moon and then stands on it. Is the reading of the scale his weight? Explain.
11. The four balls in FIGURE Q6.11 have been thrown straight up. They have the same size, but different masses. Air resistance is negligible. Rank in order, from largest to smallest, the magnitude of the net force acting on each ball. Some may be equal. Give your answer in the form a > b = c > d and explain your ranking.

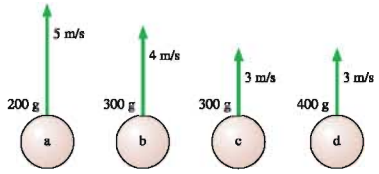


FIGURE Q6.11

12. The terms "vertical" and "horizontal" are frequently used in physics. Give *operational definitions* for these two terms. An operational definition defines a term by how it is measured or determined. Your definition should apply equally well in a laboratory or on a steep mountainside.
13. Suppose you attempt to pour out 100 g of salt, using a pan balance for measurements, while in a rocket accelerating upward. Will the quantity of salt be too much, too little, or the correct amount? Explain.
14. A box with a passenger inside is launched straight up into the air by a giant rubber band. Before launch, the passenger stood on a scale and weighed 750 N. Once the box has left the rubber band but is still moving upward, is the passenger's weight more than 750 N, 750 N, less than 750 N but not zero, or zero? Explain.
15. An astronaut orbiting the earth is handed two balls that have identical outward appearances. However, one is hollow while the other is filled with lead. How can the astronaut determine which is which? Cutting or altering the balls is not allowed.
16. A hand presses down on the book in FIGURE Q6.16. Is the normal force of the table on the book larger than, smaller than, or equal to mg?

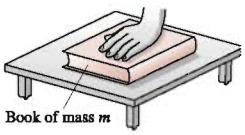


FIGURE Q6.16

17. Suppose you push a hockey puck of mass m across frictionless ice for a time Δt, starting from rest, giving the puck speed v after traveling distance d. If you repeat the experiment with a puck of mass $2m$,
    a. How long will you have to push for the puck to reach the same speed v?
    b. How long will you have to push for the puck to travel the same distance d?
18. A block pushed along the floor with velocity v_{0x} slides a distance d after the pushing force is removed.
    a. If the mass of the block is doubled but its initial velocity is not changed, what distance does the block slide before stopping?
    b. If the initial velocity is doubled to $2v_{0x}$ but the mass is not changed, what distance does the block slide before stopping?
19. Can the friction force on an object ever point in the direction of the object's motion? If yes, give an example. If no, why not?
20. A crate of fragile dishes is in the back of a pickup truck. The truck accelerates north from a stop sign, and the crate moves without slipping. Does the friction force on the crate point north or south? Or is the friction force zero? Explain.
21. The three boxes in FIGURE Q6.21 move through the air as shown. Rank in order, from largest to smallest, the magnitudes of the three drag forces D_a, D_b, and D_c acting on the boxes. Some may be equal. Give your answer in the form a > b = c and explain your ranking.

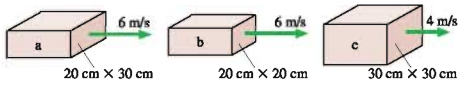


FIGURE Q6.21

22. Five balls move through the air as shown in FIGURE Q6.22. All five have the same size and shape. Air resistance is not negligible. Rank in order, from largest to smallest, the magnitudes of the accelerations a_a to a_e. Some may be equal. Give your answer in the form a > b = c > d > e and explain your ranking.

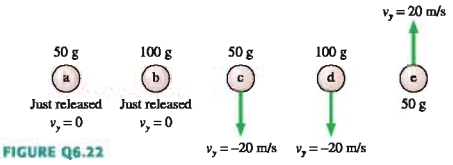


FIGURE Q6.22

EXERCISES AND PROBLEMS

Exercises

Section 6.1 Equilibrium

1. I The three ropes in FIGURE EX6.1 are tied to a small, very light ring. Two of the ropes are anchored to walls at right angles, and the third rope pulls as shown. What are T_1 and T_2, the magnitudes of the tension forces in the first two ropes?

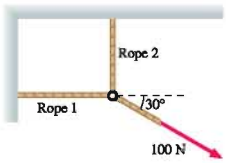


FIGURE EX6.1

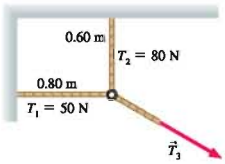


FIGURE EX6.2

2. I The three ropes in FIGURE EX6.2 are tied to a small, very light ring. Two of these ropes are anchored to walls at right angles with the tensions shown in the figure. What are the magnitude and direction of the tension $\vec{T}_3$ in the third rope?
3. II A 20 kg loudspeaker is suspended 2.0 m below the ceiling by two 3.0-m-long cables that angle outward at equal angles. What is the tension in the cables?
4. II A football coach sits on a sled while two of his players build their strength by dragging the sled across the field with ropes. The friction force on the sled is 1000 N and the angle between the two ropes is 20°. How hard must each player pull to drag the coach at a steady 2.0 m/s?

Section 6.2 Using Newton's Second Law

5. I In each of the two free-body diagrams, the forces are acting on a 2.0 kg object. For each diagram, find the values of a_x and a_y, the x- and y-components of the acceleration.

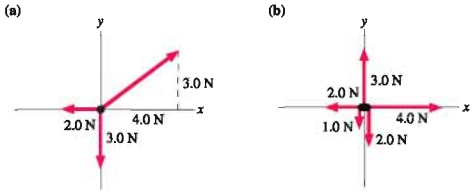


FIGURE EX6.5

6. || In each of the two free-body diagrams, the forces are acting on a 2.0 kg object. For each diagram, find the values of a_x and a_y, the x- and y-components of the acceleration.

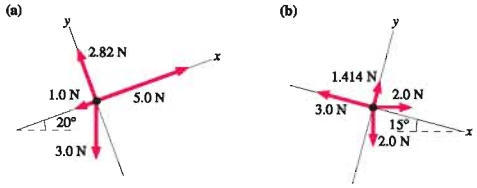


FIGURE EX6.6

7. || In each of the two free-body diagrams, the forces are acting on a 5.0 kg object. For each diagram, find the values of a_x and a_y, the x- and y-components of the acceleration.

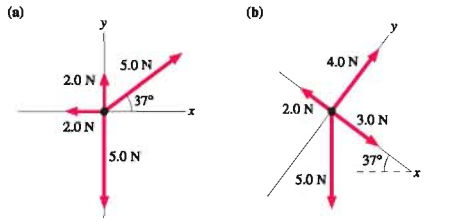


FIGURE EX6.7

8. || FIGURE EX6.8 shows the velocity graph of a 2.0 kg object as it moves along the x-axis. What is the net force acting on this object at t = 1 s? At 4 s? At 7 s?

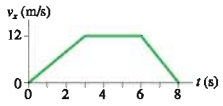


FIGURE EX6.8

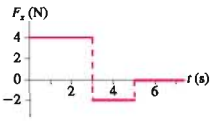


FIGURE EX6.9

9. || FIGURE EX6.9 shows the force acting on a 2.0 kg object as it moves along the x-axis. The object is at rest at the origin at t = 0 s. What are its acceleration and velocity at t = 6 s?
10. | A horizontal rope is tied to a 50 kg box on frictionless ice. What is the tension in the rope if:
    a. The box is at rest?
    b. The box moves at a steady 5.0 m/s?
    c. The box has v_x = 5.0 m/s and a_x = 5.0 m/s^2?
11. | A 50 kg box hangs from a rope. What is the tension in the rope if:
    a. The box is at rest?
    b. The box moves up at a steady 5.0 m/s?
    c. The box has v_y = 5.0 m/s and is speeding up at 5.0 m/s^2?
    d. The box has v_y = 5.0 m/s and is slowing down at 5.0 m/s^2?
12. | What thrust does a 200 g model rocket need in order to have a vertical acceleration of 10 m/s^2
    a. On Earth?
    b. On the moon, where g = 1.62 m/s^2?

Section 6.3 Mass, Weight, and Gravity

13. | An astronaut's weight while standing on earth is 800 N. What is his weight on Mars, where g = 3.76 m/s^2?
14. | A woman has a mass of 55 kg.
    a. What is her weight while standing on earth?
    b. What are her mass and her weight on the moon, where g = 1.62 m/s^2?
15. | It takes the elevator in a skyscraper 4.0 s to reach its cruising speed of 10 m/s. A 60 kg passenger gets aboard on the ground floor. What is the passenger's weight
    a. Before the elevator starts moving?
    b. While the elevator is speeding up?
    c. After the elevator reaches its cruising speed?
16. | FIGURE EX6.16 shows the velocity graph of a 75 kg passenger in an elevator. What is the passenger's weight at t = 1 s? At 5 s? At 9 s?

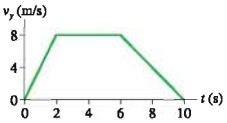


FIGURE EX6.16

Section 6.4 Friction

17. | Bonnie and Clyde are sliding a 300 kg bank safe across the floor to their getaway car. The safe slides with a constant speed if Clyde pushes from behind with 385 N of force while Bonnie pulls forward on a rope with 350 N of force. What is the safe's coefficient of kinetic friction on the bank floor?
18. | A stubborn, 120 kg mule sits down and refuses to move. To drag the mule to the barn, the exasperated farmer ties a rope around the mule and pulls with his maximum force of 800 N. The coefficients of friction between the mule and the ground are μ_s = 0.8 and μ_k = 0.5. Is the farmer able to move the mule?
19. | A 10 kg crate is placed on a horizontal conveyor belt. The materials are such that μ_s = 0.5 and μ_k = 0.3.
    a. Draw a free-body diagram showing all the forces on the crate if the conveyer belt runs at constant speed.
    b. Draw a free-body diagram showing all the forces on the crate if the conveyer belt is speeding up.
    c. What is the maximum acceleration the belt can have without the crate slipping?
20. || A 4000 kg truck is parked on a 15° slope. How big is the friction force on the truck?
21. || A 1500 kg car skids to a halt on a wet road where μ_k = 0.50. How fast was the car traveling if it leaves 65-m-long skid marks?
22. || An Airbus A320 jetliner has a takeoff mass of 75,000 kg. It reaches its takeoff speed of 82 m/s (180 mph) in 35 s. What is the thrust of the engines? You can neglect air resistance but not rolling friction.
23. || A 50,000 kg locomotive is traveling at 10 m/s when its engine and brakes both fail. How far will the locomotive roll before it comes to a stop?
24. || Estimate the size of the friction force on a baseball player sliding into second base.

Section 6.5 Drag

25. || A 75 kg skydiver can be modeled as a rectangular "box" with dimensions 20 cm × 40 cm × 180 cm. What is his terminal speed if he falls feet first?
26. || A 6.5-cm-diameter tennis ball has a terminal speed of 26 m/s. What is the ball's mass?

Problems

27. || A 5.0 kg object initially at rest at the origin is subjected to the time-varying force shown in FIGURE P6.27. What is the object's velocity at $t = 6$ s?

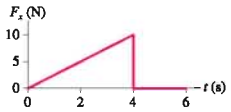


FIGURE P6.27

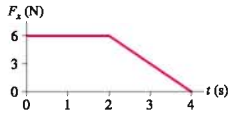


FIGURE P6.28

28. || A 2.0 kg object initially at rest at the origin is subjected to the time-varying force shown in FIGURE P6.28. What is the object's velocity at $t = 4$ s?
29. || A 1000 kg steel beam is supported by two ropes. What is the tension in each?

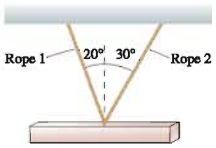


FIGURE P6.29

30. || In an electricity experiment, a 1.0 g plastic ball is suspended on a 60-cm-long string and given an electric charge. A charged rod brought near the ball exerts a horizontal electrical force $\vec{F}_{elec}$ on it, causing the ball to swing out to a 20° angle and remain there.
    a. What is the magnitude of $\vec{F}_{elec}$?
    b. What is the tension in the string?
31. || A 500 kg piano is being lowered into position by a crane while two people steady it with ropes pulling to the sides. Bob's rope pulls to the left, 15° below horizontal, with 500 N of tension. Ellen's rope pulls toward the right, 25° below horizontal.
    a. What tension must Ellen maintain in her rope to keep the piano descending at a steady speed?
    b. What is the tension in the main cable supporting the piano?
32. Henry gets into an elevator on the 50th floor of a building and it begins moving at $t = 0$ s. The figure shows his weight over the next 12 s.

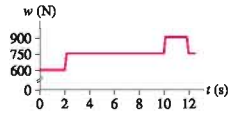


FIGURE P6.32

    a. Is the elevator's initial direction up or down? Explain how you can tell.
    b. What is Henry's mass?
    c. How far has Henry traveled at $t = 12$ s?
33. || Zach, whose mass is 80 kg, is in an elevator descending at 10 m/s. The elevator takes 3.0 s to brake to a stop at the first floor.
    a. What is Zach's weight before the elevator starts braking?
    b. What is Zach's weight while the elevator is braking?
34. || You've always wondered about the acceleration of the elevators in the 101-story-tall Empire State Building. One day, while visiting New York, you take your bathroom scale into the elevator and stand on it. The scale reads 150 lb as the door closes. The reading varies between 120 lb and 170 lb as the elevator travels 101 floors. What conclusions can you draw?
35. || An accident victim with a broken leg is being placed in traction. The patient wears a special boot with a pulley attached to the sole. The foot and boot together have a mass of 4.0 kg, and the doctor has decided to hang a 6.0 kg mass from the rope. The boot is held suspended by the ropes and does not touch the bed.

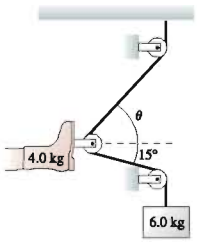


FIGURE P6.35

    a. Determine the amount of tension in the rope by using Newton's laws to analyze the hanging mass.
    b. The net traction force needs to pull straight out on the leg. What is the proper angle θ for the upper rope?
    c. What is the net traction force pulling on the leg?

    Hint: If the pulleys are frictionless, which we will assume, the tension in the rope is constant from one end to the other.
36. || Seat belts and air bags save lives by reducing the forces exerted on the driver and passengers in an automobile collision. Cars are designed with a "crumple zone" in the front of the car. In the event of an impact, the passenger compartment decelerates over a distance of about 1 m as the front of the car crumples. An occupant restrained by seat belts and air bags decelerates with the car. By contrast, an unrestrained occupant keeps moving forward with no loss of speed (Newton's first law!) until hitting the dashboard or windshield. These are unyielding surfaces, and the unfortunate occupant then decelerates over a distance of only about 5 mm.
    a. A 60 kg person is in a head-on collision. The car's speed at impact is 15 m/s. Estimate the net force on the person if he or she is wearing a seat belt and if the air bag deploys.
    b. Estimate the net force that ultimately stops the person if he or she is not restrained by a seat belt or air bag.
    c. How do these two forces compare to the person's weight?
37. || Compressed air is used to fire a 50 g ball vertically upward from a 1.0-m-tall tube. The air exerts an upward force of 2.0 N on the ball as long as it is in the tube. How high does the ball go above the top of the tube?
38. || A rifle with a barrel length of 60 cm fires a 10 g bullet with a horizontal speed of 400 m/s. The bullet strikes a block of wood and penetrates to a depth of 12 cm.
    a. What resistive force (assumed to be constant) does the wood exert on the bullet?
    b. How long does it take the bullet to come to rest?
    c. Draw a velocity-versus-time graph for the bullet in the wood.
39. || A 20,000 kg rocket has a rocket motor that generates 3.0×10^5 N of thrust.
    a. What is the rocket's initial upward acceleration?
    b. At an altitude of 5000 m the rocket's acceleration has increased to 6.0 m/s^2. What mass of fuel has it burned?

40. || A 2.0 kg steel block is at rest on a steel table. A horizontal string pulls on the block.
    a. What is the minimum string tension needed to move the block?
    b. If the string tension is 20 N, what is the block's speed after moving 1.0 m?
    c. If the string tension is 20 N and the table is coated with oil, what is the block's speed after moving 1.0 m?
41. || Sam, whose mass is 75 kg, takes off across level snow on his jet-powered skis. The skis have a thrust of 200 N and a coefficient of kinetic friction on snow of 0.10. Unfortunately, the skis run out of fuel after only 10 s.
    a. What is Sam's top speed?
    b. How far has Sam traveled when he finally coasts to a stop?
42. ||| Sam, whose mass is 75 kg, takes off down a 50-m-high, 10° slope on his jet-powered skis. The skis have a thrust of 200 N. Sam's speed at the bottom is 40 m/s. What is the coefficient of kinetic friction of his skis on snow?
43. || A baggage handler drops your 10 kg suitcase onto a conveyor belt running at 2.0 m/s. The materials are such that $\mu_s = 0.50$ and $\mu_k = 0.30$. How far is your suitcase dragged before it is riding smoothly on the belt?
44. || You and your friend Peter are putting new shingles on a roof pitched at 25°. You're sitting on the very top of the roof when Peter, who is at the edge of the roof directly below you, 5.0 m away, asks you for the box of nails. Rather than carry the 2.5 kg box of nails down to Peter, you decide to give the box a push and have it slide down to him. If the coefficient of kinetic friction between the box and the roof is 0.55, with what speed should you push the box to have it gently come to rest right at the edge of the roof?
45. || It's moving day, and you need to push a 100 kg box up a 20° ramp into the truck. The coefficients of friction for the box on the ramp are $\mu_s = 0.90$ and $\mu_k = 0.60$. Your largest pushing force is 1000 N. Can you get the box into the truck without assistance if you get a running start at the ramp? If you stop on the ramp, will you be able to get the box moving again?
46. || A 2.0 kg wood block is launched up a wooden ramp that is inclined at a 30° angle. The block's initial speed is 10 m/s.
    a. What vertical height does the block reach above its starting point?
    b. What speed does it have when it slides back down to its starting point?
47. ||| It's a snowy day and you're pulling a friend along a level road on a sled. You've both been taking physics, so she asks what you think the coefficient of friction between the sled and the snow is. You've been walking at a steady 1.5 m/s, and the rope pulls up on the sled at a 30° angle. You estimate that the mass of the sled, with your friend on it, is 60 kg and that you're pulling with a force of 75 N. What answer will you give?
48. || A horizontal rope pulls a 10 kg wood sled across frictionless snow. A 5.0 kg wood box rides on the sled. What is the largest tension force for which the box doesn't slip?
49. || A pickup truck with a steel bed is carrying a steel file cabinet. If the truck's speed is 15 m/s, what is shortest distance in which it can stop without the file cabinet sliding?
50. || You're driving along at 25 m/s with your aunt's valuable antiques in the back of your pickup truck when suddenly you see a giant hole in the road 55 m ahead of you. Fortunately, your foot is right beside the brake and your reaction time is zero! Will the antiques be as fortunate?
    a. Can you stop the truck before it falls into the hole?
    b. If your answer to part a is yes, can you stop without the antiques sliding and being damaged? Their coefficients of friction are $\mu_s = 0.60$ and $\mu_k = 0.30$.

    Hint: You're not trying to stop in the shortest possible distance. What's your best strategy for avoiding damage to the antiques?
51. || The 2.0 kg wood box in FIGURE P6.51 slides down a vertical wood wall while you push on it at a 45° angle. What magnitude of force should you apply to cause the box to slide down at a constant speed?

FIGURE P6.51

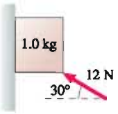


FIGURE P6.52

52. || A 1.0 kg wood block is pressed against a vertical wood wall by the 12 N force shown in FIGURE P6.52. If the block is initially at rest, will it move upward, move downward, or stay at rest?
53. || What is the terminal speed for an 80 kg skier going down a 40° snow-covered slope on wooden skis? Assume that the skier is 1.8 m tall and 0.40 m wide.
54. ||| A ball is shot from a compressed-air gun at twice its terminal speed.
    a. What is the ball's initial acceleration, as a multiple of g, if it is shot straight up?
    b. What is the ball's initial acceleration, as a multiple of g, if it is shot straight down?
    c. Draw a plausible velocity-versus-time graph for the ball that is shot straight down.
55. || An artist friend of yours needs help hanging a 500 lb sculpture from the ceiling. For artistic reasons, she wants to use just two ropes. One will be 30° from vertical, the other 60°. She needs you to determine the smallest diameter rope that can safely support this expensive piece of art. On a visit to the hardware store you find that rope is sold in increments of $\frac{1}{8}$-inch diameter and that the safety rating is 4000 pounds per square inch of cross section. What diameter rope should you buy?
56. || You've been called in to investigate a construction accident in which the cable broke while a crane was lifting a 4500 kg container. The steel cable is 2.0 cm in diameter and has a safety rating of 50,000 N. The crane is designed not to exceed speeds of 3.0 m/s or accelerations of 1.0 m/s^2, and your tests find that the crane is not defective. What is your conclusion? Did the crane operator recklessly lift too heavy a load? Or was the cable defective?
57. || You've entered a "slow ski race" where the winner is the skier who takes the *longest* time to go down a 15° slope without ever stopping. You need to choose the best wax to apply to your skis. Red wax has a coefficient of kinetic friction 0.25, yellow is 0.20, green is 0.15, and blue is 0.10. Having just finished taking physics, you realize that a wax too slippery will cause you to accelerate down the slope and lose the race. But a wax that's too

sticky will cause you to stop and be disqualified. You know that a strong headwind will apply a 50 N horizontal force against you as you ski, and you know that your mass is 75 kg. Which wax do you choose?

58. || A 1.0 kg ball hangs from the ceiling of a truck by a 1.0-m-long string. The back of the truck, where you are riding with the ball, has no windows and has been completely soundproofed. The truck travels along an exceedingly smooth test track, and you feel no bumps or bounces as it moves. Your only instruments are a meter stick, a protractor, and a stopwatch.
    a. The driver tells you, over a loudspeaker, that the truck is either at rest, or it is moving forward at a steady speed of 5 m/s. Can you determine which it is? If so, how? If not, why not?
    b. Next, the driver tells you that the truck is either moving forward with a steady speed of 5 m/s, or it is accelerating at 5 m/s^2. Can you determine which it is? If so, how? If not, why not?
    c. Suppose the truck has been accelerating forward at 5 m/s^2 long enough for the ball to achieve a steady position. Does the ball have an acceleration? If so, what are the magnitude and direction of the ball's acceleration?
    d. Draw a free-body diagram that shows all forces acting on the ball as the truck accelerates.
    e. Suppose the ball makes a 10° angle with the vertical. If possible, determine the truck's velocity. If possible, determine the truck's acceleration.
59. || Imagine *hanging* from a big spring scale as it moves vertically with acceleration a_y. Show that Equation 6.10 is the correct expression for your weight.
60. || A particle of mass m moving along the x-axis experiences the net force $F_x = ct$, where c is a constant. The particle has velocity v_{0x} at $t = 0$. Find an algebraic expression for the particle's velocity v_x at a later time t.
61. || Astronauts in space "weigh" themselves by oscillating on a spring. Suppose the position of an oscillating 75 kg astronaut is given by $x = (0.30\text{ m})\sin((\pi\text{ rad/s})\cdot t)$, where t is in s. What force does the spring exert on the astronaut at (a) $t = 1.0$ s and (b) 1.5 s. Note that the angle of the sine function is in radians.
62. ||| An object moving in a liquid experiences a *linear* drag force: $\vec{D} = (bv$, direction opposite the motion), where b is a constant called the *drag coefficient*. For a sphere of radius R, the drag constant can be computed as $b = 6\pi\eta R$, where η is the *viscosity* of the liquid.
    a. Find an algebraic expression for the terminal speed v_{term} of a spherical particle of radius R and mass m falling through a liquid of viscosity η.
    b. Water at 20°C has viscosity $\eta = 1.0 \times 10^{-3}$ Ns/m^2. Sand grains have density 2400 kg/m^3. Suppose a 1.0-mm-diameter sand grain is dropped into a 50-m-deep lake whose water is a constant 20°C. If the sand grain reaches terminal speed almost instantly (a quite good approximation), how long will it take the sand grain to settle to the bottom of the lake?

Problems 63 through 65 show a free-body diagram. For each:

a. Write a realistic dynamics problem for which this is the correct free-body diagram. Your problem should ask a question that can be answered with a value of position or velocity (such as "How far?" or "How fast?"), and should give sufficient information to allow a solution.
b. Solve your problem!

63.

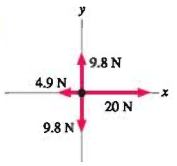


FIGURE P6.63

64.

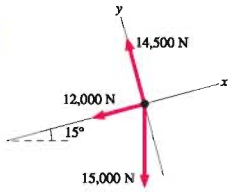


FIGURE P6.64

65.

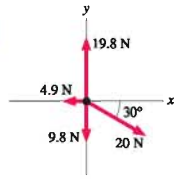


FIGURE P6.65

In Problems 66 through 68 you are given the dynamics equations that are used to solve a problem. For each of these, you are to

a. Write a realistic problem for which these are the correct equations.
b. Draw the free-body diagram and the pictorial representation for your problem.
c. Finish the solution of the problem.

66. $-0.80n = (1500\text{ kg})a_x$
    $n - (1500\text{ kg})(9.80\text{ m/s}^2) = 0$
67. $T - 0.20n - (20\text{ kg})(9.80\text{ m/s}^2)\sin 20° = (20\text{ kg})(2.0\text{ m/s}^2)$
    $n - (20\text{ kg})(9.80\text{ m/s}^2)\cos 20° = 0$
68. $(100\text{ N})\cos 30° - f_k = (20\text{ kg})a_x$
    $n + (100\text{ N})\sin 30° - (20\text{ kg})(9.80\text{ m/s}^2) = 0$
    $f_k = 0.20n$

Challenge Problems

69. Try this! Hold your right hand out with your palm perpendicular to the ground, as if you were getting ready to shake hands. You can't hold anything in your palm this way because it would fall straight down. Use your left hand to hold a small object, such as a ball or a coin, against your outstretched palm, then let go as you quickly swing your hand to the left across your body, parallel to the ground. You'll find that the object stays against your palm; it doesn't slip or fall.
    a. Is the condition for keeping the object against your palm one of maintaining a certain minimum velocity v_{min}? Or one of maintaining a certain minimum acceleration a_{min}? Explain.
    b. Suppose the object's mass is 50 g, with $\mu_s = 0.80$ and $\mu_k = 0.40$. Determine either v_{min} or a_{min}, whichever you answered in part a.

70. A machine has an 800 g steel shuttle that is pulled along a square steel rail by an elastic cord. The shuttle is released when the elastic cord has 20 N tension at a 45° angle. What is the initial acceleration of the shuttle?

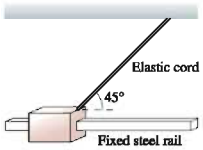


FIGURE CP6.70

71. The figure shows an *accelerometer*, a device for measuring the horizontal acceleration of cars and airplanes. A ball is free to roll on a parabolic track described by the equation $y = x^2$, where both x and y are in meters. A scale along the bottom is used to measure the ball's horizontal position x.
    a. Find an expression that allows you to use a measured position x (in m) to compute the acceleration a_x (in m/s^2). (For example, $a_x = 3x$ is a possible expression.)
    b. What is the acceleration if $x = 20$ cm?

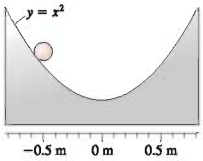


FIGURE CP6.71

72. A testing laboratory wants to determine if a new widget can withstand large accelerations and decelerations. To find out, they glue a 5.0 kg widget to a test stand that will drive it vertically up and down. The graph shows its acceleration during the first second, starting from rest.

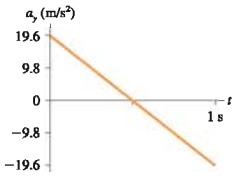


FIGURE CP6.72

    a. Identify the forces acting on the widget and draw a free-body diagram.
    b. Determine the value of n_y, the y-component of the normal force acting on the widget, during the first second of motion. Give your answer as a graph of n_y versus t.
    c. Your answer to part b should show an interval of time during which n_y is negative. How can this be? Explain what it means physically for n_y to be negative.
    d. At what time is the weight of the widget a maximum? What is the acceleration at this time?
    e. Is the weight of the widget ever zero? If so, at what time does this happen? What is the acceleration at that time?
    f. Suppose the technician forgets to glue the widget to the test stand. Will the widget remain on the test stand throughout the first second, or will it fly off the stand at some instant of time? If so, at what time will this occur?

73. An object moving in a liquid experiences a *linear* drag force: $\vec{D} = (bv, \text{direction opposite the motion})$, where b is a constant called the *drag coefficient*. For a sphere of radius R, the drag constant can be computed as $b = 6\pi\eta R$, where η is the *viscosity* of the liquid.
    a. Find an algebraic expression for $v_x(t)$, the x-component of velocity as a function of time, for a spherical particle of radius R and mass m that is shot horizontally with initial speed v_0 through a liquid of viscosity η.
    b. Water at 20°C has viscosity $\eta = 1.0 \times 10^{-3}$ Ns/m^2. Suppose a 4.0-cm-diameter, 33 g ball is shot horizontally into a tank of 20°C water. How long will it take for the horizontal speed to decrease to 50% of its initial value?

74. An object moving in a liquid experiences a *linear* drag force: $\vec{D} = (bv, \text{direction opposite the motion})$, where b is a constant called the *drag coefficient*. For a sphere of radius R, the drag constant can be computed as $b = 6\pi\eta R$, where η is the *viscosity* of the liquid.
    a. Use what you've learned in calculus to prove that
$$a_x = v_x \frac{dv_x}{dx}$$
    b. Find an algebraic expression for $v_x(x)$, the x-component of velocity as a function of distance traveled, for a spherical particle of radius R and mass m that is shot horizontally with initial speed v_0 through a liquid of viscosity η.
    c. Water at 20°C has viscosity $\eta = 1.0 \times 10^{-3}$ Ns/m^2. Suppose a 1.0-cm-diameter, 1.0 g marble is shot horizontally into a tank of 20°C water at 10 cm/s. How far will it travel before stopping?

75. An object with cross section A is shot horizontally across frictionless ice. Its initial velocity is v_{0x} at $t_0 = 0$ s. Air resistance is not negligible.
    a. Show that the velocity at time t is given by the expression
$$v_x = \frac{v_{0x}}{1 + Av_{0x}t/4m}$$
    b. A 1.6-m-wide, 1.4-m-high, 1500 kg car hits a very slick patch of ice while going 20 m/s. If friction is neglected, how long will it take until the car's speed drops to 10 m/s? To 5 m/s?
    c. Assess whether or not it is reasonable to neglect kinetic friction.

STOP TO THINK ANSWERS

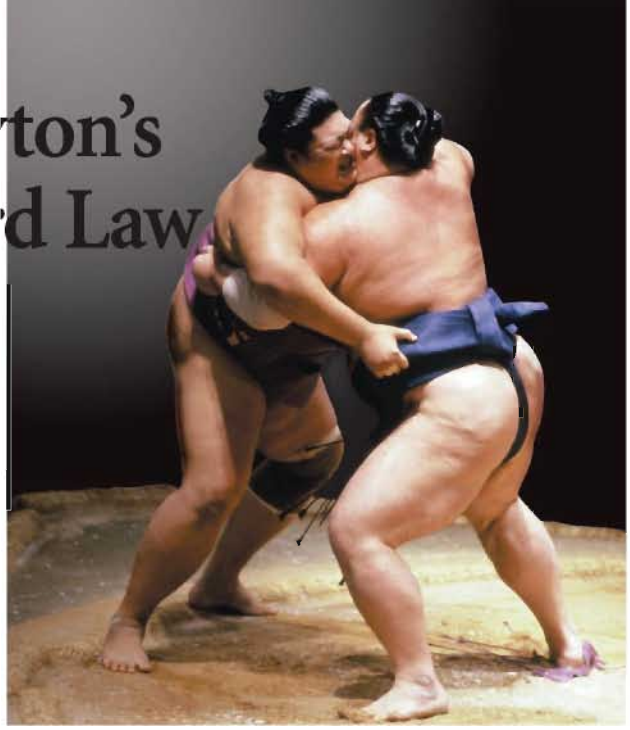
Stop to Think 6.1: a. The lander is descending and slowing. The acceleration vector points upward, and so $\vec{F}_{net}$ points upward. This can be true only if the thrust has a larger magnitude than the weight.

Stop to Think 6.2: a. You are descending and slowing, so your acceleration vector points upward and there is a net upward force on you. The floor pushes up against your feet harder than gravity pulls down.

Stop to Think 6.3: $f_b > f_c = f_d = f_e > f_a$. Situations c, d, and e are all kinetic friction, which does not depend on either velocity or acceleration. Kinetic friction is smaller than the maximum static friction that is exerted in b. $f_a = 0$ because no friction is needed to keep the object at rest.

Stop to Think 6.4: d. The ball is shot *down* at 30 m/s, so $v_{0y} = -30$ m/s. This exceeds the terminal speed, so the upward drag force is *larger* than the downward weight force. Thus the ball *slows down* even though it is "falling." It will slow until $v_y = -15$ m/s, the terminal velocity, then maintain that velocity.

7 Newton's Third Law



These two sumo wrestlers are *interacting* with each other.

Rather than a single particle responding to a well-defined force, these sumo wrestlers are *interacting* with each other. The harder one sumo wrestler pushes, the harder the other pushes back. A hammer and a nail, your foot and a soccer ball, and the earth-moon system are other examples of interacting objects.

Newton's second law is not sufficient to explain what happens when two or more objects interact. Newton's second law, the essence of single-particle dynamics, treats an object as an isolated entity acted upon by external forces. Chapter 7 will introduce a new law of physics, Newton's *third* law, that describes how two objects interact with each other.

Newton's third law brings us to the pinnacle of Newton's theory of forces and motion. The tools you will have learned when you finish this chapter can be used to solve complex but realistic dynamics problems.

▶ Looking Ahead

The goal of Chapter 7 is to use Newton's third law to understand interacting objects. In this chapter you will learn to:

- Identify action/reaction pairs of forces in interacting objects.
- Understand and use Newton's third law.
- Use an expanded problem-solving strategy for dynamics problems.
- Understand the role of strings, ropes, and pulleys.

◀ Looking Back

This chapter further develops the concept of force. Please review:

- Sections 5.1–5.3 The basic concept of force and the atomic-level view of tension.
- Section 6.2 The basic problem-solving strategy for dynamics.

7.1 Interacting Objects

Our goal is to understand how two objects interact. Think about the hammer and nail in FIGURE 7.1 on the next page. The hammer certainly exerts a force on the nail as it drives the nail forward. At the same time, the nail exerts a force on the hammer. If you are not sure that it does, imagine hitting the nail with a glass hammer. It's the force of the nail on the hammer that causes the glass to shatter.

If you stop to think about it, any time that object A pushes or pulls on object B, object B pushes or pulls back on object A. As sumo wrestler A pushes on sumo wrestler B, B pushes back on A. (If A pushed forward without B pushing back, A would fall over in the same way you do if someone suddenly opens a door you're

FIGURE 7.1 The hammer and nail are interacting with each other.

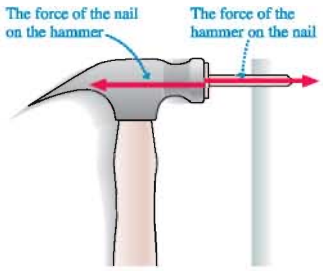
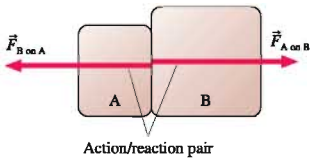


FIGURE 7.2 An action/reaction pair of forces.



Action/reaction pair

leaning against.) Your chair pushes upward on you (a normal force) while, at the same time, you push down on the chair. These are examples of what we call an *interaction.* An **interaction** is the mutual influence of two objects on each other.

To be more specific, if object A exerts a force $\vec{F}_{A\text{ on }B}$ on object B, then object B exerts a force $\vec{F}_{B\text{ on }A}$ on object A. This pair of forces, shown in FIGURE 7.2, is called an **action/reaction pair.** Two objects interact by exerting an action/reaction pair of forces on each other. Notice the very explicit subscripts on the force vectors. The first letter is the *agent,* the second letter is the object on which the force acts. $\vec{F}_{A\text{ on }B}$ is a force exerted *by* A *on* B. The distinction is important, and we will use this explicit notation for much of this chapter.

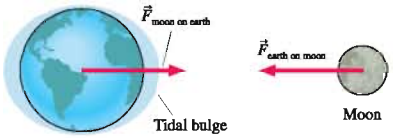
NOTE ▶ The name "action/reaction pair" is somewhat misleading. The forces occur simultaneously, and we cannot say which is the "action" and which the "reaction." Neither is there any implication about cause and effect; the action does not *cause* the reaction. **An action/reaction pair of forces exists as a pair, or not at all.** In identifying action/reaction pairs, the labels are the key. Force $\vec{F}_{A\text{ on }B}$ is paired with force $\vec{F}_{B\text{ on }A}$. ◀

The sumo wrestlers and the hammer and nail interact through contact forces. The same idea holds true for long-range forces. You probably have played with kitchen magnets or bar magnets. As you hold two magnets, you can feel with your fingertips that *both* have forces pulling on them.

But what about gravity? If you release a ball, it falls because the earth's gravity exerts a downward force $\vec{F}_{\text{earth on ball}}$ on it. But does the ball also pull upward on the earth? That is, is there a force $\vec{F}_{\text{ball on earth}}$?

Newton was the first to recognize that, indeed, the ball *does* pull upward on the earth. Likewise, the moon pulls on the earth in response to the earth's gravity pulling on the moon. Newton's evidence was the tides. Scientists and astronomers had studied and timed the ocean's tides since antiquity. It was known that the tides depend on the phase of the moon, but Newton was the first to understand that the tides are the ocean's response to the gravitational pull of the moon on the earth. As FIGURE 7.3 shows, the flexible water bulges toward the moon while the relatively inflexible crust of the earth remains stationary.

FIGURE 7.3 The ocean tides are an indication of the long-range gravitational interaction of the earth and the moon.

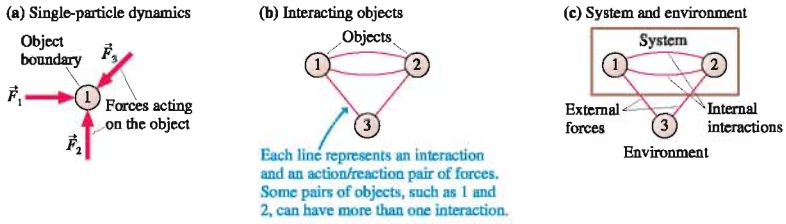


Objects, Systems, and the Environment

Chapters 5 and 6 considered forces acting on a single object that we modeled as a particle. FIGURE 7.4a shows a diagrammatic representation of single-particle dynamics. If all the forces acting on the particle are known, we can use Newton's second law, $\vec{a} = \vec{F}_{\text{net}}/m$, to determine the particle's acceleration.

We now want to extend the particle model to situations in which two or more objects, each represented as a particle, interact with each other. For example, FIGURE 7.4b shows three objects interacting via action/reaction pairs of forces. The forces can be given labels such as $\vec{F}_{1\text{ on }2}$ and $\vec{F}_{2\text{ on }1}$. How do these particles move?

FIGURE 7.4 Single-particle dynamics and a model of interacting objects.



We will often be interested in the motion of some of the objects, say objects 1 and 2, but not of others. For example, objects 1 and 2 might be the hammer and the nail, while object 3 is the earth. The earth interacts with both the hammer and the nail via gravity, but in a practical sense the earth remains "at rest" while the hammer and nail move. Let's define the **system** as those objects whose motion we want to analyze and the **environment** as objects external to the system.

FIGURE 7.4c is a new kind of diagram, an **interaction diagram,** in which we've enclosed the objects of the system in a box and represented interactions as lines connecting objects. This is a rather abstract, schematic diagram, but it captures the essence of the interactions. Notice that interactions with objects in the environment are called **external forces.** For the hammer and nail, the gravitational force on each—an interaction with the earth—is an external force.

NOTE ▶ The system–environment distinction is a practical matter, not a fundamental distinction. If object A pushes or pulls on object B, then B pushes or pulls on A. *Every* force is one member of an action/reaction pair, and there is no such thing as a true "external force." What we call an external force is an interaction between an object of interest, one we've chosen to place inside the system, and an object whose motion is not of interest. ◀

Newton's second law, $\vec{a} = \vec{F}_{\text{net}}/m$, applies *separately* to objects 1 and 2 in Figure 7.4c:

$$\text{Object 1:} \quad \vec{a}_1 = \frac{\vec{F}_{1\,\text{net}}}{m_1} = \frac{1}{m_1}\sum \vec{F}_{\text{on }1}$$

$$\text{Object 2:} \quad \vec{a}_2 = \frac{\vec{F}_{2\,\text{net}}}{m_2} = \frac{1}{m_2}\sum \vec{F}_{\text{on }2} \qquad (7.1)$$

The net force on object 1, denoted $\sum \vec{F}_{\text{on }1}$, is the sum of *all* forces acting *on* object 1. The sum includes both forces due to object 2 ($\vec{F}_{2\text{ on }1}$) and any external forces originating in the environment.

NOTE ▶ Forces exerted *by* object 1, such as $\vec{F}_{1\text{ on }2}$, do *not* appear in the equation for object 1. Objects change their motion in response to forces exerted *on* them, not to forces exerted *by* them. ◀

The bat and the ball are interacting with each other.

7.2 Analyzing Interacting Objects

The key steps for analyzing interacting objects are (1) the identification of the action/reaction pairs of forces and (2) the drawing of free-body diagrams. The interaction diagram will be our primary tool.

TACTICS BOX 7.1 Analyzing interacting objects

1. **Represent each object as a circle.** Place each in the correct position relative to other objects.
   - Give each a name and a label.
   - The surface of the earth (contact forces) and the entire earth (long-range forces) should be considered separate objects. Label the entire earth EE.
   - Ropes and pulleys often need to be considered objects.
2. **Identify interactions.** Draw connecting lines between the circles to represent interactions.
   - Draw *one* line for each interaction. Label it with the type of force.
   - Every interaction line connects two and only two objects.
   - There can be at most two interactions at a surface: a force parallel to the surface (e.g., friction) and a force perpendicular to the surface (e.g., a normal force).
   - The entire earth interacts only by the long-range gravitational force.
3. **Identify the system.** Identify the objects of interest; draw and label a box enclosing them. This completes the interaction diagram.
4. **Draw a free-body diagram for each object in the system.** Include only the forces acting *on* each object, not forces exerted by the object.
   - Every interaction line crossing the system boundary is one external force acting on the object. The usual force symbols, such as $\vec{n}$ and $\vec{T}$, can be used.
   - Every interaction line within the system represents an action/reaction pair of forces. There is one force vector on *each* of the objects, and these forces always point in opposite directions. Use labels like $\vec{F}_{1 \text{ on } 2}$ and $\vec{F}_{2 \text{ on } 1}$.
   - Connect the two action/reaction forces—which must be on *different* free-body diagrams—with a dashed line.

Exercises 1–7

We'll illustrate these ideas with two concrete examples. The first example will be much longer than usual because we'll go carefully through all the steps in the reasoning.

EXAMPLE 7.1 Pushing a crate

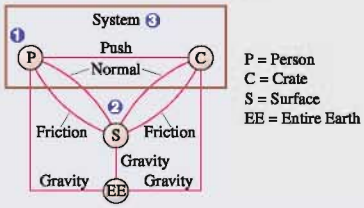
FIGURE 7.5 shows a person pushing a large crate across a rough surface. Identify all interactions, show them on an interaction diagram, then draw free-body diagrams of the person and the crate.

FIGURE 7.5 A person pushes a crate across a rough floor.

VISUALIZE The interaction diagram of FIGURE 7.6 starts by representing every object as a circle in the correct position but separated from all other objects. The person and the crate are obvious objects. The earth is also an object that both exerts and experiences forces, but it's necessary to distinguish between the surface, which exerts contact forces, and the entire earth, which exerts the long-range gravitational force.

Figure 7.6 also identifies the various interactions. Some, like the pushing interaction between the person and the crate, are fairly obvious. The interactions with the earth are a little trickier. Gravity, a long-range force, is an interaction between each object (including the surface!) and the earth as a whole. Friction forces and normal forces are contact interactions between each object and the earth's surface. These are two different interac-

FIGURE 7.6 The interaction diagram.

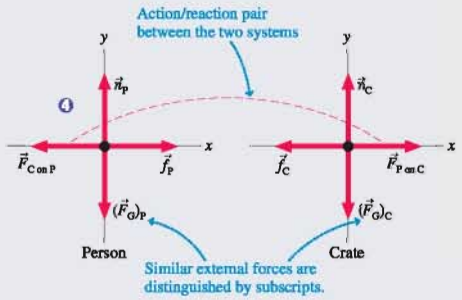


tions, so two interaction lines connect the crate to the surface and the person to the surface. Altogether, there are eight interactions. Finally, we've enclosed the person and crate in a box labeled System. These are the objects whose motion we wish to analyze.

NOTE ▶ Interactions are between two *different* objects. None of the interactions are between an object and itself. ◀

We can now draw free-body diagrams for the objects in the system, the crate and the person. **FIGURE 7.7** correctly locates the crate's free-body diagram to the right of the person's free-body diagram. For each, three interaction lines cross the system boundary and thus represent external forces. These are the gravitational force from the entire earth, the upward normal force from the surface, and a friction force from the surface. We can use familiar labels such as $\vec{n}_P$ and $\vec{f}_C$, but **it's very important to distinguish different forces with subscripts.** There's now more than one normal force. If you call both simply $\vec{n}$, you're almost certain to make mistakes when you start writing out the second-law equations.

FIGURE 7.7 Free-body diagrams of the person and the crate.



The directions of the normal forces and the gravitational forces are clear, but we have to be careful with friction. Friction force $\vec{f}_C$ is kinetic friction of the crate sliding across the surface, so it points left, opposite the motion. But what about friction between the person and the surface? It is tempting to draw force $\vec{f}_P$ pointing to the left. After all, friction forces are supposed to be in the direction opposite the motion. But if we did so, the person would have two forces to the left, $\vec{F}_{C\,on\,P}$ and $\vec{f}_P$, and none to the right, causing the person to accelerate *backward!* That is clearly not what happens, so what is wrong?

Imagine pushing a crate to the right across loose sand. Each time you take a step, you tend to kick the sand to the *left*, behind you. Thus friction force $\vec{f}_{P\,on\,S}$, the force of the person pushing against the earth's surface, is to the *left*. In reaction, the force of the earth's surface against the person is a friction force to the *right*. It is force $\vec{f}_{S\,on\,P}$, which we've shortened to $\vec{f}_P$, that causes the person to accelerate in the forward direction. Further, as we'll discuss more below, this is a *static* friction force; your foot is planted on the ground, not sliding across the surface.

Finally, we have one internal interaction. The crate is pushed with force $\vec{F}_{P\,on\,C}$. If A pushes or pulls on B, then B pushes or pulls back on A. The reaction to force $\vec{F}_{P\,on\,C}$ is $\vec{F}_{C\,on\,P}$, the crate pushing back against the person's hands. Force $\vec{F}_{P\,on\,C}$ is a force exerted on the crate, so it's shown on the crate's free-body diagram. Force $\vec{F}_{C\,on\,P}$ is exerted on the person, so it is drawn on the person's free-body diagram. **The two forces of an action/reaction pair never occur on the same object.** Notice that forces $\vec{F}_{P\,on\,C}$ and $\vec{F}_{C\,on\,P}$ are pointing in opposite directions. We've connected them with a dashed line to show that they are an action/reaction pair.

ASSESS The completed free-body diagrams of Figure 7.7 could now be the basis for a quantitative analysis.

Propulsion

The friction force $\vec{f}_P$ (force of surface on person) is an example of **propulsion.** It is the force that a system with an internal source of energy uses to drive itself forward. Propulsion is an important feature not only of walking or running but also of the forward motion of cars, jets, and rockets. Propulsion is somewhat counterintuitive, so it is worth a closer look.

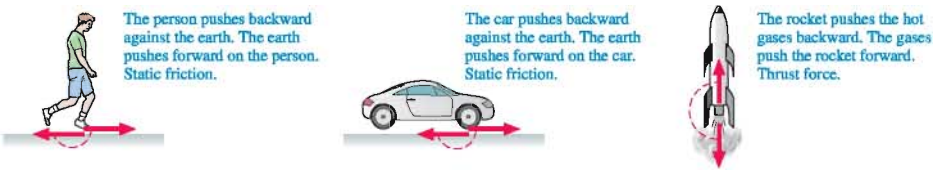
If you try to walk across a frictionless floor, your foot slips and slides *backward*. In order for you to walk, the floor needs to have friction so that your foot *sticks* to the floor as you straighten your leg, moving your body forward. The friction that prevents slipping is *static* friction. Static friction, you will recall, acts in the direction that prevents slipping. The static friction force $\vec{f}_P$ has to point in the *forward* direction to prevent your foot from slipping backward. It is this forward-directed static friction force that propels you forward! The force of your foot on the floor, the other half of the action/reaction pair, is in the opposite direction.

The distinction between you and the crate is that you have an *internal source of energy* that allows you to straighten your leg by pushing backward against the surface. In essence, you walk by pushing the earth away from you. The earth's surface responds by pushing you forward. These are static friction forces. In contrast, all the crate can do is slide, so *kinetic* friction opposes the motion of the crate.

What force causes this sprinter to accelerate?

FIGURE 7.8 shows how propulsion works. A car uses its motor to spin the tires, causing the tires to push backward against the ground. This is why dirt and gravel are kicked backward, not forward. The earth's surface responds by pushing the car forward. These are also *static* friction forces. The tire is rolling, but the bottom of the tire, where it contacts the road, is instantaneously at rest. If it weren't, you would leave one giant skid mark as you drove and would burn off the tread within a few miles.

FIGURE 7.8 Examples of propulsion.

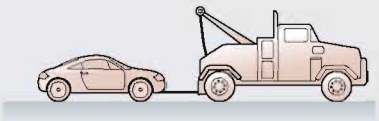


Rocket motors are somewhat different because they are not pushing *against* anything. That's why rocket propulsion works in the vacuum of space. Instead, the rocket engine pushes hot, expanding gases out of the back of the rocket. In response, the exhaust gases push the rocket forward with the force we've called *thrust*.

EXAMPLE 7.2 Towing a car

A tow truck uses a rope to pull a car along a horizontal road, as shown in FIGURE 7.9. Identify all interactions, show them on an interaction diagram, then draw free-body diagrams of each object in the system.

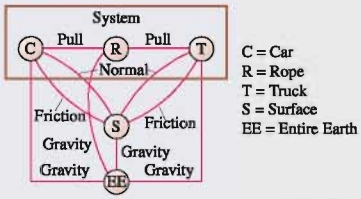
FIGURE 7.9 A truck towing a car.



VISUALIZE The interaction diagram of FIGURE 7.10 represents the objects as separate circles, but with the correct relative positions. The rope is shown as a separate object. Many of the interactions are identical to those in Example 7.1. The system—the objects in motion—consists of the truck, the rope, and the car.

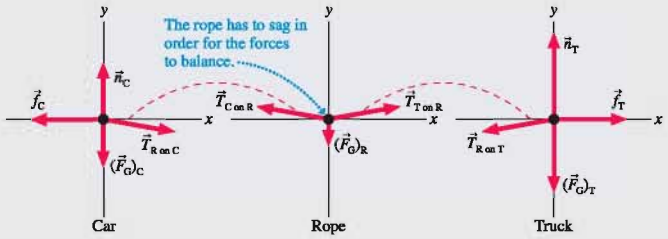
The three objects in the system require three free-body diagrams, shown in FIGURE 7.11. Gravity, friction, and normal forces at the surface are all interactions that cross the system boundary and are shown as external forces. The car is an inert object rolling along. It would slow and stop if the rope were cut, so the surface must exert a rolling friction force $\vec{f}_C$ to the left. The truck, however, has an internal source of energy. The truck's drive wheels push the ground to the left with force $\vec{f}_{T\text{ on }S}$. In reaction, the ground propels the truck forward, to the right, with force $\vec{f}_T$.

FIGURE 7.10 The interaction diagram.



We next need to identify the horizontal forces between the car, the truck, and the rope. The rope pulls on the car with a tension force $\vec{T}_{R\text{ on }C}$. You might be tempted to put the reaction force on the truck because we say that "the truck pulls the car," but the truck is not in contact with the car. The truck pulls on the rope, then the rope pulls on the car. Thus the reaction to $\vec{T}_{R\text{ on }C}$ is a force on the *rope:* $\vec{T}_{C\text{ on }R}$. These are an action/reaction pair. At the other end, $\vec{T}_{T\text{ on }R}$ and $\vec{T}_{R\text{ on }T}$ are also an action/reaction pair.

FIGURE 7.11 Free-body diagrams of Example 7.2.

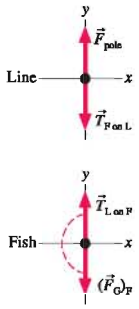


NOTE ▶ Drawing an interaction diagram helps you avoid making mistakes because it shows very clearly what is interacting with what. ◀

Notice that the tension forces of the rope *cannot* be horizontal. If they were, the rope's free-body diagram would show a net downward force and the rope would accelerate downward. The tension forces $\vec{T}_{\text{T on R}}$ and $\vec{T}_{\text{C on R}}$ have to angle slightly upward to balance the gravitational force, so any real rope has to sag at least a little in the center.

ASSESS Make sure you avoid the common error of considering $\vec{n}$ and $\vec{F}_G$ to be an action/reaction pair. These are both forces on the *same* object, whereas the two forces of an action/reaction pair are always on two *different* objects that are interacting with each other. The normal and gravitational forces are often equal in magnitude, as they are in this example, but that doesn't make them an action/reaction pair of forces.

STOP TO THINK 7.1 A fishing line of negligible mass lifts a fish upward at constant speed. The line and the fish are the system, the fishing pole is part of the environment. What, if anything, is wrong with the free-body diagrams?



7.3 Newton's Third Law

Newton was the first to recognize how the two members of an action/reaction pair of forces are related to each other. Today we know this as Newton's third law:

Newton's third law Every force occurs as one member of an action/reaction pair of forces.

- The two members of an action/reaction pair act on two *different* objects.
- The two members of an action/reaction pair are equal in magnitude but opposite in direction: $\vec{F}_{\text{A on B}} = -\vec{F}_{\text{B on A}}$.

We deduced most of the third law in Section 7.2. There we found that the two members of an action/reaction pair are always opposite in direction (see Figures 7.7 and 7.11). According to the third law, this will always be true. But the most significant portion of the third law, which is by no means obvious, is that the two members of an action/reaction pair have *equal* magnitudes. That is, $F_{\text{A on B}} = F_{\text{B on A}}$. This is the quantitative relationship that will allow you to solve problems of interacting objects.

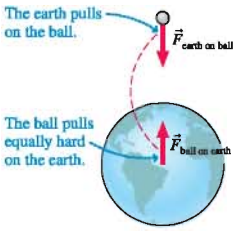
Newton's third law is frequently stated as "For every action there is an equal but opposite reaction." While this is indeed a catchy phrase, it lacks the preciseness of our preferred version. In particular, it fails to capture an essential feature of action/reaction pairs—that they each act on a *different* object.

NOTE ▶ Newton's third law extends and completes our concept of *force*. We can now recognize force as an *interaction* between objects rather than as some "thing" with an independent existence of its own. The concept of an interaction will become increasingly important as we begin to study the laws of momentum and energy. ◀

Reasoning with Newton's Third Law

Newton's third law is easy to state but harder to grasp. For example, consider what happens when you release a ball. Not surprisingly, it falls down. But if the ball and the earth exert equal and opposite forces on each other, as Newton's third law alleges, why don't you see the earth "fall up" to meet the ball?

FIGURE 7.12 The action/reaction forces of a ball and the earth are equal in magnitude.



The key to understanding this and many similar puzzles is that **the forces are equal but the accelerations are not.** Equal causes can produce very unequal effects. FIGURE 7.12 shows equal-magnitude forces on the ball and the earth. The force on ball B is simply the gravitational force of Chapter 6:

$$\vec{F}_{\text{earth on ball}} = (\vec{F}_G)_B = -m_B g\hat{j} \tag{7.2}$$

where m_B is the mass of the ball. According to Newton's second law, this force gives the ball an acceleration

$$\vec{a}_B = \frac{(\vec{F}_G)_B}{m_B} = -g\hat{j} \tag{7.3}$$

This is just the familiar free-fall acceleration.

According to Newton's third law, the ball pulls up on the earth with force $\vec{F}_{\text{ball on earth}}$. As the ball accelerates down, the earth as a whole has an upward acceleration

$$\vec{a}_E = \frac{\vec{F}_{\text{ball on earth}}}{m_E} \tag{7.4}$$

where m_E is the mass of the earth. Because $\vec{F}_{\text{earth on ball}}$ and $\vec{F}_{\text{ball on earth}}$ are an action/reaction pair, $\vec{F}_{\text{ball on earth}}$ must be equal in magnitude and opposite in direction to $\vec{F}_{\text{earth on ball}}$. That is,

$$\vec{F}_{\text{ball on earth}} = -\vec{F}_{\text{earth on ball}} = -(\vec{F}_G)_B = +m_B g\hat{j} \tag{7.5}$$

Using this result in Equation 7.4, we find the upward acceleration of the earth as a whole is

$$\vec{a}_E = \frac{\vec{F}_{\text{ball on earth}}}{m_E} = \frac{m_B g\hat{j}}{m_E} = \left(\frac{m_B}{m_E}\right) g\hat{j} \tag{7.6}$$

The upward acceleration of the earth is less than the downward acceleration of the ball by the factor m_B/m_E. If we assume a 1 kg ball, we can estimate the magnitude of $\vec{a}_E$:

$$a_E = \frac{1\text{ kg}}{6 \times 10^{24}\text{ kg}} g \approx 2 \times 10^{-24}\text{ m/s}^2$$

With this incredibly small acceleration, it would take the earth 8×10^{15} years, approximately 500,000 times the age of the universe, to reach a speed of 1 mph! So we certainly would not expect to see or feel the earth "fall up" after dropping a ball.

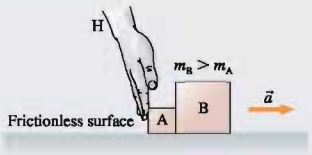
NOTE ▶ Newton's third law equates the size of two forces, not two accelerations. The acceleration continues to depend on the mass, as Newton's second law states. **In an interaction between two objects of different mass, the lighter mass will do essentially all of the accelerating even though the forces exerted on the two objects are equal.** ◀

EXAMPLE 7.3 The forces on accelerating boxes

The hand shown in FIGURE 7.13 pushes boxes A and B to the right across a frictionless table. The mass of B is larger than the mass of A.

a. Draw free-body diagrams of A, B, and the hand H, showing only the *horizontal* forces. Connect action/reaction pairs with dashed lines.
b. Rank in order, from largest to smallest, the horizontal forces shown on your free-body diagrams.

FIGURE 7.13 Hand H pushes boxes A and B.



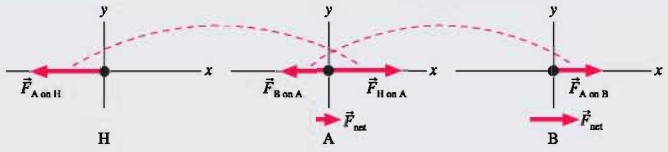
VISUALIZE a. The hand H pushes on box A, and A pushes back on H. Thus $\vec{F}_{\text{H on A}}$ and $\vec{F}_{\text{A on H}}$ are an action/reaction pair. Similarly, A pushes on B and B pushes back on A. The hand H does not touch box B, so there is no interaction between them. There is no friction. FIGURE 7.14 shows the four horizontal forces and identifies two action/reaction pairs. (We've chosen to ignore forces of the wrist or arm on the hand because our objects of interest are the boxes A and B.) Notice that each force is shown on the free-body diagram of the object that it acts *on*.

b. According to Newton's third law, $F_{\text{A on H}} = F_{\text{H on A}}$ and $F_{\text{A on B}} = F_{\text{B on A}}$. But the third law is not our only tool. Because the boxes are accelerating to the right, Newton's *second* law tells us that box A must have a net force to the right. Consequently, $F_{\text{H on A}} > F_{\text{B on A}}$. Thus

$$F_{\text{A on H}} = F_{\text{H on A}} > F_{\text{B on A}} = F_{\text{A on B}}$$

ASSESS You might have expected $F_{\text{A on B}}$ to be larger than $F_{\text{H on A}}$ because $m_B > m_A$. It's true that the *net* force on B is larger than the *net* force on A, but we have to reason more closely to judge the individual forces. Notice how we used both the second and the third laws to answer this question.

FIGURE 7.14 The free-body diagrams, showing only the horizontal forces.



STOP TO THINK 7.2 Car B is stopped for a red light. Car A, which has the same mass as car B, doesn't see the red light and runs into the back of B. Which of the following statements is true?

a. B exerts a force on A, but A doesn't exert a force on B.
b. B exerts a larger force on A than A exerts on B.
c. B exerts the same amount of force on A as A exerts on B.
d. A exerts a larger force on B than B exerts on A.
e. A exerts a force on B, but B doesn't exert a force on A.

Acceleration Constraints

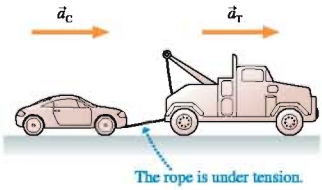
Newton's third law is one quantitative relationship you can use to solve problems of interacting objects. In addition, we frequently have other information about the motion in a problem. For example, think about the two boxes in Example 7.3. As long as they're touching, box A *has* to have exactly the same acceleration as box B. If they were to accelerate differently, either box B would take off on its own or it would suddenly slow down and box A would run over it! Our problem implicitly assumes that neither of these is happening. Thus the two accelerations are *constrained* to be equal: $\vec{a}_A = \vec{a}_B$. A well-defined relationship between the accelerations of two or more objects is called an **acceleration constraint.** It is an independent piece of information that can help solve a problem.

In practice, we'll express acceleration constraints in terms of the x- and y-components of $\vec{a}$. Consider the car being towed in FIGURE 7.15. As long as the rope is under tension, the accelerations are constrained to be equal: $\vec{a}_C = \vec{a}_T$. This is one-dimensional motion, so for problem solving we would use just the x-components a_{Cx} and a_{Tx}. In terms of these components, the acceleration constraint is

$$a_{Cx} = a_{Tx} = a_x$$

Because the accelerations of both objects are equal, we can drop the subscripts C and T and call both of them a_x.

FIGURE 7.15 The car and the truck have the same acceleration.



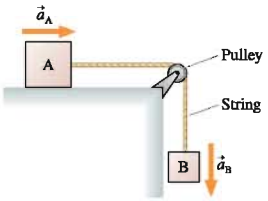


FIGURE 7.16 The string constrains the two objects to accelerate together.

Don't assume the accelerations of A and B will always have the same sign. Consider blocks A and B in FIGURE 7.16. The blocks are connected by a string, so they are constrained to move together and their accelerations have equal magnitudes. But A has a positive acceleration (to the right) in the x-direction while B has a negative acceleration (downward) in the y-direction. Thus the acceleration constraint is

$$a_{Ax} = -a_{By}$$

This relationship does *not* say that a_{Ax} is a negative number. It is simply a relational statement, saying that a_{Ax} is (-1) times whatever a_{By} happens to be. The acceleration a_{By} in Figure 7.16 is a negative number, so a_{Ax} is positive. In some problems, the signs of a_{Ax} and a_{By} may not be known until the problem is solved, but the *relationship* is known from the beginning.

A Revised Strategy for Interacting-Objects Problems

Problems of interacting objects can be solved with a few modifications to the basic problem-solving strategy we developed in Chapter 6. A revised problem-solving strategy is shown below.

NOTE ▶ We have dropped the motion diagram from the pictorial representation. Motion diagrams served a useful function in the early chapters, but by now you should be able to determine the directions of the acceleration vectors without the need for an explicit diagram. But if you are uncertain—use one! ◀

PROBLEM-SOLVING STRATEGY 7.1 Interacting-objects problems

MODEL Identify which objects are part of the system and which are part of the environment. Make simplifying assumptions.

VISUALIZE Draw a pictorial representation.

- Show important points in the motion with a sketch. You may want to give each object a separate coordinate system. Define symbols and identify what the problem is trying to find.
- Identify acceleration constraints.
- Draw an interaction diagram to identify the forces on each object and all action/reaction pairs.
- Draw a *separate* free-body diagram for each object.
- Connect the force vectors of action/reaction pairs with dashed lines. Use subscript labels to distinguish forces that act independently on more than one object.

SOLVE Use Newton's second and third laws.

- Write the equations of Newton's second law for *each* object, using the force information from the free-body diagrams.
- Equate the magnitudes of action/reaction pairs.
- Include the acceleration constraints, the friction model, and other quantitative information relevant to the problem.
- Solve for the acceleration, then use kinematics to find velocities and positions.

ASSESS Check that your result has the correct units, is reasonable, and answers the question.

You might be puzzled that the Solve step calls for the use of the third law to equate just the *magnitudes* of action/reaction forces. What about the "opposite in direction" part of the third law? You have already used it! Your free-body diagrams should show the two members of an action/reaction pair to be opposite in direction, and that information will have been utilized in writing the second-law equations. Because the directional information has already been used, all that is left is the magnitude information.

NOTE ▶ Two steps are especially important when drawing the free-body diagrams. First, draw a *separate* diagram for each object. The diagrams need not have the same coordinate system. Second, show only the forces acting *on* that object. The force $\vec{F}_{\text{A on B}}$ goes on the free-body diagram of object B, but $\vec{F}_{\text{B on A}}$ goes on the diagram of object A. The two members of an action/reaction pair *always* appear on two different free-body diagrams—*never* on the same diagram. ◀

EXAMPLE 7.4 Keep the crate from sliding

You and a friend have just loaded a 200 kg crate filled with priceless art objects into the back of a 2000 kg truck. As you press down on the accelerator, force $\vec{F}_{\text{surface on truck}}$ propels the truck forward. To keep things simple, call this just $\vec{F}_{\text{T}}$. What is the maximum magnitude $\vec{F}_{\text{T}}$ can have without the crate sliding? The static and kinetic coefficients of friction between the crate and the bed of the truck are 0.80 and 0.30. Rolling friction of the truck is negligible.

MODEL The crate and the truck are separate objects that form the system. We'll model them as particles. The earth and the road surface are part of the environment.

VISUALIZE The sketch in FIGURE 7.17 establishes a coordinate system, lists the known information, and—new to problems of interacting objects—identifies the acceleration constraint. As long as the crate doesn't slip, it must accelerate *with* the truck. Both accelerations are in the positive x-direction, so the acceleration constraint in this problem is

$$a_{\text{C}x} = a_{\text{T}x} = a_x$$

The interaction diagram of Figure 7.17 shows the crate interacting twice with the truck—a friction force parallel to the surface of the truck bed and a normal force perpendicular to this surface. The truck interacts similarly with the road surface, but notice that the crate does not interact with the ground; there's no contact between them. The two interactions within the system are each an action/reaction pair, so this is a total of four forces. You can also see four external forces crossing the system boundary, so the free-body diagrams should show a total of eight forces.

Finally, the interaction information is transferred to the free-body diagrams, where we see friction between the crate and truck as an action/reaction pair and the normal forces (the truck pushes up on the crate, the crate pushes down on the truck) as another action/reaction pair. It's easy to overlook forces such as $\vec{f}_{\text{C on T}}$, but you won't make this mistake if you first identify action/reaction pairs on an interaction diagram. Note that $\vec{f}_{\text{C on T}}$ and $\vec{f}_{\text{T on C}}$ are *static* friction forces because they are forces that prevent slipping; force $\vec{f}_{\text{T on C}}$ must point forward to prevent the crate from sliding out the back of the truck.

SOLVE Now we're ready to write Newton's second law. For the crate:

$$\sum (F_{\text{on crate}})_x = f_{\text{T on C}} = m_{\text{C}} a_{\text{C}x} = m_{\text{C}} a_x$$

$$\sum (F_{\text{on crate}})_y = n_{\text{T on C}} - (F_{\text{G}})_{\text{C}} = n_{\text{T on C}} - m_{\text{C}} g = 0$$

For the truck:

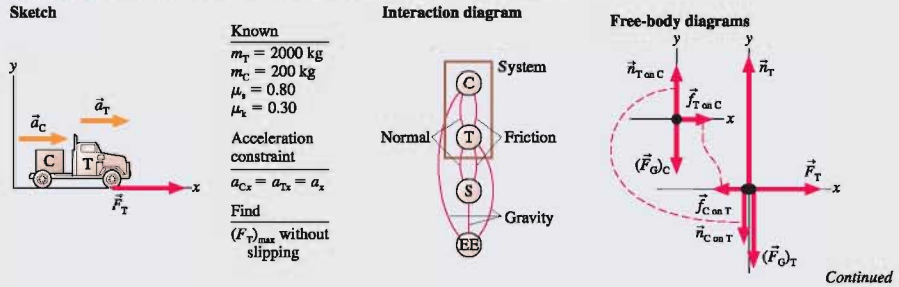
$$\sum (F_{\text{on truck}})_x = F_{\text{T}} - f_{\text{C on T}} = m_{\text{T}} a_{\text{T}x} = m_{\text{T}} a_x$$

$$\sum (F_{\text{on truck}})_y = n_{\text{T}} - (F_{\text{G}})_{\text{T}} - n_{\text{C on T}}$$

$$= n_{\text{T}} - m_{\text{T}} g - n_{\text{C on T}} = 0$$

Be sure you agree with all the signs, which are based on the free-body diagrams. The net force in the y-direction is zero because there's no motion in the y-direction. It may seem like a lot of effort to write all the subscripts, but it is very important in problems with more than one object.

FIGURE 7.17 Pictorial representation of the crate and truck in Example 7.4.



Continued

Notice that we've already used the acceleration constraint $a_{Cx} = a_{Tx} = a_x$. Another important piece of information is Newton's third law, which tells us that $f_{C\,on\,T} = f_{T\,on\,C}$ and $n_{C\,on\,T} = n_{T\,on\,C}$. Finally, we know that the maximum value of F_T will occur when the static friction on the crate reaches its maximum value:

$$f_{T\,on\,C} = f_{s\,max} = \mu_s n_{T\,on\,C}$$

The friction depends on the normal force on the crate, not the normal force on the truck.

Now we can assemble all the pieces. From the *y*-equation of the crate, $n_{T\,on\,C} = m_C g$. Thus

$$f_{T\,on\,C} = \mu_s n_{T\,on\,C} = \mu_s m_C g$$

Using this in the *x*-equation of the crate, we find that the acceleration is

$$a_x = \frac{f_{T\,on\,C}}{m_C} = \mu_s g$$

This is the crate's maximum acceleration without slipping. Now use this acceleration *and* the fact that $f_{C\,on\,T} = f_{T\,on\,C} = \mu_s m_C g$ in the *x*-equation of the truck to find

$$F_T - f_{C\,on\,T} = F_T - \mu_s m_C g = m_T a_x = m_T \mu_s g$$

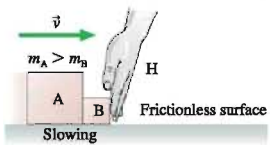
Solving for F_T, we find the maximum propulsion without the crate sliding is

$$(F_T)_{max} = \mu_s(m_T + m_C)g$$
$$= (0.80)(2200\text{ kg})(9.80\text{ m/s}^2) = 17{,}000\text{ N}$$

ASSESS This is a hard result to assess. Few of us have any intuition about the size of forces that propel cars and trucks. Even so, the fact that the forward force on the truck is a significant fraction (80%) of the combined weight of the truck and the crate seems plausible. We might have been suspicious if F_T had been only a tiny fraction of the weight or much greater than the weight.

As you can see, there are many equations and many pieces of information to keep track of when solving a problem of interacting objects. These problems are not inherently harder than the problems you learned to solve in Chapter 6, but they do require a high level of organization. Using the systematic approach of the problem-solving strategy will help you solve similar problems successfully.

STOP TO THINK 7.3 Boxes A and B are sliding to the right across a frictionless table. The hand H is slowing them down. The mass of A is larger than the mass of B. Rank in order, from largest to smallest, the *horizontal* forces on A, B, and H.



a. $F_{B\,on\,H} = F_{H\,on\,B} = F_{A\,on\,B} = F_{B\,on\,A}$
b. $F_{B\,on\,H} = F_{H\,on\,B} > F_{A\,on\,B} = F_{B\,on\,A}$
c. $F_{B\,on\,H} = F_{H\,on\,B} < F_{A\,on\,B} = F_{B\,on\,A}$
d. $F_{H\,on\,B} = F_{H\,on\,A} > F_{A\,on\,B}$

7.4 Ropes and Pulleys

Many objects are connected by strings, ropes, cables, and so on. In single-particle dynamics, we defined *tension* as the force exerted on an object by a rope or string. Now we need to think more carefully about the string itself. Just what do we mean when we talk about the tension "in" a string?

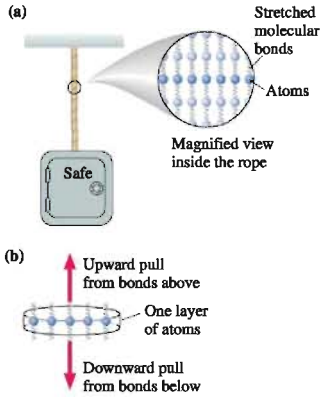
Tension Revisited

FIGURE 7.18a shows a heavy safe hanging from a rope, placing the rope under tension. If you cut the rope, the safe and the lower portion of the rope will fall. Thus there must be a force *within* the rope by which the upper portion of the rope pulls upward on the lower portion to prevent it from falling.

Chapter 5 introduced an atomic-level model in which tension is due to the stretching of spring-like molecular bonds within the rope. Stretched springs exert pulling forces, and the combined pulling force of billions of stretched molecular springs in a string or rope is what we call *tension*.

An important aspect of tension is that it pulls equally *in both directions*. **FIGURE 7.18b** is a very thin cross section through the rope. This small piece of rope is in equilibrium, so it must be pulled equally from both sides. To gain a mental picture, imagine holding your arms outstretched and having two friends pull on them. You'll remain at rest—but "in tension"—as long as they pull with equal strength in opposite directions. But if one lets go, analogous to the breaking of molecular bonds if a rope breaks or is cut, you'll fly off in the other direction!

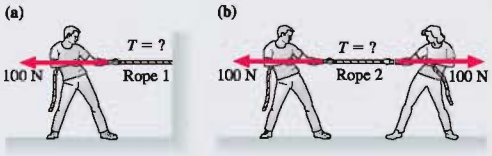
FIGURE 7.18 Tension forces within the rope are due to stretching the spring-like molecular bonds.



EXAMPLE 7.5 Pulling a rope

FIGURE 7.19a shows a student pulling horizontally with a 100 N force on a rope that is attached to a wall. In FIGURE 7.19b, two students in a tug-of-war pull on opposite ends of a rope with 100 N each. Is the tension in the second rope larger than, smaller than, or the same as that in the first rope?

FIGURE 7.19 Pulling on a rope. Which produces a larger tension?



SOLVE Surely pulling on a rope from both ends causes more tension than pulling on one end. Right? Before jumping to conclusions, let's analyze the situation carefully.

Suppose we make an imaginary slice through the rope, as shown in FIGURE 7.20a. The right half of the rope pulls on the left half with $\vec{T}_{\text{R on L}}$ while the left half pulls back on the right half with $\vec{T}_{\text{L on R}}$. These two forces are an action/reaction pair, and their magnitude is what we *mean* by "the tension in the rope." The left half of the rope is in equilibrium, so force $\vec{T}_{\text{R on L}}$ has to balance exactly the 100 N force with which the student is pulling. Thus

$$T_{\text{L on R}} = T_{\text{R on L}} = F_{\text{S on L}} = 100 \text{ N}$$

The first equality is based on Newton's third law (action/reaction pair). The second equality follows from Newton's first law (left half is in equilibrium). This reasoning leads us to the conclusion that the tension in the first rope is 100 N.

Now make an imaginary slice through the rope in FIGURE 7.20b. The left half of the rope is pulled by forces $\vec{T}_{\text{R on L}}$ and $\vec{F}_{\text{S1 on L}}$. This half of the rope is again in equilibrium because the rope is at rest, so from Newton's first law

$$T_{\text{R on L}} = F_{\text{S1 on L}} = 100 \text{ N}$$

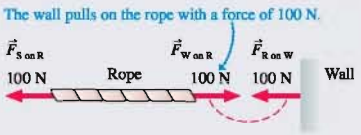
Similarly, the right half of the rope is pulled by forces $\vec{T}_{\text{L on R}}$ and $\vec{F}_{\text{S2 on R}}$. This piece of the rope is also in equilibrium, so

$$T_{\text{L on R}} = F_{\text{S2 on R}} = 100 \text{ N}$$

The tension in the rope has not changed! It is still 100 N.

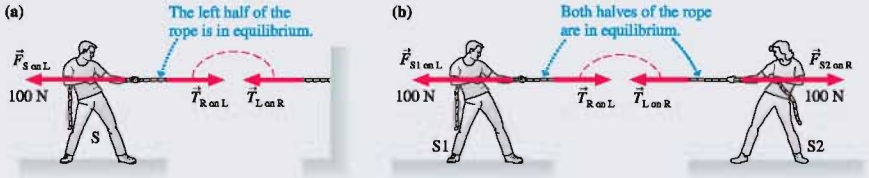
You may have *assumed* that the student on the right in Figure 7.20b is doing something to the rope that the wall in Figure 7.20a does not do. But let's look more closely. FIGURE 7.21 shows a detailed view of the point at which the rope of Figure 7.20a is tied to the wall. Because the rope pulls on the wall with force $\vec{F}_{\text{R on W}}$, the wall must pull back on the rope (action/reaction pair) with force $\vec{F}_{\text{W on R}}$. And because the rope as a whole is in equilibrium, the wall's pull to the right must balance the student's pull to the left: $F_{\text{W on R}} = F_{\text{S on R}} = 100$ N.

FIGURE 7.21 A closer look at the forces on the rope.

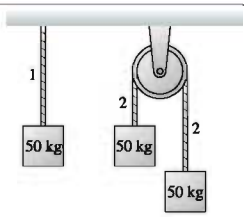


In other words, the wall in Figure 7.20a pulls the right end of the rope with a force of 100 N. The student in Figure 7.20b pulls the right end of the rope with a force of 100 N. The rope does not care whether it is pulled by a wall or by a hand. It experiences the same forces in both cases, so the rope's tension is the same 100 N in both.

FIGURE 7.20 Analysis of tension forces.



STOP TO THINK 7.4 All three 50 kg blocks are at rest. Is the tension in rope 2 greater than, less than, or equal to the tension in rope 1?



The tension in the cable pulls upward on the seat and, simultaneously, pulls downward on the motor and supports at the top of the lift.

The Massless String Approximation

The tension is constant throughout a rope that is in equilibrium, but what happens if the rope is accelerating? For example, FIGURE 7.22a shows two connected blocks being pulled by force $\vec{F}$. Is the string's tension at the right end, where it pulls back on B, the same as the tension at the left end, where it pulls on A?

FIGURE 7.22 The string's tension pulls forward on block A, backward on block B.

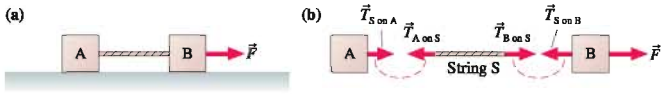


FIGURE 7.22b shows the horizontal forces acting on the blocks and the string. If the string is accelerating, then it must have a net force applied to it. The only forces acting on the string are $\vec{T}_{A\text{ on }S}$ and $\vec{T}_{B\text{ on }S}$, so Newton's second law *for the string* is

$$(F_{net})_x = T_{B\text{ on }S} - T_{A\text{ on }S} = m_s a_x \tag{7.7}$$

where m_s is the mass of the string.

If the string is accelerating, then the tensions at the two ends can *not* be the same. In fact, you can see that

$$T_{B\text{ on }S} = T_{A\text{ on }S} + m_s a_x \tag{7.8}$$

The tension at the "front" of the string is higher than the tension at the "back." This difference in the tensions is necessary to accelerate the string! On the other hand, the tension is constant throughout a string in equilibrium ($a_x = 0$). This was the situation in Example 7.5.

Often in physics and engineering problems the mass of the string or rope is much less than the masses of the objects that it connects. In such cases, we can adopt the **massless string approximation.** In the limit $m_s \to 0$, Equation 7.8 becomes

$$T_{B\text{ on }S} = T_{A\text{ on }S} \qquad \text{(massless string approximation)} \tag{7.9}$$

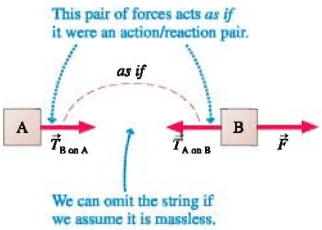
In other words, **the tension in a massless string is constant.** This is nice, but it isn't the primary justification for the massless string approximation.

Look again at Figure 7.22b. If $T_{B\text{ on }S} = T_{A\text{ on }S}$, then

$$\vec{T}_{S\text{ on }A} = -\vec{T}_{S\text{ on }B} \tag{7.10}$$

That is, the force on block A is equal and opposite to the force on block B. Forces $\vec{T}_{S\text{ on }A}$ and $\vec{T}_{S\text{ on }B}$ act *as if* they are an action/reaction pair of forces. Thus we can draw the simplified diagram of FIGURE 7.23 in which the string is missing and blocks A and B interact directly with each other through forces that we can call $\vec{T}_{A\text{ on }B}$ and $\vec{T}_{B\text{ on }A}$.

FIGURE 7.23 The massless string approximation allows objects A and B to act *as if* they are directly interacting.



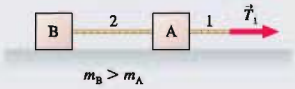
In other words, **if objects A and B interact with each other through a massless string, we can omit the string and treat forces $\vec{F}_{A\text{ on }B}$ and $\vec{F}_{B\text{ on }A}$ *as if* they are an action/reaction pair.** This is not literally true because A and B are not in contact. Nonetheless, all a massless string does is transmit a force from A to B without changing the magnitude of that force. This is the real significance of the massless string approximation.

NOTE ▶ For problems in this book, you can assume that any strings or ropes are massless unless the problem explicitly states otherwise. The simplified view of Figure 7.23 is appropriate under these conditions. But if the string has a mass, it must be treated as a separate object. ◀

EXAMPLE 7.6 Comparing two tensions

Blocks A and B in FIGURE 7.24 are connected by massless string 2 and pulled across a frictionless table by massless string 1. B has a larger mass than A. Is the tension in string 2 larger than, smaller than, or equal to the tension in string 1?

FIGURE 7.24 Blocks A and B are pulled across a frictionless table by massless strings.

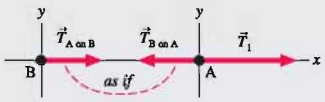


MODEL The massless string approximation allows us to treat A and B *as if* they interact directly with each other. The blocks are accelerating because there's a force to the right and no friction.

SOLVE B has a larger mass, so it may be tempting to conclude that string 2, which pulls B, has a greater tension than string 1, which pulls A. The flaw in this reasoning is that Newton's second law tells us only about the *net* force. The net force on B *is* larger than the net force on A, but the net force on A is *not* just the tension $\vec{T}_1$ in the forward direction. The tension in string 2 also pulls *backward* on A!

FIGURE 7.25 shows the horizontal forces in this frictionless situation. Forces $\vec{T}_{A\text{ on }B}$ and $\vec{T}_{B\text{ on }A}$ act *as if* they are an action/reaction pair.

FIGURE 7.25 The horizontal forces on blocks A and B.



From Newton's third law,

$$T_{A\text{ on }B} = T_{B\text{ on }A} = T_2$$

where T_2 is the tension in string 2. From Newton's second law, the net force on A is

$$(F_{A\text{ net}})_x = T_1 - T_{B\text{ on }A} = T_1 - T_2 = m_A a_{Ax}$$

The net force on A is the *difference* in tensions. The blocks are accelerating to the right, making $a_{Ax} > 0$, so

$$T_1 > T_2$$

The tension in string 2 is *smaller* than the tension in string 1.

ASSESS This is not an intuitively obvious result. A careful study of the reasoning in this example is worthwhile. An alternative analysis would note that $\vec{T}_1$ pulls *both* blocks, of combined mass $(m_A + m_B)$, whereas $\vec{T}_2$ pulls only block B. Thus string 1 must have the larger tension.

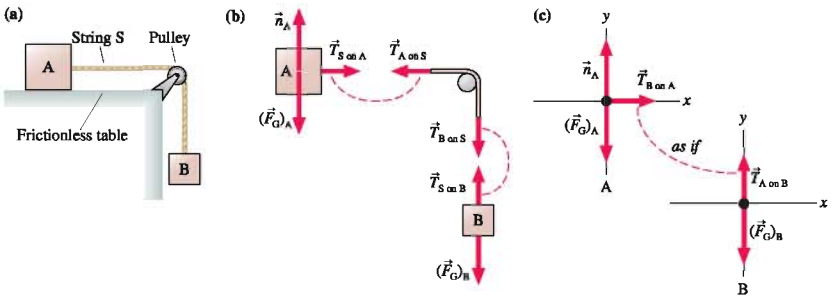
Pulleys

Strings and ropes often pass over pulleys. The application might be as simple as lifting a heavy weight or as complex as the internal cable-and-pulley arrangement that precisely moves a robot arm.

FIGURE 7.26a shows a simple situation in which block B drags block A across a frictionless table as it falls. FIGURE 7.26b shows the objects separately as well as the forces. As the string moves, static friction between the string and pulley causes the pulley to turn. If we assume that

- The string *and* the pulley are both massless, and
- There is no friction where the pulley turns on its axle,

FIGURE 7.26 Blocks A and B are connected by a string that passes over a pulley.



then no net force is needed to accelerate the string or turn the pulley. In this case,

$$T_{\text{A on S}} = T_{\text{B on S}}$$

In other words, **the tension in a massless string remains constant as it passes over a massless, frictionless pulley.**

Because of this, we can draw the simplified free-body diagram of FIGURE 7.26c on the previous page, in which the string and pulley are omitted. Forces $\vec{T}_{\text{A on B}}$ and $\vec{T}_{\text{B on A}}$ act *as if* they are an action/reaction pair, even though they are not opposite in direction. We can again say that A and B are objects that interact with each other *through the string*, and thus the force of A on B is paired with the force of B on A. The tension force gets "turned" by the pulley, which is why the two forces are not opposite each other but we can still equate their magnitudes.

STOP TO THINK 7.5 In Figure 7.26 on the previous page, is the tension in the string greater than, less than, or equal to the gravitational force acting on block B?

7.5 Examples of Interacting-Objects Problems

2.10, 2.11 Activ Online Physics

We will conclude this chapter with four extended examples. Although the mathematics will be more involved than in any of our work up to this point, we will continue to emphasize the *reasoning* one uses in approaching problems such as these. The solutions will be based on Problem-Solving Strategy 7.1. In fact, these problems are now reaching such a level of complexity that, for all practical purposes, it becomes impossible to work them unless you are following a well-planned strategy. Our earlier emphasis on identifying forces and using free-body diagrams will now really begin to pay off!

EXAMPLE 7.7 Mountain climbing

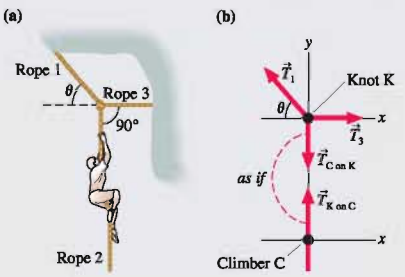
A 90 kg mountain climber is suspended from the ropes shown in FIGURE 7.27a. The maximum tension that rope 3 can withstand before breaking is 1500 N. What is the smallest that angle θ can become before the rope breaks and the climber falls into the gorge?

MODEL Climber C, who can be modeled as a particle, is one object. The other point where forces are exerted is the knot, where the three ropes are tied together. We'll consider knot K to be a second object. Both objects are in static equilibrium. We'll assume massless ropes.

VISUALIZE FIGURE 7.27b shows two free-body diagrams. Forces $\vec{T}_{\text{C on K}}$ and $\vec{T}_{\text{K on C}}$ are not, strictly speaking, an action/reaction pair because the climber is not in contact with the knot. But if the ropes are massless, $\vec{T}_{\text{C on K}}$ and $\vec{T}_{\text{K on C}}$ act *as if* they are an action/reaction pair.

SOLVE This is static equilibrium, so the net forces on the climber and on the knot are zero.

FIGURE 7.27 A mountain climber hanging from ropes.



For the climber:

$$\sum (F_{\text{on C}})_y = T_{\text{K on C}} - mg = 0$$

And for the knot:

$$\sum (F_{\text{on K}})_x = T_3 - T_1 \cos\theta = 0$$

$$\sum (F_{\text{on K}})_y = T_1 \sin\theta - T_{\text{C on K}} = 0$$

From Newton's third law,

$$T_{\text{C on K}} = T_{\text{K on C}}$$

But $T_{\text{K on C}} = mg$ from the climber's equation, so $T_{\text{C on K}} = mg$. Using this gives us the knot's equations:

$$T_1 \cos\theta = T_3$$

$$T_1 \sin\theta = mg$$

Dividing the second of these by the first gives

$$\frac{T_1 \sin\theta}{T_1 \cos\theta} = \tan\theta = \frac{mg}{T_3}$$

If angle θ is too small, tension T_3 will exceed 1500 N. The smallest possible θ, at which T_3 reaches 1500 N, is

$$\theta_{\text{min}} = \tan^{-1}\left(\frac{mg}{T_{3\,\text{max}}}\right) = \tan^{-1}\left(\frac{(90\text{ kg})(9.80\text{ m/s}^2)}{1500\text{ N}}\right) = 30°$$

EXAMPLE 7.8 The show must go on!

A 200 kg set used in a play is stored in the loft above the stage. The rope holding the set passes up and over a pulley, then is tied backstage. The director tells a 100 kg stagehand to lower the set. When he unties the rope, the set falls and the unfortunate man is hoisted into the loft. What is the stagehand's acceleration?

MODEL The system is the stagehand M and the set S, which we will model as particles. Assume a massless rope and a massless, frictionless pulley.

VISUALIZE FIGURE 7.28 shows the pictorial representation. The man's acceleration a_{My} is positive, while the set's acceleration a_{Sy} is negative. These two accelerations have the same magnitude because the two objects are connected by a rope, but they have opposite signs. Thus the acceleration constraint is $a_{\text{Sy}} = -a_{\text{My}}$. Forces $\vec{T}_{\text{M on S}}$ and $\vec{T}_{\text{S on M}}$ are not literally an action/reaction pair, but they act *as if* they are because the rope is massless and the pulley is massless and frictionless. Notice that the pulley has "turned" the tension force so that $\vec{T}_{\text{M on S}}$ and $\vec{T}_{\text{S on M}}$ are *parallel* to each other rather than opposite, as members of a true action/reaction pair would have to be.

SOLVE Newton's second law for the man and the set are

$$\sum (F_{\text{on M}})_y = T_{\text{S on M}} - m_{\text{M}}g = m_{\text{M}}a_{\text{My}}$$

$$\sum (F_{\text{on S}})_y = T_{\text{M on S}} - m_{\text{S}}g = m_{\text{S}}a_{\text{Sy}} = -m_{\text{S}}a_{\text{My}}$$

Only the y-equations are needed. Notice that we used the acceleration constraint in the last step. Newton's third law is

$$T_{\text{M on S}} = T_{\text{S on M}} = T$$

where we can drop the subscripts and call the tension simply T. With this substitution, the two second-law equations can be written

$$T - m_{\text{M}}g = m_{\text{M}}a_{\text{My}}$$

$$T - m_{\text{S}}g = -m_{\text{S}}a_{\text{My}}$$

These are simultaneous equations in the two unknowns T and a_{My}. We can eliminate T by subtracting the second equation from the first to give

$$(m_{\text{S}} - m_{\text{M}})g = (m_{\text{S}} + m_{\text{M}})a_{\text{My}}$$

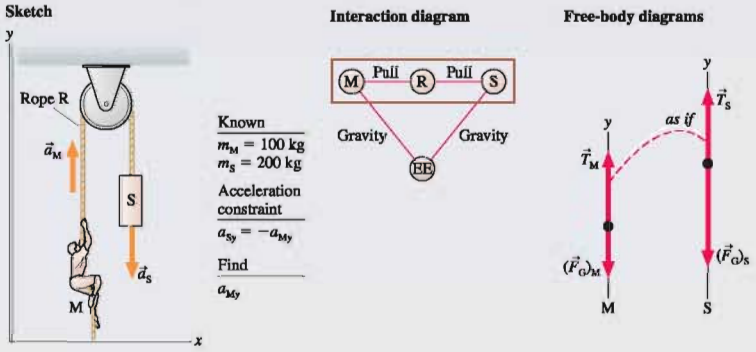
Finally, we can solve for the hapless stagehand's acceleration:

$$a_{\text{My}} = \frac{m_{\text{S}} - m_{\text{M}}}{m_{\text{S}} + m_{\text{M}}}g = \frac{100\text{ kg}}{300\text{ kg}}9.80\text{ m/s}^2 = 3.27\text{ m/s}^2$$

This is also the acceleration with which the set falls. If the rope's tension was needed, we could now find it from $T = m_{\text{M}}a_{\text{My}} + m_{\text{M}}g$.

ASSESS If the stagehand weren't holding on, the set would fall with free-fall acceleration g. The stagehand acts as a *counterweight* to reduce the acceleration.

FIGURE 7.28 Pictorial representation for Example 7.8.



EXAMPLE 7.9 A not-so-clever bank robbery

Bank robbers have pushed a 1000 kg safe to a second-story floor-to-ceiling window. They plan to break the window, then lower the safe 3.0 m to their truck. Not being too clever, they stack up 500 kg of furniture, tie a rope between the safe and the furniture, and place the rope over a pulley. Then they push the safe out the window. What is the safe's speed when it hits the truck? The coefficient of kinetic friction between the furniture and the floor is 0.50.

MODEL This is a continuation of the situation that we analyzed in Figures 7.16 and 7.26, which are worth reviewing. The system is the safe S and the furniture F, which we will model as particles. We will assume a massless rope and a massless, frictionless pulley.

VISUALIZE The safe and the furniture are tied together, so their accelerations have the same magnitude. The safe has a y-component of acceleration a_{Sy} that is negative because the safe accelerates in the negative y-direction. The furniture has an x-component a_F that is positive. Thus the acceleration constraint is

$$a_{Fx} = -a_{Sy}$$

The free-body diagrams of **FIGURE 7.29** are modeled after Figure 7.26 but now include a kinetic friction force on the furniture. Forces $\vec{T}_{F\,on\,S}$ and $\vec{T}_{S\,on\,F}$ act *as if* they are an action/reaction pair, so they have been connected with a dashed line.

SOLVE We can write Newton's second law directly from the free-body diagrams. For the furniture,

$$\sum (F_{on\,F})_x = T_{S\,on\,F} - f_k = T - f_k = m_F a_{Fx} = -m_F a_{Sy}$$

$$\sum (F_{on\,F})_y = n - m_F g = 0$$

And for the safe,

$$\sum (F_{on\,S})_y = T - m_S g = m_S a_{Sy}$$

Notice how we used the acceleration constraint in the first equation. We also went ahead and made use of Newton's third law: $T_{F\,on\,S} = T_{S\,on\,F} = T$. We have one additional piece of information, the model of kinetic friction:

$$f_k = \mu_k n = \mu_k m_F g$$

where we used the y-equation of the furniture to deduce that $n = m_F g$. Substitute this result for f_k into the x-equation of the furniture, then rewrite the furniture's x-equation and the safe's y-equation:

$$T - \mu_k m_F g = -m_F a_{Sy}$$

$$T - m_S g = m_S a_{Sy}$$

We have succeeded in reducing our knowledge to two simultaneous equations in the two unknowns a_{Sy} and T. Subtract the second equation from the first to eliminate T:

$$(m_S - \mu_k m_F)g = -(m_S + m_F)a_{Sy}$$

Finally, solve for the safe's acceleration:

$$a_{Sy} = -\left(\frac{m_S - \mu_k m_F}{m_S + m_F}\right)g$$

$$= -\frac{1000\text{ kg} - 0.5(500\text{ kg})}{1000\text{ kg} + 500\text{ kg}}9.80\text{ m/s}^2 = -4.9\text{ m/s}^2$$

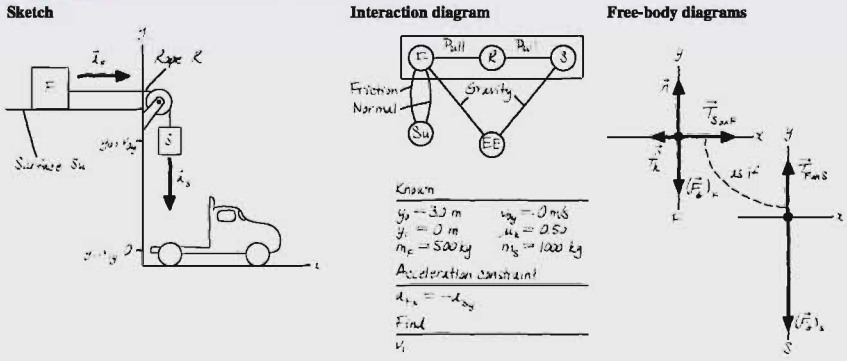
Now we need to calculate the kinematics of the falling safe. Because the time of the fall is not known or needed, we can use

$$v_{1y}^2 = v_{0y}^2 + 2a_{Sy}\Delta y = 0 + 2a_{Sy}(y_1 - y_0) = -2a_{Sy}y_0$$

$$v_1 = \sqrt{-2a_{Sy}y_0} = \sqrt{-2(-4.9\text{ m/s}^2)(3.0\text{ m})} = 5.4\text{ m/s}$$

The value of v_{1y} is negative, but we only needed to find the speed so we took the absolute value. It seems unlikely that the truck will survive the impact of the 1000 kg safe!

FIGURE 7.29 Pictorial representation for Example 7.9.



EXAMPLE 7.10 Pushing a package

A 40 kg boy works at his dad's hardware store. One of the boy's jobs is to unload the delivery truck. He places each package on a 30° ramp and shoves it up the ramp into the storeroom. He needs to shove the package with an acceleration of at least 1.0 m/s² in order for the package to make it to the top of the ramp. One day the ground is wet with rain and he's wearing slick leather-soled shoes. The coefficient of static friction between his shoes and the ground is only 0.25. The largest package of the day is 15 kg, and its coefficient of kinetic friction on the ramp is 0.40. Can he give the package a big enough shove to reach the top of the ramp without his feet slipping?

MODEL The system is the boy B and the package P, which we will model as particles.

VISUALIZE There's a lot of information in this problem, so the pictorial representation of **FIGURE 7.30** is essential. The package is moving up an incline, whereas the boy, if he slips, will move horizontally. Consequently, it is useful to give them different coordinate systems. The free-body diagrams show that the boy pushes the package with force $\vec{F}_{\text{B on P}}$ and the package pushes back with force $\vec{F}_{\text{P on B}}$. If static friction does its job, it must point *forward* to prevent the boy's feet from slipping backward. To answer the question, we'll first calculate how much static friction is needed for the boy to push the package with an acceleration of 1.0 m/s². Then we'll compare that to the maximum possible static friction $f_{s\,\max}$.

SOLVE Now we're ready to write Newton's second law. The boy is in static equilibrium, with $\vec{F}_{\text{net}} = \vec{0}$, so his equations are

$$\sum (F_{\text{on B}})_x = f_s - F_{\text{P on B}} \cos\theta = f_s - F\cos\theta = 0$$
$$\sum (F_{\text{on B}})_y = n_B - m_B g - F\sin\theta = 0$$

The package is accelerating up the ramp as he pushes it, so the package's equations are

$$\sum (F_{\text{on P}})_x = F - f_k - m_P g\sin\theta = m_P a_x$$
$$\sum (F_{\text{on P}})_y = n_P - m_P g\cos\theta = 0$$

We went ahead and made use of Newton's third law: $F_{\text{P on B}} = F_{\text{B on P}} = F$. The package's y-equation tells us that $n_P = m_P g\cos\theta$, so the kinetic friction on the package is

$$f_k = \mu_k n_P = \mu_k m_P g\cos\theta$$

If we substitute this into the package's x-equation, we can solve for force F:

$$F - \mu_k m_P g\cos\theta - m_P g\sin\theta = m_P a_x$$
$$F = m_P(a_x + g\sin\theta + \mu_k g\cos\theta) = 139\ \text{N}$$

This is the size of the force that will accelerate the package up the ramp at 1.0 m/s². If we now use this in the boy's x-equation, we find

$$f_s = F\cos\theta = 120\ \text{N}$$

The boy *needs* this much static friction to push the package without slipping. But needing 120 N of friction doesn't mean that 120 N is available. The maximum possible static friction is $f_{s\,\max} = \mu_s n_B$. In this situation, the normal force acting on the boy is not simply F_G but is affected by the vertical component of $\vec{F}_{\text{P on B}}$. From his y-equation we find

$$n_B = m_B g + F\sin\theta = 462\ \text{N}$$

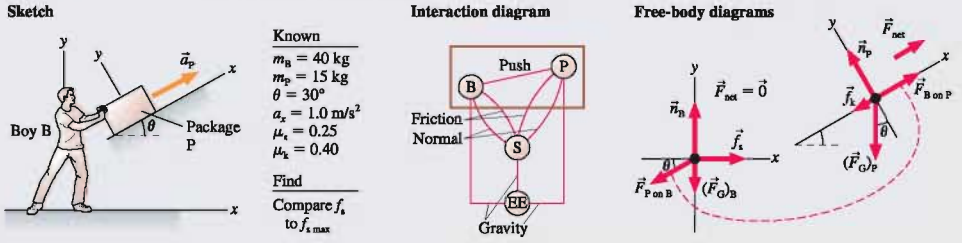
Thus the maximum static friction without slipping is

$$f_{s\,\max} = \mu_s n_B = 115\ \text{N}$$

Consequently, the boy *cannot* shove hard enough without slipping.

ASSESS This is an excellent illustration of how crucial it is to focus on clarifying information, identifying forces, and drawing free-body diagrams. The rest of the problem was not trivial, but we could work our way through it with confidence after having identified the interactions and found the free-body diagrams. It would be hopeless, even for an experienced physicist, to try to go directly to Newton's laws without this analysis.

FIGURE 7.30 Pictorial representation for Example 7.10.



STOP TO THINK 7.6 A small car is pushing a larger truck that has a dead battery. The mass of the truck is larger than the mass of the car. Which of the following statements is true?

a. The car exerts a force on the truck, but the truck doesn't exert a force on the car.
b. The car exerts a larger force on the truck than the truck exerts on the car.
c. The car exerts the same amount of force on the truck as the truck exerts on the car.
d. The truck exerts a larger force on the car than the car exerts on the truck.
e. The truck exerts a force on the car, but the car doesn't exert a force on the truck.

SUMMARY

The goal of Chapter 7 has been to learn to use Newton's third law to understand interacting objects.

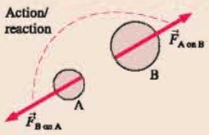
General Principles

Newton's Third Law

Every force occurs as one member of an **action/reaction pair** of forces. The two members of an action/reaction pair:

- Act on two *different* objects.
- Are equal in magnitude but opposite in direction:

$$\vec{F}_{\text{A on B}} = -\vec{F}_{\text{B on A}}$$



Solving Interacting-Objects Problems

MODEL Choose the objects of interest.

VISUALIZE
- Draw a pictorial representation.
  - Sketch and define coordinates.
  - Identify acceleration constraints.
  - Draw an interaction diagram.
  - Draw a separate free-body diagram for each object.
  - Connect action/reaction pairs with dashed lines.

SOLVE Write Newton's second law for each object.
- Include *all* forces acting *on* each object.
- Use Newton's third law to equate the magnitudes of action/reaction pairs.
- Include acceleration constraints and friction.

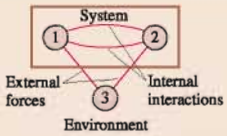
ASSESS Is the result reasonable?

Important Concepts

Objects, systems, and the environment

Objects whose motion is of interest are the system.
Objects whose motion is not of interest form the environment.
The objects of interest interact with the environment, but those interactions can be considered external forces.

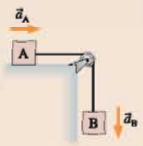
Interaction diagram



Applications

Acceleration constraints

Objects that are constrained to move together must have accelerations of equal magnitude: $a_A = a_B$.
This must be expressed in terms of components, such as $a_{Ax} = -a_{By}$.

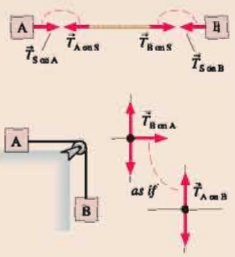


Strings and pulleys

The tension in a string or rope pulls in both directions. The tension is constant in a string if the string is:

- Massless, or
- In equilibrium

Objects connected by massless strings passing over massless, frictionless pulleys act *as if* they interact via an action/reaction pair of forces.



Terms and Notation

interaction	environment	propulsion	acceleration constraint
action/reaction pair	interaction diagram	Newton's third law	massless string approximation
system	external force		

For homework assigned on MasteringPhysics, go to www.masteringphysics.com
Problem difficulty is labeled as | (straightforward) to ||| (challenging).

Problems labeled can be done on a Dynamics Worksheet.

CONCEPTUAL QUESTIONS

1. You find yourself in the middle of a frozen lake with a surface so slippery ($\mu_s = \mu_k = 0$) you cannot walk. However, you happen to have several rocks in your pocket. The ice is extremely hard. It cannot be chipped, and the rocks slip on it just as much as your feet do. Can you think of a way to get to shore? Use pictures, forces, and Newton's laws to explain your reasoning.
2. How do you paddle a canoe in the forward direction? Explain. Your explanation should include diagrams showing forces on the water and forces on the paddle.
3. How does a rocket take off? What is the upward force on it? Your explanation should include diagrams showing forces on the rocket and forces on the parcel of hot gas that was just expelled from the rocket's exhaust.
4. How do basketball players jump straight up into the air? Your explanation should include pictures showing forces on the player and forces on the ground.
5. A mosquito collides head-on with a car traveling 60 mph. Is the force of the mosquito on the car larger than, smaller than, or equal to the force of the car on the mosquito? Explain.
6. A mosquito collides head-on with a car traveling 60 mph. Is the magnitude of the mosquito's acceleration larger than, smaller than, or equal to the magnitude of the car's acceleration? Explain.
7. A small car is pushing a large truck. They are speeding up. Is the force of the truck on the car larger than, smaller than, or equal to the force of the car on the truck?
8. A very smart 3-year-old child is given a wagon for her birthday. She refuses to use it. "After all," she says, "Newton's third law says that no matter how hard I pull, the wagon will exert an equal but opposite force on me. So I will never be able to get it to move forward." What would you say to her in reply?
9. Teams red and blue are having a tug-of-war. According to Newton's third law, the force with which the red team pulls on the blue team exactly equals the force with which the blue team pulls on the red team. How can one team ever win? Explain.
10. Will hanging a magnet in front of the iron cart in FIGURE Q7.10 make it go? Explain.

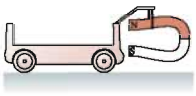


FIGURE Q7.10

11. FIGURE Q7.11 shows two masses at rest. The string is massless and the pulley is frictionless. The spring scale reads in kg. What is the reading of the scale?

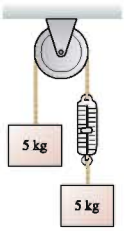


FIGURE Q7.11

12. FIGURE Q7.12 shows two masses at rest. The string is massless and the pulley is frictionless. The spring scale reads in kg. What is the reading of the scale?

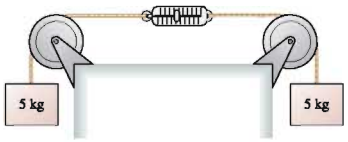


FIGURE Q7.12

13. The hand in FIGURE Q7.13 is pushing on the back of block A. Blocks A and B, with $m_B > m_A$, are connected by a massless string and slide on a frictionless surface. Is the force of the string on B larger than, smaller than, or equal to the force of the hand on A? Explain.

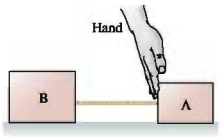


FIGURE Q7.13

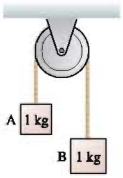


FIGURE Q7.14

14. Blocks A and B in FIGURE Q7.14 are connected by a massless string over a massless, frictionless pulley. The blocks have just been released from rest. Will the pulley rotate clockwise, counterclockwise, or not at all? Explain.
15. In case a in FIGURE Q7.15, block A is accelerated across a frictionless table by a hanging a 10 N weight (1.02 kg). In case b, block A is accelerated across a frictionless table by a steady 10 N tension in the string. The string is massless, and the pulley is massless and frictionless. Is A's acceleration in case b greater than, less than, or equal to its acceleration in case a? Explain.

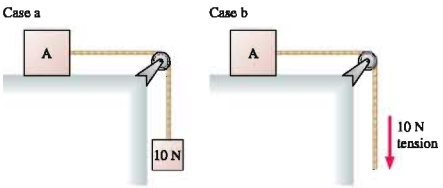


FIGURE Q7.15

EXERCISES AND PROBLEMS

Exercises

Section 7.2 Analyzing Interacting Objects

Exercises 1 through 6 describe a situation. For each:

a. Draw an interaction diagram, following the steps of Tactics Box 7.1.
b. Identify the "system" on your interaction diagram.
c. Draw a free-body diagram for each object in the system. Use dashed lines to connect the members of an action/reaction pair.

1. I A weightlifter stands up from a squatting position while holding a heavy barbell across his shoulders.
2. I A soccer ball and a bowling ball have a head-on collision. Rolling friction is negligible.
3. I A mountain climber is using a rope to pull a bag of supplies up a 45° slope. The rope is not massless.
4. I A battery-powered toy car pushes a stuffed rabbit across the floor.
5. II Block A in FIGURE EX7.5 is heavier than block B and is sliding down the incline. All surfaces have friction. The rope is massless, and the massless pulley turns on frictionless bearings. The rope and the pulley are among the interacting objects, but you'll have to decide if they're part of the system.

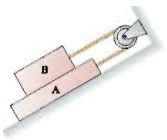


FIGURE EX7.5

FIGURE EX7.6

6. II Block A in FIGURE EX7.6 is sliding down the incline. The rope is massless, and the massless pulley turns on frictionless bearings, but the surface is not frictionless. The rope and the pulley are among the interacting objects, but you'll have to decide if they're part of the system.

Section 7.3 Newton's Third Law

7. I a. How much force does an 80 kg astronaut exert on his chair while sitting at rest on the launch pad?
   b. How much force does the astronaut exert on his chair while accelerating straight up at 10 m/s^2?
8. II FIGURE EX7.8 shows two strong magnets on opposite sides of a small table. The long-range attractive force between the magnets keeps the lower magnet in place.

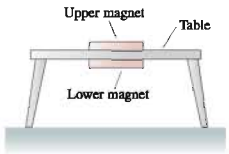


FIGURE EX7.8

   a. Draw an interaction diagram and draw free-body diagrams for both magnets and the table. Use dashed lines to connect the members of an action/reaction pair.
   b. Suppose the weight of the table is 20 N, the weight of each magnet is 2.0 N, and the magnetic force on the lower magnet is three times its weight. Find the magnitude of each of the forces shown on your free-body diagrams.
9. II A 1000 kg car pushes a 2000 kg truck that has a dead battery. When the driver steps on the accelerator, the drive wheels of the car push against the ground with a force of 4500 N. Rolling friction can be neglected.
   a. What is the magnitude of the force of the car on the truck?
   b. What is the magnitude of the force of the truck on the car?
10. II Blocks with masses of 1 kg, 2 kg, and 3 kg are lined up in a row on a frictionless table. All three are pushed forward by a 12 N force applied to the 1 kg block.
   a. How much force does the 2 kg block exert on the 3 kg block?
   b. How much force does the 2 kg block exert on the 1 kg block?
11. II A massive steel cable drags a 20 kg block across a horizontal, frictionless surface. A 100 N force applied to the cable causes the block to reach a speed of 4.0 m/s in a distance of 2.0 m. What is the mass of the cable?

Section 7.4 Ropes and Pulleys

12. I What is the tension in the rope of FIGURE EX7.12?

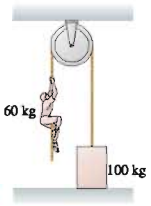


FIGURE EX7.12

13. I Jimmy has caught two fish in Yellow Creek. He has tied the line holding the 3.0 kg steelhead trout to the tail of the 1.5 kg carp. To show the fish to a friend, he lifts upward on the carp with a force of 60 N.
   a. Draw separate free-body diagrams for the trout and the carp. Label all forces, then use dashed lines to connect action/reaction pairs or forces that act as if they are a pair.
   b. Rank in order, from largest to smallest, the magnitudes of all the forces shown on your free-body diagrams. Explain your reasoning.
14. I A 2-m-long, 500 g rope pulls a 10 kg block of ice across a horizontal, frictionless surface. The block accelerates at 2.0 m/s^2. How much force pulls forward on (a) the ice, (b) the rope?

15. I The cable cars in San Francisco are pulled along their tracks by an underground steel cable that moves along at 9.5 mph. The cable is driven by large motors at a central power station and extends, via an intricate pulley arrangement, for several miles beneath the city streets. The length of a cable stretches by up to 100 ft during its lifetime. To keep the tension constant, the cable passes around a 1.5-m-diameter "tensioning pulley" that rolls back and forth on rails, as shown in FIGURE EX7.15. A 2000 kg block is attached to the tensioning pulley's cart, via a rope and pulley, and is suspended in a deep hole. What is the tension in the cable car's cable?

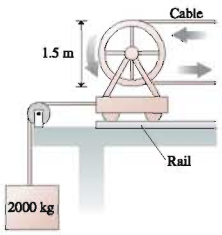


FIGURE EX7.15

16. I A 2.0 kg rope hangs from the ceiling. What is the tension at the midpoint of the rope?

17. II A mobile at the art museum has a 2.0 kg steel cat and a 4.0 kg steel dog suspended from a lightweight cable, as shown in FIGURE EX7.17. It is found that $\theta_1 = 20°$ when the center rope is adjusted to be perfectly horizontal. What are the tension and the angle of rope 3?

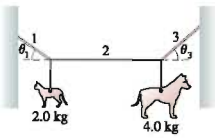


FIGURE EX7.17

Problems

18. II Block B in FIGURE P7.18 rests on a surface for which the static and kinetic coefficients of friction are 0.60 and 0.40, respectively. The ropes are massless. What is the maximum mass of block A for which the system is in equilibrium?

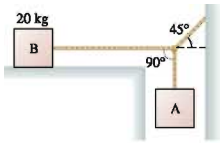


FIGURE P7.18

19. II An 80 kg spacewalking astronaut pushes off a 640 kg satellite, exerting a 100 N force for the 0.50 s it takes him to straighten his arms. How far apart are the astronaut and the satellite after 1.0 min?

20. II A massive steel cable drags a 20 kg block across a horizontal, frictionless surface. A 100 N force applied to the cable causes the block to reach a speed of 4.0 m/s in 2.0 s. What is the difference in tension between the two ends of the cable?

21. II A 1.0-m-long massive steel cable drags a 20 kg block across a horizontal, frictionless surface. A 100 N force applied to the cable causes the block to travel 4.0 m in 2.0 s. Graph the tension in the cable as a function of position along the cable, starting at the point where the cable is attached to the block.

22. II A 3.0-m-long, 2.2 kg rope is suspended from the ceiling. Graph the tension in the rope as a function of position along the rope, starting from the bottom.

23. II The sled dog in FIGURE P7.23 drags sleds A and B across the snow. The coefficient of friction between the sleds and the snow is 0.10. If the tension in rope 1 is 150 N, what is the tension in rope 2?

FIGURE P7.23

24. II While driving to work last year, I was holding my coffee mug in my left hand while changing the CD with my right hand. Then the cell phone rang, so I placed the mug on the flat part of my dashboard. Then, believe it or not, a deer ran out of the woods and on to the road right in front of me. Fortunately, my reaction time was zero, and I was able to stop from a speed of 20 m/s in a mere 50 m, just barely avoiding the deer. Later tests revealed that the static and kinetic coefficients of friction of the coffee mug on the dash are 0.50 and 0.30, respectively; the coffee and mug had a mass of 0.50 kg; and the mass of the deer was 120 kg. Did my coffee mug slide?

25. II a. Why can a car accelerate but a house cannot? Your explanation should be in terms of forces and their properties.
b. Two-thirds of the weight of a 1500 kg car rests on the drive wheels. What is the maximum acceleration of this car on a concrete surface?

26. II A Federation starship (2.0×10^6 kg) uses its tractor beam to pull a shuttlecraft (2.0×10^4 kg) aboard from a distance of 10 km away. The tractor beam exerts a constant force of 4.0×10^4 N on the shuttlecraft. Both spacecraft are initially at rest. How far does the starship move as it pulls the shuttlecraft aboard?

27. II Bob, who has a mass of 75 kg, can throw a 500 g rock with a speed of 30 m/s. The distance through which his hand moves as he accelerates the rock from rest until he releases it is 1.0 m.
a. What constant force must Bob exert on the rock to throw it with this speed?
b. If Bob is standing on frictionless ice, what is his recoil speed after releasing the rock?

28. II You see the boy next door trying to push a crate down the sidewalk. He can barely keep it moving, and his feet occasionally slip. You start to wonder how heavy the crate is. You call to ask the boy his mass, and he replies "50 kg." From your recent physics class you estimate that the static and kinetic coefficients of friction are 0.8 and 0.4 for the boy's shoes, and 0.5 and 0.2 for the crate. Estimate the mass of the crate.

29. II Two packages at UPS start sliding down the 20° ramp shown in FIGURE P7.29. Package A has a mass of 5.0 kg and a coefficient of friction of 0.20. Package B has a mass of 10 kg and a coefficient of friction of 0.15. How long does it take package A to reach the bottom?

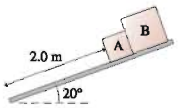


FIGURE P7.29

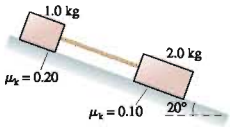


FIGURE P7.30

30. II The two blocks in FIGURE P7.30 are sliding down the incline. What is the tension in the massless string?

31. II FIGURE P7.31 shows two 1.0 kg blocks connected by a rope. A second rope hangs beneath the lower block. Both ropes have a mass of 250 g. The entire assembly is accelerated upward at 3.0 m/s^2 by force $\vec{F}$.
a. What is F?
b. What is the tension at the top end of rope 1?
c. What is the tension at the bottom end of rope 1?
d. What is the tension at the top end of rope 2?

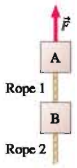


FIGURE P7.31

32. || The 1.0 kg block in FIGURE P7.32 is tied to the wall with a rope. It sits on top of the 2.0 kg block. The lower block is pulled to the right with a tension force of 20 N. The coefficient of kinetic friction at both the lower and upper surfaces of the 2.0 kg block is $\mu_k = 0.40$.
a. What is the tension in the rope holding the 1.0 kg block to the wall?
b. What is the acceleration of the 2.0 kg block?

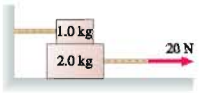


FIGURE P7.32

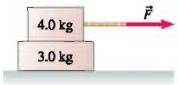


FIGURE P7.33

33. || The coefficient of static friction is 0.60 between the two blocks in FIGURE P7.33. The coefficient of kinetic friction between the lower block and the floor is 0.20. Force $\vec{F}$ causes both blocks to cross a distance of 5.0 m, starting from rest. What is the least amount of time in which this motion can be completed without the top block sliding on the lower block?

34. ||| The lower block in FIGURE P7.34 is pulled on by a rope with a tension force of 20 N. The coefficient of kinetic friction between the lower block and the surface is 0.30. The coefficient of kinetic friction between the lower block and the upper block is also 0.30. What is the acceleration of the 2.0 kg block?

FIGURE P7.34

35. ||| A rope attached to a 20 kg wood sled pulls the sled up a 20° snow-covered hill. A 10 kg wood box rides on top of the sled. If the tension in the rope steadily increases, at what value of the tension does the box slip?

36. || The 100 kg block in FIGURE P7.36 takes 6.0 s to reach the floor after being released from rest. What is the mass of the block on the left?

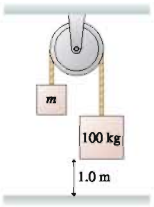


FIGURE P7.36

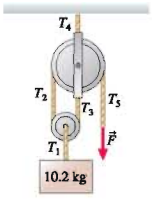


FIGURE P7.37

37. || The 10.2 kg block in FIGURE P7.37 is held in place by the massless rope passing over two massless, frictionless pulleys. Find the tensions T_1 to T_5 and the magnitude of force $\vec{F}$.

38. || The coefficient of kinetic friction between the 2.0 kg block in FIGURE P7.38 and the table is 0.30. What is the acceleration of the 2.0 kg block?

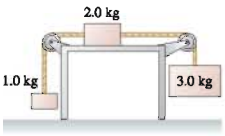


FIGURE P7.38

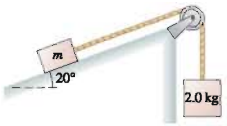


FIGURE P7.39

39. || FIGURE P7.39 shows a block of mass m resting on a 20° slope. The block has coefficients of friction $\mu_s = 0.80$ and $\mu_k = 0.50$ with the surface. It is connected via a massless string over a massless, frictionless pulley to a hanging block of mass 2.0 kg.
a. What is the minimum mass m that will stick and not slip?
b. If this minimum mass is nudged ever so slightly, it will start being pulled up the incline. What acceleration will it have?

40. || A 4.0 kg box is on a frictionless 35° slope and is connected via a massless string over a massless, frictionless pulley to a hanging 2.0 kg weight. The picture for this situation is similar to Figure P7.39.
a. What is the tension in the string if the 4.0 kg box is *held* in place, so that it cannot move?
b. If the box is then released, which way will it move on the slope?
c. What is the tension in the string once the box begins to move?

41. || The 1.0 kg physics book in FIGURE P7.41 is connected by a string to a 500 g coffee cup. The book is given a push up the slope and released with a speed of 3.0 m/s. The coefficients of friction are $\mu_s = 0.50$ and $\mu_k = 0.20$.
a. How far does the book slide?
b. At the highest point, does the book stick to the slope, or does it slide back down?

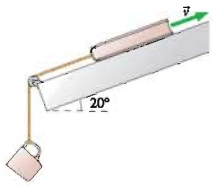


FIGURE P7.41

42. || The 2000 kg cable car shown in FIGURE P7.42 descends a 200-m-high hill. In addition to its brakes, the cable car controls its speed by pulling an 1800 kg counterweight up the other side of the hill. The rolling friction of both the cable car and the counterweight are negligible.
a. How much braking force does the cable car need to descend at constant speed?
b. One day the brakes fail just as the cable car leaves the top on its downward journey. What is the runaway car's speed at the bottom of the hill?

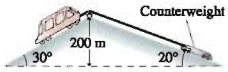


FIGURE P7.42

43. || The century-old *ascensores* in Valparaiso, Chile, are small cable cars that go up and down the steep hillsides. As FIGURE P7.43 shows, one car ascends as the other descends. The cars use a two-cable arrangement to compensate for friction; one cable passing around a large pulley connects the cars, the second is pulled by a small motor. Suppose the mass of both cars (with passengers) is 1500 kg, the coefficient of rolling friction is 0.020, and the cars move at constant speed. What is the tension in the (a) the connecting cable and (b) the cable to the motor?

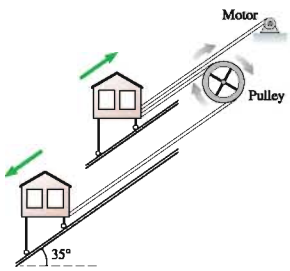


FIGURE P7.43

44. || A house painter uses the chair and pulley arrangement of FIGURE P7.44 to lift himself up the side of a house. The painter's mass is 70 kg and the chair's mass is 10 kg. With what force must he pull down on the rope in order to accelerate upward at 0.20 m/s^2?

FIGURE P7.44

45. ||| A 70 kg tightrope walker stands at the center of a rope. The rope supports are 10 m apart and the rope sags 10° at each end. The tightrope walker crouches down, then leaps straight up with an acceleration of 8.0 m/s^2 to catch a passing trapeze. What is the tension in the rope as he jumps?

46. || Find an expression for the magnitude of the horizontal force F in FIGURE P7.46 for which m_1 does not slip either up or down along the wedge. All surfaces are frictionless.

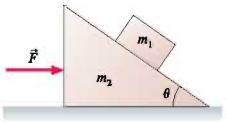


FIGURE P7.46

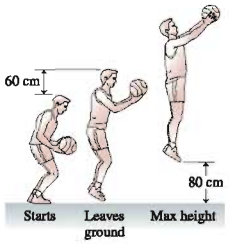


FIGURE P7.47

47. || A 100 kg basketball player can leap straight up in the air to a height of 80 cm, as shown in FIGURE P7.47. You can understand how by analyzing the situation as follows:
   a. The player bends his legs until the upper part of his body has dropped by 60 cm, then he begins his jump. Draw separate free-body diagrams for the player and for the floor *as* he is jumping, but before his feet leave the ground.
   b. Is there a net force on the player as he jumps (before his feet leave the ground)? How can that be? Explain.
   c. With what speed must the player leave the ground to reach a height of 80 cm?
   d. What was his acceleration, assumed to be constant, as he jumped?
   e. Suppose the player jumps while standing on a bathroom scale that reads in newtons. What does the scale read before he jumps, as he is jumping, and after his feet leave the ground?

Problems 48 and 49 show the free-body diagrams of two interacting systems. For each of these, you are to
   a. Write a realistic problem for which these are the correct free-body diagrams. Be sure that the answer your problem requests is consistent with the diagrams shown.
   b. Finish the solution of the problem.

48.

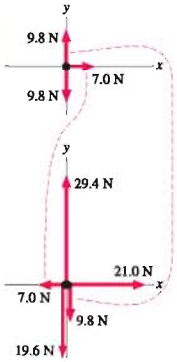


FIGURE P7.48

49.

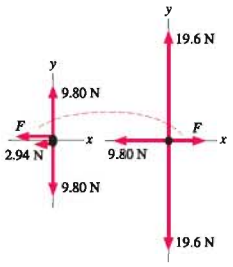


FIGURE P7.49

Challenge Problems

50. A 100 g ball of clay is thrown horizontally with a speed of 10 m/s toward a 900 g block resting on a frictionless surface. It hits the block and sticks. The clay exerts a constant force on the block during the 10 ms it takes the clay to come to rest relative to the block. After 10 ms, the block and the clay are sliding along the surface as a single system.
   a. What is their speed after the collision?
   b. What is the force of the clay on the block during the collision?
   c. What is the force of the block on the clay?

   NOTE ▶ This problem can be worked using the conservation laws you will be learning in the next few chapters. However, here you're asked to solve the problem using Newton's laws. ◀

51. In FIGURE CP7.51, find an expression for the acceleration of m_1. Assume the table is frictionless.
   Hint: Think carefully about the acceleration constraint.

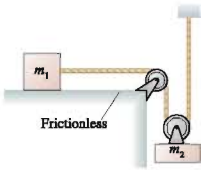


FIGURE CP7.51

52. What is the acceleration of the 2.0 kg block in FIGURE CP7.52 across the frictionless table?
Hint: Think carefully about the acceleration constraint.

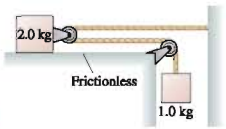


FIGURE CP7.52

53. FIGURE CP7.53 shows a 200 g hamster sitting on an 800 g wedge-shaped block. The block, in turn, rests on a spring scale.
a. Initially, static friction is sufficient to keep the hamster from moving. In this case, the hamster and the block are effectively a single 1000 g mass and the scale should read 9.8 N. Show that this is the case by treating the hamster and the block as *separate* objects and analyzing the forces.

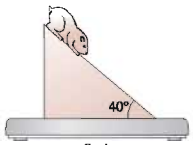


FIGURE CP7.53

b. An extra-fine lubricating oil having $\mu_s = \mu_k = 0$ is sprayed on the top surface of the block, causing the hamster to slide down. Friction between the block and the scale is large enough that the block does *not* slip on the scale. What does the scale read as the hamster slides down?

54. FIGURE CP7.54 shows three hanging masses connected by massless strings over two massless, frictionless pulleys.
a. Find the acceleration constraint for this system. It is a single equation relating a_{1y}, a_{2y}, and a_{3y}.
Hint: y_A isn't constant.
b. Find an expression for the tension in string A.
Hint: You should be able to write four second-law equations. These, plus the acceleration constraint, are five equations in five unknowns.

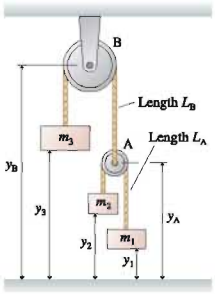


FIGURE CP7.54

c. Suppose: $m_1 = 2.5$ kg, $m_2 = 1.5$ kg, and $m_3 = 4.0$ kg. Find the acceleration of each.
d. The 4.0 kg mass would appear to be in equilibrium. Explain why it accelerates.

STOP TO THINK ANSWERS

Stop to Think 7.1: The gravitational force and the tension force are incorrectly identified as an action/reaction pair. Force $\vec{T}_{\text{L on F}}$ should be paired with force $\vec{T}_{\text{F on L}}$. Gravity is the pull of the entire earth, so $(\vec{F}_G)_F$ should be paired with a force pulling up on the entire earth.

Stop to Think 7.2: c. Newton's third law says that the force of A on B is *equal* and opposite to the force of B on A. This is always true. The speed of the objects isn't relevant.

Stop to Think 7.3: b. $F_{\text{B on H}} = F_{\text{H on B}}$ and $F_{\text{A on B}} = F_{\text{B on A}}$ because these are action/reaction pairs. Box B is slowing down and therefore must have a net force to the left. So from Newton's second law we also know that $F_{\text{H on B}} > F_{\text{A on B}}$.

Stop to Think 7.4: Equal to. Each block is hanging in equilibrium, with no net force, so the upward tension force is mg.

Stop to Think 7.5: Less than. Block B is *accelerating* downward, so the net force on B must point down. The only forces acting on B are the tension and gravity, so $T_{\text{S on B}} < (F_G)_B$.

Stop to Think 7.6: c. Newton's third law says that the force of A on B is *equal* and opposite to the force of B on A. This is always true. The mass of the objects isn't relevant.